## Eureka Math ${ }^{2}$ Level 6 Correlation to <br> Connecticut Model Curriculum

## Level 6: Ratios and Rates

| Model Unit Name | Model Unit Standards | Lessons | Pacing <br> Lessons that address concepts in more than one unit are only counted once |
| :---: | :---: | :---: | :---: |
| Operating with Positive Rational Numbers | 6.NS.A. 1 | Module 2: Operations with Fractions and Multi-Digit Numbers <br> Topic B: Dividing Fractions <br> Lesson 6: Dividing a Whole Number by a Fraction <br> Lesson 7: Dividing a Fraction by a Whole Number <br> Lesson 8: Dividing Fractions by Making Common Denominators <br> Topic C: Dividing Fractions Fluently <br> Lesson 9: Dividing Fractions by Using Tape Diagrams <br> Lesson 10: Dividing Fractions by Using the Invert and Multiply Strategy <br> Lesson 11: Applications of Fraction Division <br> Lesson 12: Fraction Operations in a Real-World Situation | 28 days |
|  | 6.NS.B. 2 | Module 2: Operations with Fractions and Multi-Digit Numbers <br> Topic E: Division of Multi-Digit Numbers <br> Lesson 17: Partial Quotients <br> Lesson 18: The Standard Division Algorithm <br> Lesson 19: Expressing Quotients as Decimals |  |
|  | 6.NS.B. 3 | Module 2: Operations with Fractions and Multi-Digit Numbers <br> Topic D: Decimal Addition, Subtraction, and Multiplication <br> Lesson 13: Decimal Addition and Subtraction <br> Lesson 14: Patterns in Multiplying Decimals <br> Lesson 15: Decimal Multiplication |  |


| Operating with Positive Rational Numbers (cont.) | 6.NS.B. 4 | Topic F: Decimal Division <br> Lesson 21: Dividing a Decimal by a Whole Number <br> Lesson 22: Dividing a Decimal by a Decimal Greater Than 1 <br> Lesson 23: Dividing a Decimal by a Decimal Less Than 1 <br> Lesson 24: Living on Mars <br> Module 2: Operations with Fractions and Multi-Digit Numbers <br> Topic A: Factors, Multiples, and Divisibility <br> Lesson 1: Factors and Multiples <br> Lesson 2: Divisibility <br> Lesson 3: The Greatest Common Factor <br> Lesson 4: The Least Common Multiple <br> Lesson 5: The Euclidean Algorithm (Optional) <br> Module 5: Area, Surface Area, and Volume <br> Topic D: Volumes of Right Rectangular Prisms Lesson 15: Exploring Volume Lesson 16: Applying Volume Formulas Lesson 17: Problem Solving with Volume Lesson 18: Volumes of Composite Solids Lesson 19: Volume and Surface Area in Real-World Situations |  |
| :---: | :---: | :---: | :---: |
| Understanding Positive and Negative Numbers | 6.NS.C. 5 | Module 3: Rational Numbers <br> Topic A: Integers and Rational Numbers <br> Lesson 1: Positive and Negative Numbers <br> Lesson 4: Rational Numbers in Real-World Situations <br> Module 3: Rational Numbers <br> Topic A: Integers and Rational Numbers <br> Lesson 2: Integers <br> Lesson 3: Rational Numbers <br> Lesson 4: Rational Numbers in Real-World Situations <br> Topic C: The Coordinate Plane <br> Lesson 10: The Four Quadrants of the Coordinate Plane Lesson 11: Plotting Points in the Coordinate Plane | 17 days |




| Using Expressions and Equations (cont.) | 6.EE.B.6 | Module 4: Expressions and One-Step Equations <br> Topic B: Expressions and Real-World Problems <br> Lesson 9: Addition and Subtraction Expressions from Real-World Situations <br> Lesson 10: Multiplication and Division Expressions from Real-World Situations <br> Lesson 11: Modeling Real-World Situations with Expressions <br> Topic C: Equivalent Expressions Using the Properties of Operations <br> Lesson 16: Equivalent Algebraic Expressions |  |
| :---: | :---: | :---: | :---: |
|  | 6.EE.B. 7 | Module 4: Expressions and One-Step Equations <br> Topic D: Equational and Inequalities <br> Lesson 17: Equations and Solutions <br> Lesson 19: Solving Equations with Addition and Subtraction <br> Lesson 20: Solving Equations with Multiplication and Division <br> Lesson 21: Solving Problems with Equations |  |
|  | 6.EE.B. 8 | Module 4: Expressions and One-Step Equations <br> Topic D: Equations and Inequalities <br> Lesson 18: Inequalities and Solutions |  |
| Applications of Geometry | 6.G.A. 1 | Module 5: Area, Surface Area, and Volume <br> Topic A: Areas of Polygons <br> Lesson 1: The Area of a Parallelogram <br> Lesson 2: The Area of a Right Triangle <br> Lesson 3: The Area of a Triangle <br> Lesson 4: Areas of Triangles in Real-World Situations <br> Topic B: Problem Solving with Area <br> Lesson 5: Perimeter and Area in the Coordinate Plane <br> Lesson 6: Problem Solving with Area in the Coordinate Plane <br> Lesson 7: Area of Trapezoids and Other Polygons <br> Lesson 8: Areas of Composite Figures in Real-World Situations | 13 days |


| Applications of Geometry (cont.) | 6.G.A. 3 | Module 5: Area, Surface Area, and Volume <br> Topic B: Problem Solving with Area <br> Lesson 5: Perimeter and Area in the Coordinate Plane <br> Lesson 6: Problem Solving with Area in the Coordinate Plane <br> Module 5: Area, Surface Area, and Volume <br> Topic C: Nets and Surface Area <br> Lesson 9: Properties of Solids <br> Lesson 10: Discovering Nets of Solids <br> Lesson 11: Constructing Nets of Solids <br> Lesson 12: From Nets to Surface Area <br> Topic D: Volumes of Right Rectangular Prisms <br> Lesson 19: Volume and Surface Area in Real-World Situations |  |
| :---: | :---: | :---: | :---: |
| Ratios and Rates | 6.RP.A. 1 | Module 1: Ratios, Rates, and Percents <br> Topic A: Ratios <br> Lesson 2: Introduction to Ratios <br> Lesson 3: Introduction to Ratios <br> Lesson 4: Exploring Ratios by Making Batches <br> Lesson 5: Equivalent Ratios <br> Topic B: Collections of Equivalent Ratios <br> Lesson 8: Addition Patterns in Ratio Relationships <br> Lesson 10: Multiplicative Reasoning in Ratio Relationships <br> Lesson 11: Applications of Ratio Reasoning <br> Module 5: Area, Surface Area, and Volume <br> Topic C: Comparing Ratio Relationships <br> Lesson 15: The Value of the Ratio <br> Topic D: Rates <br> Lesson 16: Speed <br> Lesson 17: Rates | 26 days |



| Ratios and Rates (cont.) |  | Lesson 24: Finding a Part <br> Lesson 25: Finding the Whole <br> Lesson 26: Solving Percent Problems |  |
| :---: | :---: | :---: | :---: |
| Algebraic Reasoning | 6.EE.B. 6 | Module 4: Expressions and One-Step Equations <br> Topic B: Expressions and Real-World Problems <br> Lesson 9: Addition and Subtraction Expressions from Real-World Situations Lesson 10: Multiplication and Division Expressions from Real-World Situations Lesson 11: Modeling Real-World Situations with Expressions <br> Topic C: Equivalent Expressions Using the Properties of Operations Lesson 16: Equivalent Algebraic Expressions | 4 days |
|  | 6.EE.B. 7 | Module 4: Expressions and One-Step Equations <br> Topic D: Equations and Inequalities <br> Lesson 17: Equations and Solutions <br> Lesson 19: Solving Equations with Addition and Subtraction <br> Lesson 20: Solving Equations with Multiplication and Division <br> Lesson 21: Solving Problems with Equations |  |
|  | 6.EE.C. 9 | Module 4: Expressions and One-Step Equations <br> Topic E: Relating Variables by Using Tables, Graphs, and Equations Lesson 22: Relationship Between Two Variables <br> Lesson 23: Graphs of Ratio Relationships <br> Lesson 24: Graphs of Non-Ratio Relationships Lesson 25: The Statue of Liberty |  |
| Statistics and Distributions | 6.SP.A. 1 | Module 6: Statistics <br> Topic A: Understanding Distributions <br> Lesson 1: Posing Statistical Questions <br> Lesson 6: Selecting a Data Display <br> Topic D: Answering Statistical Questions by Analyzing Data Lesson 17: Developing a Statistical Project | 22 days |


| Statistics and Distributions (cont.) | 6.SP.A. 2 | Module 6: Statistics <br> Topic A: Understanding Distributions <br> Lesson 2: Describing a Data Distribution <br> Lesson 3: Creating a Dot Plot <br> Lesson 4: Creating a Histogram <br> Topic B: Mean and Mean Absolute Deviation <br> Lesson 9: Variability in a Data Distribution <br> Topic C: Median, Interquartile Range, and Box Plots <br> Lesson 14: Using a Box Plot to Summarize a Distribution <br> Topic D: Answering Statistical Questions by Analyzing Data <br> Lesson 18: Connecting Graphical Representations and Summary Measures |
| :---: | :---: | :---: |
|  | 6.SP.A. 3 | Module 6: Statistics <br> Topic B: Mean and Mean Absolute Deviation <br> Lesson 7: Using the Mean to Describe the Center <br> Lesson 8: The Mean as a Balance Point <br> Lesson 9: Variability in a Data Distribution <br> Lesson 10: The Mean Absolute Deviation <br> Lesson 11: Using the Mean and Mean Absolute Deviation <br> Topic C: Median, Interquartile Range, and Box Plots <br> Lesson 12: Using the Median to Describe the Center <br> Lesson 13: Using the Interquartile Range to Describe Variability <br> Lesson 15: More Practice with Box Plots <br> Lesson 16: Interpreting Box Plots <br> Topic D: Answering Statistical Questions by Analyzing Data <br> Lesson 19: Comparing Data Distributions <br> Lesson 22: Presenting Statistical Projects |
|  | 6.SP.B. 4 | Module 6: Statistics <br> Topic A: Understanding Distributions <br> Lesson 3: Creating a Dot Plot |



## Eureka Math ${ }^{2}$ Scope and Sequence: Year at a Glance <br> Level 6: Ratios and Rates

If a district uses this resource to implement the state model curriculum for grade 6, the following scope and sequence should be followed to ensure alignment and attention to the progressions of mathematics.

| Module 1 <br> Ratios, Rates, and Percents | Module 2 <br> Operations with Fractions and Multi-Digit Numbers | Module 3 <br> Rational Numbers | Module 4 <br> Expressions and One-Step Equations | Module 5 <br> Area, Surface Area, and Volume | Module 6 <br> Statistics |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Topic A: Ratios <br> Lesson 1: Jars of Jelly Beans <br> - Use multiplicative reasoning to estimate the solution to a real-world problem. <br> 6.RP.A.3, MP3, 6.Mod1.AD3 <br> Lesson 2: Introduction to Ratios <br> - Write ratios that relate two quantities as an ordered pair of numbers. <br> - Use ratio language to compare two quantities. <br> 6.RP.A.1, MP2, 6.Mod1.AD1 <br> Lesson 3: Ratios and Tape <br> Diagrams <br> - Write multiple ratios to describe the same situation. <br> - Represent ratios with tape diagrams. <br> 6.RP.A.1, 6.RP.A.3, MP6, <br> 6.Mod1.AD1, 6.Mod1.AD3 | Topic A: Factors, Multiples, and Divisibility <br> Lesson 1: Factors and Multiples <br> - Use visual models to determine common factors and common multiples of pairs of numbers. <br> 6.NS.B.4, MP8, 6.Mod2.AD12, <br> 6.Mod2.AD13 <br> Lesson 2: Divisibility <br> - Determine whether numbers are divisible by other numbers. <br> 6.NS.B.4, MP3, 6.Mod2.AD12, <br> 6.Mod2.AD13 <br> Lesson 3: The Greatest Common <br> Factor <br> - Determine the greatest common factor of two whole numbers less than or equal to 100 . <br> 6.NS.B.4, MP7, 6.Mod2.AD12 | Topic A: Integers and Rational Numbers <br> Lesson 1: Positive and Negative <br> Numbers <br> - Represent quantities in real-world situations by using positive and negative numbers. <br> - Plot positive numbers, negative numbers, and 0 on horizontal and vertical number lines. <br> 6.NS.C.5, MP2, 6.Mod3.AD1 <br> Lesson 2: Integers <br> - Plot integers and their opposites on horizontal and vertical number lines and identify 0 as its own opposite. <br> - Identify the opposite of the opposite of a number. <br> 6.NS.C.6.a, MP7, 6.Mod3.AD2, 6.Mod3.AD3 | Topic A: Numerical <br> Expressions <br> Lesson 1: Expressions with <br> Addition and Subtraction <br> - Evaluate expressions with addition and subtraction. <br> 6.EE.A.1, MP6, 6.Mod4.AD3 <br> Lesson 2: Expressions with Multiplication and Division <br> - Evaluate expressions with multiplication and division. <br> 6.EE.A.1, MP7, 6.Mod4.AD3 <br> Lesson 3: Exploring Exponents <br> - Write numerical expressions by using exponential notation. <br> 6.EE.A.1, MP3, 6.Mod4.AD3 | Topic A: Areas of Polygons <br> Lesson 1: The Area of a Parallelogram <br> - Compose parallelograms into rectangles to derive the formula for the area of a parallelogram. <br> - Compute the area of a parallelogram by using the formula $A=b h$. <br> 6.EE.A.2.c, 6.G.A.1, MP8, <br> 6.Mod4.AD6, 6.Mod5.AD1 <br> Lesson 2: The Area of a Right <br> Triangle <br> - Compose two identical right triangles into a rectangle to derive the formula for the area of a right triangle. <br> - Compute the area of a right triangle by using the formula $A=\frac{1}{2} b h$. <br> 6.EE.B.7, 6.G.A.1, MP3, 6.Mod4.AD13, 6.Mod5.AD1, <br> 6.Mod5.AD2 | Topic A: Understanding Distributions <br> Lesson 1: Posing Statistical <br> Questions <br> - Identify and write statistical questions. <br> - Identify the types of data that can be collected to answer a statistical question. <br> 6.SP.A.1, 6.SP.B.5.b, MP6, <br> 6.Mod6.AD1, 6.Mod6.AD6 <br> Lesson 2: Describing a Data <br> Distribution <br> - Given a dot plot, describe the center, spread, and other characteristics of the data distribution. <br> 6.SP.A.2, 6.SP.B.5.a, MP2, <br> 6.Mod6.AD2, 6.Mod6.AD5 <br> Lesson 3: Creating a Dot Plot <br> - Create a dot plot and describe a data distribution. <br> 6.SP.A.2, 6.SP.B.4, MP1, <br> 6.Mod6.AD2, 6.Mod6.AD4 |

## Lesson 4: Exploring Ratios by

Making Batches

- Create ratios by making batches of different quantities.
- Use tape diagrams to determine
unknown quantities in ratios


## 6.RP.A.1, 6.RP.A.3, MP8,

## 6.Mod1.AD1, 6.Mod1.AD3

## Lesson 5: Equivalent Ratios

- Find equivalent ratios by multiplying both numbers in a given ratio by the same nonzero number.
- Use equivalent ratios to find unknown quantities.
6.RP.A.1, 6.RP.A.3, MP2,
6.Mod1.AD1, 6.Mod1.AD3

Topic B: Collections of Equivalent Ratios

Lesson 6: Ratio Tables and Double Number Lines

- Represent equivalent ratios by using ratio tables and double number lines.
- Use representations of ratio relationships to solve problems. 6.RP.A.3, 6.RP.A.3.a, MP7, 6.Mod1.AD3, 6.Mod1.AD4


## Lesson 7: Graphs of Ratio

Relationships

- Plot points in the coordinate plane that each represent a ratio.
- Identify characteristics of graphs, tables, and double number lines representing ratio relationships. 6.RP.A.3.a, MP2, 6.Mod1.AD4 Multiple
- Find the least common multiple of two whole numbers less than or equal to 12.
6.NS.B.4, MP6, 6.Mod2.AD13

Lesson 5: The Euclidean Algorithm (Optional)

- Find the greatest common factor of large numbers by using the Euclidean algorithm.
- Find the least common multiple of large numbers by using the greatest
factor
common factor.


## 6.NS.B.4, MP7, 6.Mod2.AD12

## 6.Mod2.AD13

## Topic B: Dividing Fractions

## Lesson 6: Dividing a Whole

Number by a Fraction

- Divide a whole number by a fraction by using tape diagrams and reasoning about division.


## 6.NS.A.1, MP2, 6.Mod2.AD4

6.Mod2.AD5, 6.Mod2.AD6

Lesson 7: Dividing a Fraction by a Whole Number

- Divide a fraction by a whole number.
- Divide a mixed number by a whole
number.
6.NS.A.1, MP1, 6.Mod2.AD4,
6.Mod2.AD5, 6.Mod2.AD6


## Lesson 3: Rational Numbers

- Plot rational numbers on horizonta
and vertical number lines.
- Identify the locations of rational numbers plotted on horizontal and vertical number lines.


## 6.NS.C.6.a, 6.NS.C.6.c, MP3

## 6.Mod3.AD3, 6.Mod3.AD6

## Lesson 4: Rational Numbers in

 Real-World Situations- Represent opposite quantities in realworld situations by using rational numbers.


## 6.NS.C.5, 6.NS.C.6.a, MP6,

 6.Mod3.AD1, 6.Mod3.AD2Topic B: Ordering and Magnitude

Lesson 5: Comparing Rational

## Numbers

- Write and interpret statements of comparison about rational numbers. - Compare rational numbers in realworld situations.
6.NS.C.7, 6.NS.C.7.a, 6.NS.C.7.b, MP3, 6.Mod3.AD8, 6.Mod3.AD9, 6.Mod3.AD10


## Lesson 6: Ordering Rational

Numbers

- Order rational numbers.
- Write, interpret, and explain statements of order for rational statements of order for rational 6.NS.C.7, 6.NS.C.7.a, 6.NS.C.7.b, MP1, 6.Mod3.AD8, 6.Mod3.AD9, 6.Mod3.AD10

Lesson 4: Evaluating Expressions with Exponents

- Evaluate numerical expressions written in exponential notation. 6.EE.A.1, MP7, 6.Mod4.AD3

Lesson 5: Exploring Order of Operations

- Identify the relationships between operations and apply those expressions.
6.EE.A.1, MP6, 6.Mod4.AD3

Lesson 6: Order of Operations

- Evaluate numerical expressions with exponents by using the conventional order of operations. 6.EE.A.1, MP1, 6.Mod4.AD3

Topic B: Expressions and Real-World Problems

Lesson 7: Algebraic Expressions with Addition and Subtraction

- Write algebraic expressions to
represent descriptions involving addition and subtraction.
- Write descriptions of algebraic expressions involving addition and subtraction.
6.EE.A.2.a, 6.EE.A.2.b, MP8, 6.Mod4.AD4, 6.Mod4.AD5

Lesson 3: The Area of a Triangle - Compose two identical triangles into a parallelogram to derive the formula for the area of a triangle.

- Compute the area of any triangle by
using the formula $A=\frac{1}{2} b h$.
6.EE.A.2.c, 6.G.A.1, MP7,
6.Mod4.AD6, 6.Mod5.AD1

Lesson 4: Areas of Triangles in Real-World Situations

- Use composition or decomposition to write equivalent expressions that
represent the area of a triangle.
- Solve real-world and mathematic problems involving the areas of triangles
6.EE.A.3, 6.G.A.1, MP2,
6.Mod4.AD7, 6.Mod5.AD1, 6.Mod5.AD2

Topic B: Problem Solving with Area

Lesson 5: Perimeter and Area in the Coordinate Plane

- Determine the perimeters of rectangles and polygons graphed in the coordinate plane.
- Determine the areas of
parallelograms, rectangles, and polygons graphed in the coordinate plane.
6.NS.C.8, 6.G.A.1, 6.G.A.3, MP7,
6.Mod3.AD14, 6.Mod5.AD1,
6.Mod5.AD5

Lesson 4: Creating a Histogram - Use a frequency table to construct a frequency histogram for a data distribution
6.SP.A.2, 6.SP.B.4, MP2,
6.Mod6.AD2, 6.Mod6.AD4

Lesson 5: Comparing Data Displays

- Identify the differences between bar graphs and histograms.
- Construct relative frequency histograms.
6.SP.B.4, 6.SP.B.5.b, MP5, 6.Mod6.AD4, 6.Mod6.AD6

Lesson 6: Selecting a Data Display - Display data by using a dot plot or a histogram and describe the data histogram an
distribution.
6.SP.A.1, 6.SP.B.4, MP5, 6.Mod6.AD1, 6.Mod6.AD4

Topic B: Mean and Mean Absolute Deviation

Lesson 7: Using the Mean to Describe the Center

- Describe the center of a data distribution by using an equal share value called the mean
- Connect the concept of equal shares with the mathematical formula fo
SP A 3 6 SP B 5
6.SP.A.3, 6.SP.B.5.c, MP2 6.Mod6.AD3, 6.Mod6.AD7


## esson 8: Addition Patterns in

 Ratio Relationships- Use addition patterns in tables and graphs of equivalent ratios to describe ratio relationships and find unknown quantities.


## 6.RP.A.1, 6.RP.A.3, 6.RP.A.3.a,

 MP7, 6.Mod1.AD1, 6.Mod1.AD3,
## 6.Mod1.AD4

Lesson 9: Multiplication Patterns in Ratio Relationships

- Use graphs and tables to explore multiplication patterns in ratio relationships.
- Use multiplication to complete ratio tables.
6.RP.A.3, 6.RP.A.3.a, MP7,
6.Mod1.AD3, 6.Mod1.AD4

Lesson 10: Multiplicative Reasoning in Ratio Relationships

- Write and use equivalent ratios when one of the numbers in the ratio is 1 . 6.RP.A.1, 6.RP.A.3, 6.RP.A.3.a, MP8, 6.Mod1.AD1, 6.Mod1.AD3, 6.Mod1.AD4

Lesson 11: Applications of Ratio Reasoning

- Solve multi-step ratio problems by reasoning about equivalent ratios. 6.RP.A.1, 6.RP.A.3, 6.RP.A.3.a, MP1, 6.Mod1.AD1, 6.Mod1.AD3, 6.Mod1.AD4

Lesson 8: Dividing Fractions by Making Common Denominators - Divide a fraction by a fraction by using a common denominator.

- Divide a mixed number by a fraction by using a common denominator 6.NS.A.1, MP7, 6.Mod2.AD3, 6.Mod2.AD4, 6.Mod2.AD6

Topic C: Dividing Fractions Fluently

Lesson 9: Dividing Fractions by Using Tape Diagrams

- Use a tape diagram to divide a fraction by a fraction.
- Relate division of a fraction by a fraction to an unknown factor problem. 6.NS.A.1, MP8, 6.Mod2.AD4, 6.Mod2.AD5, 6.Mod2.AD6

Lesson 10: Dividing Fractions by Using the Invert and Multiply Strategy - Use the invert and multiply strategy to divide a fraction by a fraction.

## 6.NS.A.1, MP7, 6.Mod2.AD4

## 6.Mod2.AD6

## Lesson 11: Applications of Fraction

 Division- Solve real-world problems by dividing fractions and mixed numbers. 6.NS.A.1, MP1, 6.Mod2 AD5

Lesson 12: Fraction Operations in a Real-World Situation

- Add, subtract, multiply, and divide fractions and mixed numbers to solve 6.NS.A.1, MP2, 6.M


## Lesson 7: Absolute Value

- Determine the absolute values of rational numbers.


## 6.NS.C.7.c, MP8, 6.Mod3.AD11,

 6.Mod3.AD12
## Lesson 8: Absolute Value and

 Order- Explain the relationship between the order of rational numbers and the order of their absolute values.
- Order and compare the absolute values of rational numbers and the values of rational numbers and the 6.NS.C.7, 6.NS.C.7.d, MP2, 6.Mod3.AD8, 6.Mod3.AD13

Lesson 9: Interpreting Order and Distance in Real-World Situations - Distinguish between comparisons of absolute value and statements of order in real-world situations. - Determine and interpret distance between rational numbers 6.NS.C.7.d, MP1, 6.Mod3.AD13

Topic C: The Coordinate Plane

Lesson 10: The Four Quadrants of the Coordinate Plane

- Use ordered pairs to identify the locations of points in the coordinate plane.
- Relate the signs of $x$ - and $y$ coordinates to each of the four quadrants of the coordinate plane. 6.NS.C.6.b, MP7, 6.Mod3.AD4

Lesson 8: Algebraic Expressions with Addition, Subtraction, Multiplication, and Division - Write algebraic expressions to represent descriptions involving addition, subtraction, multiplication, and division.

- Write descriptions of algebraic expressions involving addition, subtraction, multiplication, and division.
6.EE.A.2.a, 6.EE.A.2.b, 6.EE.A.2.c, MP6, 6.Mod4.AD4, 6.Mod4.AD5, 6.Mod4.AD6

Lesson 9: Addition and Subtraction Expressions from Real-World Situations

- Define variables precisely
- Write algebraic expressions involving addition and subtraction to represent real-world situations.
6.EE.A.2.a, 6.EE.A.2.b, 6.EE.B.6, MP6, 6.Mod4.AD4, 6.Mod4.AD5, 6.Mod4.AD11

Lesson 10: Multiplication and Division Expressions from RealWorld Situations

- Write and interpret algebraic expressions involving multiplication and division that represent real-world situations
6.EE.B.6, MP2, 6.Mod4.AD11

Lesson 11: Modeling Real-World Situations with Expressions

- Write algebraic expressions with two terms to represent real-world situations - involving addition and multiplication. 6.EE.A.2.b, 6.EE.A.2.c, 6.EE.B.6, MP2, 6.Mod4.AD5, 6.Mod4.AD6, 6.Mod4.AD11

Lesson 6: Problem Solving with Area in the Coordinate Plane - Determine the areas of triangles graphed in the coordinate plane - Determine the areas of polygons composed of triangles and parallelograms graphed in the coordinate plane.
6.EE.A.3, 6.G.A.1, 6.G.A.3, MP1, 6.Mod4.AD7, 6.Mod5.AD1, 6.Mod5.AD5

Lesson 7: Areas of Trapezoids and Other Polygons

- Calculate the areas of trapezoids and other polygons by using composition and decomposition.
- Use composition or decomposition to write equivalent expressions for the areas of polygons.
6.EE.A.3, 6.EE.A.4, 6.G.A.1, MP3,
6.Mod4.AD7, 6.Mod4.AD8, 6.Mod5.AD1

Lesson 8: Areas of Composite Figures in Real-World Situations - Determine the areas of real-world composite figures.

- Solve problems in real-world
situations involving rates and areas
6.RP.A.3.b, 6.G.A.1, MP4,
6.Mod1.AD6, 6.Mod5.AD1, 6.Mod5.AD2

Topic C: Nets and Surface Area

Lesson 9: Properties of Solids

- Identify the shapes of the faces of right prisms and pyramids.
- Name parallel and perpendicular
edges and faces of solids.
6.G.A.4, MP6, 6.Mod5.AD6

Lesson 8: The Mean as a Balance Point

- Describe the center of a distribution by using the mean and interpret the mean as a balance point.
6.SP.A.3, 6.SP.B.5.c, MP2, 6.Mod6.AD3, 6.Mod6.AD7

Lesson 9: Variability in a Data Distribution

- Describe a data distribution by using the mean and variability. 6.SP.A.2, 6.SP.A.3, MP2 6.Mod6.AD2, 6.Mod6.AD3

Lesson 10: The Mean Absolute Deviation

- Calculate and interpret the mean absolute deviation for a data distribution. 6.SP.A.3, 6.SP.B.5.c, MP8, 6.Mod6.AD3, 6.Mod6.AD7

Lesson 11: Using the Mean and Mean Absolute Deviation

- Use the mean and mean absolute deviation to describe a data distribution.
6.SP.A.3, 6.SP.B.5.c, MP6 6.Mod6.AD3, 6.Mod6.AD7

Topic C: Median, Interquartile Range, and Box Plots

Lesson 12: Using the Median to Describe the Center

- Calculate and interpret the median of a data distribution.
6.SP.A.3, 6.SP.B.5.c, MP6, 6.Mod6.AD3, 6.Mod6.AD7


## Topic C: Comparing Ratio

 Relationships
## Lesson 12: Multiple Ratio

Relationships

- Compare ratio relationships by using graphs, tables, and double number lines.
6.RP.A.3.a, MP5, 6.Mod1.AD4, 6.Mod1.AD5

Lesson 13: Comparing Ratio
Relationships, Part 1

- Compare ratio relationships by using ratio tables.
6.RP.A.3.a, MP7, 6.Mod1.AD5

Lesson 14: Comparing Ratio Relationships, Part 2

- Compare ratio relationships by creating equivalent ratios.
6.RP.A.3.a, MP3, 6.Mod1.AD5

Lesson 15: The Value of the Ratio

- Compare ratio relationships by using - Compare ratue of the ratio.
6.RP.A.2, 6.RP.A.3.a, MP6, 6.Mod1.AD2, 6.Mod1.AD5


## Topic D: Rates

## Lesson 16: Speed

- Find distance and time
corresponding to a given speed.
- Identify real-world examples of rates and interpret their meanings in context.
6.RP.A.2, 6.RP.A.3.a, 6.RP.A.3.b, MP2, 6.Mod1.AD2, 6.Mod1.AD4, 6.Mod1.AD6

Topic D: Decimal Addition,
Subtraction, and
Multiplication
Lesson 13: Decimal Addition and Subtraction

- Add and subtract decimals by using the standard algorithms for each operation.
6.NS, 6.NS.B.3, MP5,
6.Mod2.AD2, 6.Mod2.AD9

Lesson 14: Patterns in Multiplying Decimals

- Recognize and apply patterns in
factors when multiplying whole
numbers and decimals.
6.NS.B.3, MP8, 6.Mod2.AD10

Lesson 15: Decimal Multiplication

- Multiply decimals by using the standard algorithm.
6.NS, 6.NS.B.3, MP6,
6.Mod2.AD2, 6.Mod2.AD10

Lesson 16: Applications of Decimal Operations

- Create a model of a building and use decimal operations to calculate cost, revenue, and profit or loss. 6.NS, MP4, 6.Mod2.AD2

Lesson 11: Plotting Points in th Coordinate Plane

- Use ordered pairs to plot points in the coordinate plane.
6.NS.C.6.b, 6.NS.C.6.c, MP6, 6.Mod3.AD4, 6.Mod3.AD7

Lesson 12: Reflections in the Coordinate Plane

- Graph points and their reflections in the coordinate plane.
- Recognize that when two ordered pairs differ only by the sign of one or both coordinates, the locations of the points are related by reflections points are related by ref
6.NS.C.6.b, 6.NS.C.6.c, MP8
6.Mod3.AD4, 6.Mod3.AD5
6.Mod3.AD7


## Lesson 13: Constructing the

 Coordinate Plane- Draw and label a coordinate plane, choosing a reasonable scale for a given set of points. Plot points and describe how a graph changes when the scale changes.
6.NS.C.6.b, 6.NS.C.6.c, MP5,
6.Mod3.AD4, 6.Mod3.AD7

Lesson 14: Modeling with the
Coordinate Plane

- Create time graphs in the coordinate plane.
- Solve real-world problems by using time graphs.


## 6.NS.C.8, MP4, 6.Mod3.AD14

Topic C: Equivalent
Expressions Using the
Properties of Operations

Lesson 12: Applying Properties to Multiplication and Division Expressions

- Write and identify equivalent
algebraic expressions involving multiplication and division by using the properties of operations.
- Write algebraic expressions that
represent real-world situations.
6.EE.A.2.c, 6.EE.A.3, 6.EE.A.4, MP3, 6.Mod4.AD6, 6.Mod4.AD7, 6.Mod4.AD8


## Property

- Use the distributive property to write the product of two factors as a sum or difference.
6.NS.B.4, 6.EE.A.3, 6.EE.A.4, MP7, 6.Mod4.AD2, 6.Mod4.AD7
6.Mod4.AD8

Lesson 14: Using the Distributive Property to Factor Expressions

- Use the distributive property to write a sum or difference as the product of two factors.
6.NS.B.4, 6.EE.A.3, 6.EE.A.4, MP7 6.Mod4.AD2, 6.Mod4.AD7, 6.Mod4.AD8 Solids
- Represent solids by using nets
composed of triangles and rectangles
6.G.A.4, MP6, 6.Mod5.AD6

Lesson 11: Constructing Nets of Solids

- Draw and label nets for threedimensional objects.
- Determine the surface area of a solid by using its net.
6.G.A.4, MP7, 6.Mod5.AD6, 6.Mod5.AD7

Lesson 12: From Nets to Surface Area

- Determine the surface area of a solid
- Develop the surface area formula for right rectangular prisms and use it to calculate surface area.
6.EE.A.2.c, 6.EE.A.4, 6.G.A.4, MP8, 6.Mod4.AD6, 6.Mod4.AD8, 6.Mod5.AD6

Lesson 13: Surface Area in RealWorld Situations

- Solve real-world problems involving rates and surface area of right prisms and pyramids
6.RP.A.3.b, 6.EE.A.2.c, 6.G.A. 4

MP1, 6.Mod1.AD6, 6.Mod4.AD6, 6.Mod5.AD7

Lesson 14: Designing a Box

- Design different boxes for a product and calculate each box's surface are 6.EE.A.2.c, 6.G.A.4, MP4, 6.Mod4.AD6, 6.Mod5.AD7

Lesson 13: Using the Interquartile Range to Describe Variability

- Calculate quartiles of a data
distribution and describe the variability by using the interquartile range.
6.SP.A.3, 6.SP.B.5.c, MP6, 6.Mod6.AD3, 6.Mod6.AD7

Lesson 14: Using a Box Plot to
Summarize a Distribution

- Describe a data distribution by using the five-number summary and the interquartile range.
- Construct and interpret a box plot from a five-number summary 6.SP.A.2, 6.SP.B.4, MP7, 6.Mod6.AD2, 6.Mod6.AD4

Lesson 15: More Practice with Box Plots

- Construct and use box plots to
analyze data distributions.
6.SP.A.3, 6.SP.B.4, MP7,
6.Mod6.AD3, 6.Mod6.AD4

Lesson 16: Interpreting Box Plots

- Summarize a data distribution by using a box plot, the median, and the interquartile range
- Use box plots to compare two data distributions.
6.SP.A.3, 6.SP.B.4, MP7,
6.Mod6.AD3, 6.Mod6.AD4


## Lesson 17: Rates

- Identify rates and unit rates. - Calculate one quantity when given another quantity and a constant rate


## 6.RP.A.2, 6.RP.A.3.b, MP2,

## 6.Mod1.AD2, 6.Mod1.AD6

Lesson 18: Comparing Rates - Compare rates with like units of measurement by using unit rate.
6.RP.A.2, 6.RP.A.3.a, 6.RP.A.3.b,

## MP2,6.Mod1.AD2, 6.Mod1.AD5,

## 6.Mod1.AD6

Lesson 19: Using Rates to Convert Units

- Convert units of measurement by applying rate reasoning. 6.RP.A.2, 6.RP.A.3.b, 6.RP.A.3.d, MP6, 6.Mod1.AD2, 6.Mod1.AD6, 6.Mod1.AD9

Lesson 20: Solving Rate Problems

- Apply rate reasoning to solve realworld ratio problems involving speed, unit pricing, and unit conversions
- Find an unknown quantity when given a rate and a known quantity. 6.RP.A.2, 6.RP.A.3.b, 6.RP.A.3.d, MP1, 6.Mod1.AD2, 6.Mod1.AD6,
6.Mod1.AD9


## Topic E: Percents

## Lesson 21: Solving Multi-Step Rate

 Problems- Solve problems involving multiple constant rates.
6.RP.A.3.b, 6.RP.A.3.d, MP4 6.Mod1.AD6, 6.Mod1.AD9


## Topic E: Division of Multi-

 Digit Numbers
## Lesson 17: Partial Quotients

- Divide multi-digit whole numbers by using the partial quotients method, and express quotients as mixed numbers.


## 6.NS.B, 6.NS.B. 2 MP8,

6.Mod2.AD7, 6.Mod2.AD8

Lesson 18: The Standard Division Algorithm

- Divide multi-digit whole numbers by using the standard algorithm. 6.NS.B.2, MP7, 6.Mod2.AD8


## Lesson 19: Expressing Quotients as

 DecimalsDivide multi-digit whole numbers by using the standard algorithm, and express quotients as decimals. 6.NS.B.2, MP6, 6.Mod2.AD8

Lesson 20: Real-World Division Problems

- Create and solve real-world division problems
6.NS, MP2, 6.Mod2.AD1

Topic F: Decimal Division
Lesson 21: Dividing a Decimal by a Whole Number

- Divide a decimal by a multi-digit
whole number by using the standard division algorithm.
6.NS.B.3, MP6, 6.Mod2.AD11

Topic D: Solving Problems in the Coordinate Plane

Lesson 15: Distance in the Coordinate Plane

- Find the lengths of horizontal and vertical line segments with rational number coordinates as endpoints in the coordinate plane by counting the number of units between endpoints and by using absolute value. 6.NS.C.6.c, 6.NS.C.8, MP8 6.Mod3.AD7, 6.Mod3.AD14

Lesson 16: Figures in the
Coordinate Plane

- Graph geometric figures in all four quadrants of the coordinate plane. - Use distance and symmetry to solve geometric problems in the coordinate plane
6.NS.C.6.c, 6.NS.C.8, MP7 6.Mod3.AD7, 6.Mod3.AD14

Lesson 17: Problem Solving with the Coordinate Plane

- Solve geometric and real-world problems by using the coordinate plane.
6.NS.C.6.c, 6.NS.C.8, MP1, 6.Mod3.AD7, 6.Mod3.AD14

Lesson 15: Combining Like Terms by Using the Distributive Property - Add and subtract like terms by using the distributive property.

- Write an algebraic expression that represents a geometric situation. 6.EE.A.3, 6.EE.A.4, MP7, 6.Mod4.AD7, 6.Mod4.AD8


## Lesson 16: Equivalent Algebraic

## Expressions

- Write equivalent expressions by using the properties of operations and combining like term
- Write algebraic expressions that represent real-world situations. 6.EE.A.3, 6.EE.A.4, 6.EE.B.6, MP2, 6.Mod4.AD7, 6.Mod4.AD8


## 6.Mod4.AD11

Topic D: Equations and Inequalities

Lesson 17: Equations and Solutions - Determine whether a number

Dentence is true.
solution to an equation by using substitution.
6.EE.A.2.c, 6.EE.B.5, 6.EE.B.7, MP2, 6.Mod4.AD6, 6.Mod4.AD9, 6.Mod4.AD13

Lesson 18: Inequalities and Solutions

- Represent solutions to inequalities on
number lines.
- Identify whether a number is a solution to an inequality by using substitution
6.EE.B.5, 6.EE.B.8, MP2,
6.Mod4.AD10, 6.Mod4.AD14,
6.Mod4.AD15

Topic D: Volumes of Right Rectangular Prisms

Lesson 15: Exploring Volume - Find the volumes of right rectangular prisms that have fractional edge lengths by packing with cubes that have fractional edge lengths. 6.G.A.2, MP7, 6.Mod5.AD3

Lesson 16: Applying Volume Formulas

- Solve real-world and mathematical problems by applying the formulas $V=l w h$ and $V=B h$ to find volumes of right rectangular prisms with
fractional edge lengths.
6.EE.A.2.c, 6.G.A.2, MP3,
6.Mod4.AD6, 6.Mod5.AD3, 6.Mod5.AD4

Lesson 17: Problem Solving with Volume

- Solve real-world and mathematical problems by applying ratio reasoning to find volumes of right rectangular prisms.
6.EE.A.4, 6.G.A.2, MP8,
6.Mod4.AD8, 6.Mod5.AD4

Less
Solids

- Determine the volumes of solids composed of right rectangular prisms. 6.G.A.2, MP5, 6.Mod5.AD4

Topic D: Answering Statistical Questions by Analyzing Data

Lesson 17: Developing a Statistica Project

- Develop a statistical question to guide - data collection.
- Develop a plan to collect a data set to answer a proposed statistical question 6.SP.A.1, 6.SP.B.5.b, MP4 6.Mod6.AD1, 6.Mod6.AD6

Lesson 18: Connecting Graphical Representations and Summary Measures

- Find exact and approximate features of data distributions from data displays.
- Compare the effectiveness of data displays at communicating different features of data distributions
6.SP.A.2, 6.SP.B.5.c, MP3
6.Mod6.AD2, 6.Mod6.AD7

Lesson 19: Comparing Data Distributions

- Compare data distributions by using relative frequency histograms and box plots.
6.SP.A.3, 6.SP.B.4, MP7
6.Mod6.AD3, 6.Mod6.AD4

Lesson 20: Choosing a Measure of Center

- Choose a measure of center for a data distribution.
- Justify the choice of a measure of center based on the shape of the distribution and the context

Lesson 22: Introduction to Percents

- Relate percents to a part-to-whole
relationship where the whole is 100 . - Model percents and write percents in fraction and decimal forms.
6.RP.A.3.c, MP8, 6.Mod1.AD7

Lesson 23: Finding the Percent

- Calculate a percent when given a part and the whole.
- Discover that if multiple parts make a whole, then the percent representing each of the parts should total $100 \%$
6.RP.A.3.c, MP8, 6.Mod1.AD7,


## 6.Mod1.AD8

Lesson 24: Finding a Part

- Calculate a part when given the whole and a percent.
6.RP.A.3.c, MP3, 6.Mod1.AD8

Lesson 25: Finding the Whole

- Calculate the whole when given a part and a percent.
6.RP.A.3.c, MP1, 6.Mod1.AD8

Lesson 26: Solving Percent Problems - Solve multi-step percent problems. 6.RP.A.3.c, MP1, 6.Mod1.AD7, 6.Mod1.AD8

Lesson 22: Dividing a Decimal by a Decimal Greater Than 1

- Divide a decimal by a decimal greater than 1 by using the standard algorithm.


## 6.NS.B.3, MP3, 6.Mod2.AD11

Lesson 23: Dividing a Decimal by a Decimal Less Than 1

- Divide a decimal by a decimal less than 1 by using the standard algorithm.
Solve real-world problems by dividing a decimal by a decimal.
6.NS.B.3, MP1, 6.Mod2.AD1


## Lesson 24: Living on Mars

- Solve real-world problems by
performing operations with decimals.
6.NS.B.3, MP1, 6.Mod2.AD2
6.Mod2.AD11

Lesson 19: Solving Equations with Addition and Subtraction

- Solve addition and subtraction equations by using tape diagrams and algebraic reasoning.


## 6.EE.B.5, 6.EE.B.7, MP7,

 6.Mod4.AD9, 6.Mod4.AD12Lesson 20: Solving Equations with Multiplication and Division

- Solve multiplication and division
equations by using tape diagrams and algebraic reasoning
6.EE.B.5. 6.EE.B.7, MP6,
6.Mod4.AD9, 6.Mod4.AD12

Lesson 21: Solving Problems with Equations

- Solve problems by writing and solving equations.
6.EE.B.7, MP1, 6.Mod4.AD12
6.Mod4.AD13

Topic E: Relating Variables by Using Tables, Graphs, and Equations

Lesson 22: Relationship Between Two Variables

- Represent a ratio relationship with a
table and two-variable equation
- Identify the independent and
dependent variables in a real-world or mathematical situation.
6.RP.A.3, 6.EE.C.9, MP3,
6.Mod4.AD1, 6.Mod4.AD16,
6.Mod4.AD17

Lesson 19: Volume and Surface Area in Real-World Situations

- Solve real-world problems that involve surface area and volume. 6.G.A.2. 6.G.A.4, MP2, 6.Mod5.AD4, 6.Mod5.AD7

Lesson 21: Comparing Measures of Variability

- Recognize measurement variability and its causes
- Assess variability visually and by using the range, mean absolute deviation, and interquartile range. 6.SP.B.5.b, 6.SP.B.5.c, MP6, 6.Mod6.AD6, 6.Mod6.AD7

Lesson 22: Presenting Statistical Projects

- Present statistical projects that use the investigative process and critique the work of others by using the tools learned in this module
6.SP.A.3, 6.SP.B.4, MP4,
6.Mod6.AD3, 6.Mod6.AD4



## Year-Long Curriculum Overview: Levels 6-8

Trimester and quarter indicators are provided as a guide for pacing. A few optional lessons in each grade level are included in total number of lessons. About thirty additional days are allotted at each level for assessment and responsive teaching.

|  |  | STORY OF RATIOS |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Level 6 <br> Ratios and Rates | Level 7 <br> Ratios and Proportionality | Level 8 <br> Ratios and Linearity |
|  |  | Module 1: Ratios, Rates, and Percents <br> 5 Topics \| 26 Lessons | Module 1: Ratios and Proportional Relationships <br> 3 Topics \| 20 Lessons | Module 1: Scientific Notation, Exponents, and Irrational Numbers <br> 5 Topics \| 24 Lessons |
|  |  | Module 2: Operations with Fractions and Multi-Digit Numbers <br> 6 Topics \| 24 Lessons | Module 2: Operations with Rational Numbers <br> 5 Topics \| 26 Lessons | Module 2: Rigid Motions and Congruent Figures 4 Topics \| 22 Lessons |
|  |  | Module 3: Rational Numbers <br> 4 Topics \| 17 Lessons | Module 3: Expressions, Equations, and Inequalities 4 Topics \| 23 Lessons | Module 3: Dilations and Similar Figures 4 Topics \| 17 Lessons |
|  | $\bigcirc$ | Module 4: Expressions and One-Step Equations 5 Topics \| 25 Lessons | Module 4: Geometry <br> 5 Topics \| 26 Lessons | Module 4: Linear Equations in One and Two Variables 6 Topics \| 27 Lessons |
|  |  | Module 5: Area, Surface Area, and Volume 4 Topics \| 19 Lessons | Module 5: Percent and Applications of Percent | Module 5: Systems of Linear Equations 3 Topics \| 14 Lessons |
|  |  | Module 6: Statistics <br> 4 Topics \| 22 Lessons | Module 6: Probability and Populations 4 Topics \| 19 Lessons | Module 6: Functions and Bivariate Statistics 5 Topics \| 25 Lessons |
|  |  | TOTAL: <br> 28 Topics \| 133 Lessons | TOTAL: <br> 26 Topics \| 138 Lessons | TOTAL: <br> 27 Topics \| 129 Lessons |

## Supports of Diversity, Equity and Inclusion

## Providing Culturally Responsive Instruction

Eureka Math ${ }^{2}$ values the funds of knowledge that students bring into the classroom and acknowledges that deep learning happens when all students are able to leverage their diverse life experiences while learning mathematics.

One of the ways Eureka Math ${ }^{2}$ invites students into mathematics and celebrates the diversity present in every classroom is by highlighting for teachers those specific lesson moments that can be tailored to bring students' experiences from their home and communities into the classroom. For example, a strategically placed Universal Design for Learning (UDL) margin note in grade 7 module 5 lesson 15 highlights that providing students with a restaurant menu allows them to choose the meal they would like to order and promotes relevance because students can draw on their own experiences to understand the problem.

In grade 6 module 6 lesson 6, students work in pairs to choose a statistical question and decide how to display their data set. They create a poster and participate in a gallery walk to provide feedback to their peers. This UDL margin note suggests that teachers promote relevance by leveraging students' life experiences and allowing them to choose the context for the statistical question.

In grade 6 module 1 lesson 4, students complete a digital lesson in which they use tape diagrams to
understand how to make specific color batches of paint. This content provides the teacher with the opportunity to connect to students' home lives and learn more about the experiences they've had with painting.

Students are diverse, and any one classroom can have students from either an individualist frame of reference or a collectivist frame of reference. The teacher-writers of Eureka Math ${ }^{2}$ considered both frames of reference in intentionally balancing activities that build off individualism as well as collectivism.


Example of placed UDL margin note

In her book Culturally Responsive Teaching and the Brain, Zaretta Hammond references collectivism as emphasizing relationships, interdependence within a community, and cooperative learning (page 25). In Eureka Math ${ }^{2}$, a collectivist approach to learning mathematics is present in the embedded cooperative learning structures in open-middle and open-ended tasks. Specifically, the instructional routines Numbered Heads and Co-construction are rooted in students working cooperatively in groups to deepen their mathematical conceptual understanding. See grade 6 module 6 lesson 16 for an example of how students use the Co-construction routine to write statistical questions.

Beyond the instructional routines, Eureka Math ${ }^{2}$ leverages the power of student relationships and interdependence through frequent partner and group work. For any partner or group work referenced in the instructional materials, teachers may make use of strategic, flexible groupings that build off students' strengths, including home language. A Language Support margin note in the first lesson of every module serves to remind teachers to leverage students' cultural perspectives when strategically placing students in partners.

Hammond references individualism as emphasizing individual achievement and independence (page 25). In Eureka Math ${ }^{2}$, an individualist approach to learning mathematics may be seen in the embedded systems for independent practice in every lesson, such as Exit Tickets and Practice Sets. Additionally, the instructional routines Critique a Flawed Response and Take a Stand both start with students working on a math problem individually before engaging in student discourse. See grade 7 module 1 lesson 4 for an example of students engaging in the Take a Stand routine to discuss whether the statement "Graphed lines represent proportional relationships" is always, sometimes, or never true.

Beyond balancing individualism and collectivism, Eureka Math ${ }^{2}$ activities and problems provide students with mirrors in which to see their own cultural perspectives reflected, as well as windows through which to view others' cultural perspectives.

Eureka Math ${ }^{2}$ is an inclusive mathematics curriculum that represents diverse doers of math. The curriculum's images, fine art, and pictures of people represent diversity through problems and exercises related to real-life experiences, perspectives, and contributions of people from various cultures, ethnicities, and gender identities. These representations affirm student identities while rejecting the stereotypes and biases that have excluded many students from mathematical learning in favor of a more robust and inclusive perspective. Representing a diverse array of doers of mathematics in the curriculum inspires all students to think of themselves as mathematicians.

For example, Eureka Math ${ }^{2}$ includes various mathematical activities that involve counting on hands or simulating a number line with one's fingers. In images throughout the curriculum, care was taken to include a variety of body types and skin tones.

The names used in word problems and for sample students in the lesson vignettes are intentionally diverse to represent the wide variety of students who use the curriculum. The names in student-facing word problems are also designed for readability to ensure that they are not a barrier to accessing the math.

According to CAST, "individuals are engaged by information and activities that are relevant and valuable to their interests and goals." (UDL Guidelines, Engagement, Checkpoint 7.2) Eureka Math ${ }^{2}$ also leverages students' experiences, goals, and interests through Math Pasts (described below), art connections, and wordless context videos.

To honor the diverse contributions to the development of the field of mathematics, to build knowledge about our shared math history, and to empower every child to see themselves as able to do mathematics, nearly every module in Eureka Math includes a feature called Math Past. Each Math Past tells the history of some big ideas in the module, recounting the story of the mathematics through artifacts, discoveries, and other contributions from cultures around the world. Math Past also provides ideas about how to engage students in the history of mathematics. Math Past counters the traditional Eurocentric perspective and celebrates the many contributions of Black, Indigenous, and People of Color communities to the history of mathematics.


For example, in grade 6 module 5 , students are highly engaged in studying area by examining a sketch
from the Codex Vergara, a document written around 1540 CE to show the landholdings of families in Aztec villages. Students decode the symbols to determine the side lengths, perimeters, and areas
of the fields. Students then work in pairs to draw polygons composed of rectangles in the coordinate plane. Partners find more than one way to determine the area of various polygons shown in the grid. When students determine the area of a polygon by decomposing it into rectangles and apply their knowledge of the area of a rectangle, they look for and make use of structure, addressing a mathematical habit of mind. The Math Past Teacher Resource includes information about how the Aztecs recorded the perimeters and areas of fields. It also includes other ancient area problems for students to engage in and solve.

Later, in grade 8 module 5, students learn to write and solve systems of equations for problems that use ancient Chinese numbers. The problems in the lessons were translated from one of the most influential mathematical texts of all time: Jiuzhang Suanshu, translated as Nine Chapters on the Mathematical Art. The chapter titled "Fangcheng," or "Rectangular Arrays," contains 18 problems that address issues of trade, crop yield, number of animals, and other situations that can be solved with systems of linear equations. When students use systems of equations to represent real-world contexts involving comparison of weights and the trading of livestock, they reason abstractly and quantitatively (another mathematical habit of mind). The Math Past Teacher Resource guides teachers to prompt students to explore answers to questions such as the following: Are the techniques used today for solving systems of linear equations like those used in the past? What are some alternative ways to represent a system of linear equations? Are there alternative ways to solve systems of linear equations?

In a similar vein, Eureka Math ${ }^{2}$ connects works of fine art to the standards of each grade level. Each Teach book opens with a stunning work of fine art that has a connection to the math learned in the grade. There is also a wide variety of additional pieces of art embedded in each grade's lessons. For example, in grade 8 module 3 lesson 13 (pages 248-270), students examine Composition 8, a piece by Wassily Kandinsky. Students hypothesize whether the figures in the painting are similar figures, and then use what they know about rigid motions and dilations to justify their thinking.

Wordless context-building videos highlight how we use math to solve problems in our everyday lives and make sense of the world around us. Eureka Math ${ }^{2}$ lessons include more than 190 videos. The curriculum offers three types of highly engaging, wordless math context videos: character animation, collage animation, and live action.

Students can identify with the diverse set of actors and characters in the videos, which helps them visualize how math is part of everyday life. Through these videos, students will more readily realize that math surrounds them and that they, too, can engage in mathematical pursuits. The videos allow students to see themselves in the math problems they encounter, which lowers the barrier to engagement and makes the math classroom a more welcoming place.

Wordless videos in lessons serve many other purposes as well, such as the following:

- They make the context for a given problem come alive, putting all students on the same footing by giving them the requisite background knowledge.
- They remove any language and reading barriers to the written word problem.
- They raise the accessibility of mathematics through accurate and inclusive representation.
- They show the many ways in which we interact with math in the world around us and how these interactions spark curiosity and joy.
- They help students see the delight and wonder associated with being a mathematician.
- They create excitement and buzz in the classroom about the content of the new word problem.
- They invite students to tell the story of the math problem, to notice, to wonder, and to drive the discussion.
- Examples include
- Grade 6 module 1 lesson 3: Batches of Paint Part 1
- Grade 7 module 1 lesson 4: Bulk Almonds Part 1
- Grade 8 module 3 lesson 14: Mirror Height

Specific instructional prompts, engaging word problems, accessible and engaging tasks, art connections, Math Past connections, and context videos throughout Eureka Math ${ }^{2}$ work together to create a powerful curriculum that welcomes all students and invites them to become doers of mathematics.

## Addressing Learner Variance

To ensure success of all learners, every Eureka Math ${ }^{2}$ lesson includes Universal Design for Learning (UDL) strategies and scaffolds that address learner variance. These suggestions promote flexibility with engagement, representation, and action and expression, the three UDL principles described by CAST. These strategies and scaffolds are complements to the curriculum's overall alignment with the UDL Guidelines and were designed to support educators in effectively teaching students who experience difficulty in mathematics. The strategies are based on research specific to mathematics instruction.

According to Teaching Mathematics Meaningfully: Solutions for Reaching Struggling Learners, Second Edition, (page 71) "Students who have learning difficulties that affect their ability to do well in mathematics come from a variety of backgrounds and experiences. Although each of these students is individual and unique, students often demonstrate one or more of the nine learning characteristics..." The nine learning characteristics described include: learned helplessness, passive learning, knowledge and skills gaps, math anxiety, memory disabilities, attention disabilities, metacognitive thinking disabilities, processing disabilities, and reading disabilities. Some of these characteristics can affect all students who may be struggling in math regardless of whether they have learning-related disabilities (learned helplessness, passive learning, knowledge and skills gaps, math anxiety). Other characteristics result from learning-related disabilities (memory disabilities, attention disabilities, metacognitive thinking disabilities, processing disabilities, and reading disabilities). These learning characteristics as well as curriculum factors can result in common mathematics performance traits of students who struggle in mathematics.

According to Allsopp et. al (2018), "Mathematics visuals appear to be most effective when used in conjunction with other effective instructional practices. An example of this is the use of explicit instruction techniques in conjunction with visuals. (page 192)." "Explicit cueing techniques can be utilized with visuals in ways that help students attend to the visual's most important features and its representation of the mathematical idea. Simple techniques, such as color-coding, using
directional arrows, and highlighting, can help students focus on what is most relevant." An example of this is found in grade 7 module 5 lesson 7 .

A variety of other strategies suggested in the literature are the foundation of all UDL margin notes found in Eureka Math ${ }^{2}$. Each margin note is aligned to a strategy found to minimize the impacts of one of the nine learning characteristics listed above. Strategies include, but are not limited to:

- Break down tasks into manageable chunks
- Demonstrate the belief that students can be successful.
- Visually organize to cue student to important aspects of concept.
- Teach students to change their frame of thinking.
- Embed math in relevant contexts.
- Help students make connections to prior knowledge
- Engage students by addressing interests.
- Celebrate progress and success.
- Cultivate a growth mindset.
- Relate math to students' lives.
- Use concrete materials
- Associate content with meaningful context.
- Use a variety (visual, auditory, tactile or kinesthetic) of strategies.
- Provide visual organizers.
- Provide think alouds.
- Use novel learning contexts.
- Help students focus on what is important rather than on things that are irrelevant.

Eureka Math ${ }^{2}$ embeds differentiation through the simple-to-complex sequencing of lesson and Practice problems. This logical sequence gradually reduces scaffolds and builds in complexity, allowing teachers to differentiate assignments for either individual or small-group work. For all students, including those working above grade level, the gradual reduction of support and increase in complexity builds independent thinking and encourages productive struggle. Problems toward the end of the Problem Set (a lesson's daily independent practice) are often open-ended, at Depth of Knowledge (DOK) levels 2 and 3, and integrate two or more standards and/or Standards for Mathematical Practice. Teachers can assign problems of different complexities to students according to their needs or allow students to select problems in the 10-minute (approximate) timeframe. Lessons provide differentiation suggestions at the point of instruction to support a wide variety of learners. Differentiation margin notes found in the Teach book offer guidance for adapting instruction so that all students can successfully access grade-level content. There are two types of Differentiation margin notes: Support and Challenge. Challenge boxes suggest ways to keep students working at a more advanced level engaged by providing opportunities for extension.

In this example from grade 8 module 6 lesson 2 the Differentiation margin note offers a suggestion for students to interact with the purpose of the Learn segment of determining whether tables represent functions at a deeper level of complexity by having students create their own tables and trade with a partner.

## Differentiation: Challenge

If students finish early, consider asking them to create a toble of values. Have them trade the table with a partner and ask their partner to identify whether the table of values represents a function.

## Supporting Multilanguage Learners

Eureka Math ${ }^{2}$ writers relied on language development research to outline and build in the language support needed for multilanguage learners to engage with the language-rich lessons. With the goal of supporting the clear, concise, and precise use of reading, writing, speaking, and listening in English, Eureka Math ${ }^{2}$ supports multilanguage learners through each lesson's instructional design. It does this by including instructional best practices, support for mathematical discourse, and support for the different tiers of terminology. Additionally, Language Support margin notes provide just-intime, targeted instructional recommendations to support multilanguage learners.

## Instructional Best Practices

The following table outlines the instructional best practices included in Eureka Math ${ }^{2}$.

| Practice |  |
| :--- | :--- |
| Activate prior knowledge <br> (mathematics content, terminology, contexts) | The daily Fluency and Launch lesson components activate prior <br> knowledge to prepare students for new learning. Context videos <br> demonstrate math concepts in a concrete or real-world context. |
| Provide multiple entry points to the mathematics | Recurring Notice and Wonder routines and frequent open-middle and <br> open-ended tasks provide multiple points of entry for students to <br> participate. The inclusion of fine art and Math Past history components <br> engages students with math in the real world. |
| Use clear, concise student-facing language | Readability guidelines ensure that words are never an obstacle to math <br> learning. |
| Provide strategic active processing time | Frequent mathematical discourse, core instructional routines, and the <br> 10/2 principle expand opportunities for students to synthesize and <br> process new information. |
| Illustrate multiple modes and formats | Varied physical and visual models, such as digital interactives, context <br> videos, and graphic organizers, help students make connections and <br> deepen understanding. |
| Provide opportunities for strategic review | Daily fluency activities, distributed practice Remember problems, Exit <br> Tickets, and comprehensive assessments provide frequent <br> opportunities for strategic review. |

## Mathematical Discourse

To support all learners, lessons provide ample authentic and engaging opportunities for students to read, write, speak, and listen. Eureka Math ${ }^{2}$ supports teachers in creating language-rich classrooms by modeling teacher-student discourse and by providing suggestions for supported student-tostudent discourse. Because curricula in general have an abundance of receptive language experiences (reading and listening), Eureka Math ${ }^{2}$ focuses specific supports on language production (speaking and writing) in mathematics.

The instructional routines that promote discourse are aligned with Stanford's Language Design Principles of supporting sense-making, optimizing output, cultivating conversation, and maximizing linguistic and cognitive meta-awareness.

Eureka Math ${ }^{2}$ periodically includes Language Support notes that suggest specific sentence frames and sentence starters to support multilanguage learners in student-tostudent discussions, such as those used in instructional routines. General sentence frames and sentence starters are provided in the Talking Tool which is referenced often during times of student-to-student discourse.

Talking Tool

|  | I did it thin wer becouse. The aniwer li__becouse My chrwing shows. |
| :---: | :---: |
| Agree or Disagree品 | I agree becoute ... <br> That is true becouse . . . <br> I disogree because.... <br> That is not the becouse . . . . <br> Do you ogree or divogree wh $\qquad$ 3 Wr ? |
| Ask for Reasening | whyddyou ...? <br> Conyuvesplon...? <br> Whot con we do fint? <br> Howls _nemated to - |
|  | I heord you say. ... _soid.... Another way to sor that is What does that mean? |

## Terminology

Eureka Math ${ }^{2}$ lessons give students experience with a new mathematical concept before naming it with a precise mathematical term. Students may see a mathematical concept come to life in a digital interactive, manipulate counters in groups, or use an instructional routine to engage in mathematical discourse before the teacher gives that concept a name. In addition, teachers are provided with educative guidance, either in the body of the lesson or in a Language Support margin note, to support students in pairing the written term with a visual representation. Eureka Math ${ }^{2}$ highlights domain-specific terms from previous lessons in the current lesson, along with instructional recommendations for supporting those terms. These instructional recommendations focus on previewing the meaning of the terms before students are expected to interact with them in the mathematics of the lesson. Additionally, domain-specific terms from previous lessons are also supported by pairing the written term with a visual representation. For each grade, the
academic verbs needed to engage with the mathematics were considered. Each grade in Eureka Math ${ }^{2}$ offers a carefully curated list of targeted academic verbs that appear in the lessons for students o preview before they are expected to understand and use the language. For example, before students are asked to verify in grade 8, lessons preview the meaning of the academic verb, supporting the meaning of the term in a class discussion emphasizing the use of synonyms of that verb.

Multiple-meaning terms encompass homophones like very and vary, and homographs, like scale and scale, (see image from grade 7 module 1 lesson 19) and other pronunciation-based challenges, like the difference between approximate (as an adjective, as in, "What is the approximate value?") and approximate (as a verb, as in, "Approximate the sum."). Lessons call out multiplemeaning terms that could affect multilanguage learners' understanding of the mathematics. Lessons also include Language Support notes to preview the meaning of the term in the lesson. These previews include pairing the term with a visual, with real items, or with a video to highlight the different meanings of the term and emphasize the specific meaning used in the lesson.

## Language Support Boxes

Consider the following figures. which all depict the werd scol. Which one best
relates to the woik of the modulue? Whry? What doces the figures show?
C.

A Language Support margin note appears in the first lesson of every module to prompt teachers to consider using strategic, flexible grouping in each activity of the module to support multilanguage learners. These grouping suggestions invite teachers to use students' knowledge and home language by pairing students in different ways. Each of these different ways of pairing students has specific benefits for multilanguage learners. The Language Support margin notes highlight either discourse, language or terminology supports.

To learn more, please visit the Great Minds MLL blog: https://gm.greatminds.org/how-to-support-multilingual-learners-in-engaging-in-math-conversations-in-the-classroom

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