Carnegie Learning Course 2 Alignment to Connecticut Mathematics Model Curricula

Middle School Math Solution Course 2			
Model Unit Name	**Model Unit Standards**	**Lessons**	**Pacing** *Lessons that address concepts in more than one Unit are only counted once.*
Operating with Rational Numbers (Addition & Subtraction)	**7.NS.A.1**	**MATHbook** **Module 2: Operating with Signed Numbers** **Topic 1: Adding and Subtracting Rational Numbers** Lesson 1: Math Football: Using Models to Understand Integer Addition Lesson 2: Walk the Line: Adding Integers, Part I Lesson 3: Two-Color Counters: Adding Integers, Part II Lesson 4: What's the Difference?: Subtracting Integers **Topic 2: Multiplying and Dividing Rational Numbers** Lesson 3: Properties Schmoperties: Using Number Properties to Interpret Expressions with Signed Numbers **MATHia** **Unit 1: Using Models to Understand Integers** ● Understanding Opposites **Unit 2: Adding and Subtracting Integers** ● Adding and Subtracting Negative Integers ● Using Number Lines to Add and Subtract Integers ● Developing Algorithms for Adding and Subtracting Integers **Unit 6: Using Number Properties to Interpret Expressions with Signed Numbers** ● Operating with Signed Decimals ● Operating with Signed Fractions	10 days

Operating with Rational Numbers (Addition & Subtraction) *(Continued)*	**7.NS.A.3**	MATHbook **Module 2: Operating with Signed Numbers** **Topic 1: Adding and Subtracting Rational Numbers** Lesson 5: All Mixed Up: Adding and Subtracting Rational Numbers **Topic 2: Multiplying and Dividing Rational Numbers** Lesson 1: Equal Groups: Multiplying and Dividing Integers Lesson 3: Properties Schmoperties: Using Number Properties to Interpret Expressions with Signed Numbers MATHia **Unit 5: Rewriting Numeric Expressions** ● Operating with Numeric Expressions ● Evaluating Simple Numeric Expressions with Integers ● Evaluating Numeric Expressions Involving Integers with Parentheses and Exponents ● Evaluating Simple Numeric Expressions with Rational Numbers ● Evaluating Complex Numeric Expressions with Rational Numbers **Unit 6: Using Number Properties to Interpret Expressions with Signed Numbers** ● Problem Solving with Rational Numbers	
Operating with Rational Numbers (Multiplication & Division)	**7.NS.A.2**	MATHbook **Module 2: Operating with Signed Numbers** **Topic 2: Multiplying and Dividing Rational Numbers** Lesson 1: Equal Groups: Multiplying and Dividing Integers Lesson 2: Be Rational!: Quotients of Integers Lesson 3: Properties Schmoperties: Using Number Properties to Interpret Expressions with Signed Numbers MATHia **Unit 3: Multiplying and Dividing Integers** ● Integer Products and Quotients **Unit 6: Using Number Properties to Interpret Expressions with Signed Numbers** ● Operating with Signed Decimals ● Operating with Signed Fractions	19 days

Operating with Rational Numbers (Multiplication & Division) ***(Continued)***	**7.NS.A.3**	MATHbook **Module 2: Operating with Signed Numbers** **Topic 1: Adding and Subtracting Rational Numbers** Lesson 5: All Mixed Up: Adding and Subtracting Rational Numbers **Topic 2: Multiplying and Dividing Rational Numbers** Lesson 1: Equal Groups: Multiplying and Dividing Integers Lesson 3: Properties Schmoperties: Using Number Properties to Interpret Expressions with Signed Numbers MATHia **Unit 5: Rewriting Numeric Expressions** ● Operating with Numeric Expressions ● Evaluating Simple Numeric Expressions with Integers ● Evaluating Numeric Expressions Involving Integers with Parentheses and Exponents ● Evaluating Simple Numeric Expressions with Rational Numbers ● Evaluating Complex Numeric Expressions with Rational Numbers **Unit 6: Using Number Properties to Interpret Expressions with Signed Numbers** ● Problem Solving with Rational Numbers	
	7.EE.A.2	MATHbook **Module 3: Reasoning Algebraically** **Topic 2: Multiple Representations of Equations and Inequalities** Lesson 2: Stations, Stacks, and Structure: Structure of Linear Equations	
	7.EE.B.3	MATHbook **Module 3: Reasoning Algebraically** **Topic 1: Two-Step Expressions and Equations** Lesson 1: No Substitute for Hard Work: Evaluating Algebraic Expressions MATHia **Unit 9: Building Inequalities and Equations to Solve Problems** ● Using Linear Equations and Inequalities **Unit 10: Using Multiple Representations to Solve Problems** ● Solving Problems with Integers ● Solving Problems with Decimals and Fractions	

Two and Three Dimensional Geometry	**7.G.A.2**	**MATHbook** **Module 5: Constructing and Measuring** **Topic 1: Angles and Triangles** Lesson 1: Here's Lookin' at Euclid: Geometric Constructions Lesson 3: Consider Every Side: Constructing Triangles Given Sides Lesson 4: Unique or Not?: Constructing Triangles Given Angles	28 days
	7.G.A.3	**MATHbook** **Module 5: Constructing and Measuring** **Topic 2: Three-Dimensional Figures** Lesson 1: Solid Surgery: Cross-Sections of Rectangular Prisms and Pyramids **MATHia** **Unit 2: Cross-Sections** ● Visualizing Cross Sections of Three-Dimensional Shapes	
	7.G.B.4	**MATHbook** **Module 1: Thinking Proportionally** **Topic 1: Circles and Ratio** Lesson 1: Pi: The Ultimate Ratio: Exploring the Ratio of Circle Circumference to Diameter Lesson 2: That's a Spicy Pizza!: Area of Circles Lesson 3: Circular Reasoning: Solving Area and Circumference Problems **MATHia** **Unit 1: Exploring the Ratio of Circle Circumference to Diameter** ● Investigating Circles **Unit 2: Solving Area and Circumference Problems** ● Developing the Area Formula for Circles ● Calculating Circumference and Area of Circles	

Two and Three Dimensional Geometry *(Continued)*	**7.G.B.5**	**MATHbook** **Module 5: Constructing and Measuring** **Topic 1: Angles and Triangles** Lesson 2: Special Delivery: Special Angle Relationships **MATHia** **Unit 1: Special Angle Relationships** • Calculating Angles • Exploring Angle Relationships • Solving for Angle Measures	
	7.G.B.6	**MATHbook** **Module 1: Thinking Proportionally** **Topic 3: Proportional Relationships** Lesson 4: More Ups and Downs: Percent Increase and Percent Decrease **Module 5: Constructing and Measuring** **Topic 2: Three-Dimensional Figures** Lesson 2: Hey Mister, Got Some Bird Seed?: Volume of Pyramids Lesson 3: The Sound of Surface Area: Surface Area of Pyramids Lesson 4: More Than Four Sides of the Story: Volume and Surface Area of Prisms and Pyramids **MATHia** **Unit 3: Volume of Prisms and Pyramids** • Calculating Volume of Right Prisms • Using Volume Formulas for Right Prisms • Using Volume of Right Prisms • Relating Volumes of Prisms and Pyramids • Calculating Volume of Pyramids **Unit 4: Surface Area of Pyramids and Prisms** • Volume and Surface Area of Prisms and Pyramids	

Proportional Reasoning	**7.RP.A.1**	**MATHbook** **Module 1: Thinking Proportionally** **Topic 1: Circles and Ratio** Lesson 2: That's a Spicy Pizza!: Area of Circles **Topic 2: Proportionality** Lesson 1: Poultry in Motion: Solving Problems with Ratios of Fractions **MATHia** **Unit 3: Ratios of Fractions** ● Fractional Rates ● Determining and Comparing Fractional Rates	33 days

Proportional Reasoning *(Continued)*	**7.RP.A.2**	MATHbook **Module 1: Thinking Proportionally** **Topic 2: Proportionality** Lesson 2: How Does Your Garden Grow?: Defining Proportional Relationships Lesson 3: Tagging Sharks: Solving Proportions Using Formal Strategies Lesson 4: Complying with Title IX: Constant of Proportionality Lesson 5: Fish-Inches: Identifying the Constant of Proportionality in Graphs Lesson 6: Minding Your Ps and Qs: Constant of Proportionality in Multiple Representations MATHia **Unit 4: Ration Representations** ● Recognizing Proportional Relationships ● Determining Characteristics of Graphs of Proportional Relationships **Unit 5: Ratios of Fractions** ● Solving Proportions Using Equivalent Ratios **Unit 6: Defining Proportional Relationships** ● Exploring Proportions **Unit 7: Solving Proportions Using Formal Strategies** ● Rewriting Proportions as Products ● Solving Proportions Using Means and Extremes **Unit 8: Determining the Constant of Proportionality** ● Writing Proportional Relationships with Equations ● Converting Between Forms of Proportional Relationships **Unit 9: Constant of Proportionality in Multiple Representations** ● Modeling the Constant of Proportionality	

Proportional Reasoning *(Continued)*	**7.RP.A.3**	**MATHbook** **Module 1: Thinking Proportionally** **Topic 2: Proportionality** Lesson 3: Tagging Sharks: Solving Proportions Using Formal Strategies **Topic 3: Proportional Relationships** Lesson 1: Markups and Markdowns: Introducing Proportions to Solve Percent Problems Lesson 2: Perks of Work: Calculating Tips and Commission Lesson 3: No Taxation Without Calculating: Sales Tax and Fees Lesson 4: More Ups and Downs: Percent Increase and Percent Decrease **MATHia** **Unit 10: Analyzing Percent Models** ● Fractional Percent Models ● Converting with Fractional Percents **Unit 11: Introducing Proportions to Solve Percent Problems** ● Using Proportions to Solve Percent Problems ● Solving Simple Percent Problems **Unit 12: Calculating Sales Tax and Discounts** ● ECalculating Sales Tax or Discounts ● Solving Problems with Both Sales Tax and Discounts ● Analyzing Different Forms of Expressions **Unit 13: Percent Increase and Percent Decrease** ● Calculating Percent Change and Final Amounts ● Using Percents and Percent Change **MATHbook** **Module 2: Operating with Signed Numbers** **Topic 2: Multiplying and Dividing Rational Numbers** Lesson 2: Be Rational!: Quotients of Integers **Module 4: Analyzing Populations and Probabilities** **Topic 1: Introduction to Probability** Lesson 3: Toss the Cup: Determining Experimental Probability of Simple Events	

Proportional Reasoning *(Continued)*	**7.G.A.1**	**MATHbook** **Module 1: Thinking Proportionally** **Topic 1: Circles and Ratio** Lesson 4: Pound for Pound, Inch for Inch: Scale and Scale Drawings **Topic 2: Proportionality** Lesson 3: Tagging Sharks: Solving Proportions Using Formal Strategies **MATHia** **Unit 3: Scale and Scale Drawings** ● Critical Attributes of Similar Figures ● Using Scale Drawings ● Calculating Measurements Using a Scale	
Algebraic Reasoning II	**7.EE.A.1**	**MATHbook** **Module 3: Reasoning Algebraically** **Topic 1: Two-Step Expressions and Equations** Lesson 1: No Substitute for Hard Work: Evaluating Algebraic Expressions **Topic 2: Multiple Representations of Equations and Inequalities** Lesson 1: Mathematics Gymnastics: Rewriting Expressions Using the Distributive Property **MATHia** **Unit 1: Rewriting Algebraic Expressions** ● Rewriting Simple Algebraic Expressions with Integer Coefficients ● Rewriting Algebraic Expressions Involving Integer Coefficients with Four Operations ● Rewriting Algebraic Expressions Involving Integer Coefficients with Parentheses and Exponents ● Rewriting Complex Algebraic Expressions Involving Integer Coefficients ● Rewriting Algebraic Expressions Involving Integer Coefficients **Unit 6: Rewriting Variable Expressions Using the Distributive Property** ● Factoring Linear Expressions	35 days
	7.EE.A.2	**MATHbook** **Module 3: Reasoning Algebraically** **Topic 2: Multiple Representations of Equations and Inequalities** Lesson 2: Stations, Stacks, and Structure: Structure of Linear Equations	

Algebraic Reasoning II ***(Continued)***	**7.EE.A.4**	**MATHbook** **Module 3: Reasoning Algebraically** **Topic 1: Two-Step Expressions and Equations** Lesson 2: Picture Algebra: Modeling Equations and Equal Expressions Lesson 3: Expressions That Play Together. . .: Solving Equations on a Double Number Line Lesson 4: Formally Yours: Using Inverse Operations to Solve Equations Lesson 5: Put It on a Plane: Representing Equations with Tables and Graphs **Topic 2: Multiple Representations of Equations and Inequalities** Lesson 1: Mathematics Gymnastics: Rewriting Expressions Using the Distributive Property Lesson 2: Stations, Stacks, and Structure: Structure of Linear Equations Lesson 3: Be Greater Than: Solving Inequalities with Inverse Operations Lesson 4: Deep Dive: Using Equations and Inequalities to Solve Problems Lesson 5: Texas Tea: Using Multiple Representations to Solve Problems **MATHia** **Unit 2: Modeling Equations by Equal Expressions** ● Using Picture Algebra with Equations ● Identifying Attributes of Linear Relationships ● Analyzing Models of Two-Step Linear Relationships ● Modeling Two-Step Expressions ● Checking Solving to Linear Equations **Unit 3: Solving Equations on a Double Number Line** ● Exploring Two-Step Equations with Double Number Lines ● Using Double Number Lines to Solve Two-Step Equations **Unit 4: Using Inverse Operations to Solve Equations** ● Exploring Two-Step Equations ● Solving with Multiplication ● Solving with Division ● Solve Two-Step Equations **Unit 5: Representing Equations with Tables and Graphs** ● Graphs of Equations ● Using Graphs to Solve Equations	

		Unit 6: Rewriting Variable Expression Using the Distributive Property • Solving with the Distributive Property Over Multiplication • Solving with the Distributive Property Over Division **Unit 7: Analyzing Linear Equations Involving the Distributive Property** • Analyzing Models of Linear Relationships Involving the Distributive Property • Modeling Integer Rates of Change • Modeling Fractional Rates of Change • Modeling Using the Distributive Property Over Division **Unit 8: Solving Inequalities with Inverse Operations** • Graphing Inequalities with Rational Numbers • Solving One-Step Linear Inequalities • Solving Two-Step Linear Inequalities **Unit 9: Building Inequalities and Equations to Solve Problems** • Determining the Value of an Independent Variable • Writing Linear Equations and Inequalities from a Scenario • Using Linear Equations and Inequalities **Unit 10: Using Multiple Representations to Solve Problems** • Solving Problems with Integers • Solving Problems with Decimals and Fractions	

<table>
<tr><td rowspan="3">Probability</td><td>7.SP.C.5</td><td>MATHbook
Module 4: Analyzing Populations and Probabilities
Topic 1: Introduction to Probability
Lesson 1: Rolling, Rolling, Rolling . . .: Defining and Representing Probability
MATHia
Unit 1: Introduction to Probability
● Determining Probabilities</td><td rowspan="3">16 days</td></tr>
<tr><td>7.SP.C.6</td><td>MATHbook
Module 4: Analyzing Populations and Probabilities
Topic 1: Introduction to Probability
Lesson 3: Toss the Cup: Determining Experimental Probability of Simple Events
Lesson 4: A Stimulating Conversation: Simulating Simple Experiments
Topic 2: Compound Probability
Lesson 1: Evens or Odds?: Using Arrays to Organize Outcomes
Lesson 4: On a Hot Streak: Simulating Probability of Compound Events
MATHia
Unit 1: Introduction to Probability
● Comparing Experimental and Theoretical Probabilities</td></tr>
<tr><td>7.SP.C.7</td><td>MATHbook
Module 4: Analyzing Populations and Probabilities
Topic 1: Introduction to Probability
Lesson 2: Give the Models a Chance: Probability Models
Lesson 3: Toss the Cup: Determining Experimental Probability of Simple Events
Lesson 4: A Stimulating Conversation: Simulating Simple Experiments
Topic 2: Compound Probability
Lesson 1: Evens or Odds?: Using Arrays to Organize Outcomes
Lesson 3: Puppy Love: Using Tree Diagrams
Lesson 3: Pet Probability: Determining Compound Probability
MATHia
Unit 1: Introduction to Probability
● Determining Probabilities
● Modeling Simple Events
● Comparing Experimental and Theoretical Probabilities</td></tr>
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Probability ***(Continued)***	**7.SP.C.8**	**MATHbook** **Module 4: Analyzing Populations and Probabilities** **Topic 2: Compound Probability** Lesson 1: Evens or Odds?: Using Arrays to Organize Outcomes Lesson 3: Puppy Love: Using Tree Diagrams Lesson 3: Pet Probability: Determining Compound Probability Lesson 4: On a Hot Streak: Simulating Probability of Compound Events **MATHia** **Unit 1: Introduction to Probability** ● Simulating Simple Events **Unit 2: Compound Probability** ● Introduction to Compound Events ● Determining Compound Probabilities Using Data Tables ● Calculating Compound Probabilities ● Determining Compound Probabilities Using Tree Diagrams ● Simulating Compound Events	
Inferences and Populations	**7.SP.A.1**	**MATHbook** **Module 4: Analyzing Populations and Probabilities** **Topic 3: Drawing Inferences** Lesson 1: We Want to Hear From You!: Collecting Random Samples Lesson 2: Tiles, Gumballs, and Pumpkins: Using Random Samples to Draw Inferences **MATHia** **Unit 3: Drawing Inferences** ● Using Statistics to Draw Inferences About a Population	9 days
	7.SP.A.2	**MATHbook** **Module 4: Analyzing Populations and Probabilities** **Topic 3: Drawing Inferences** Lesson 2: Tiles, Gumballs, and Pumpkins: Using Random Samples to Draw Inferences **MATHia** **Unit 3: Drawing Inferences** ● Using Statistics to Draw Inferences About a Population	

Inferences and Populations *(Continued)*	**7.SP.B.3**	**MATHbook** **Module 4: Analyzing Populations and Probabilities** **Topic 3: Drawing Inferences** Lesson 3: Spicy or Dark?: Comparing Two Populations Lesson 4: Finding Your Spot to Live: Using Random Samples from Two Populations to Draw Conclusions **MATHia** **Unit 2: Compound Probability** ● Determining Compound Probabilities Using Data Tables **Unit 4: Comparing Two Populations** ● Comparing Characteristics of Data Displays ● Comparing Populations using Data Displays	
	7.SP.B.4	**MATHbook** **Module 4: Analyzing Populations and Probabilities** **Topic 3: Drawing Inferences** Lesson 4: Finding Your Spot to Live: Using Random Samples from Two Populations to Draw Conclusions **MATHia** **Unit 4: Comparing Two Populations** ● Using Random Samples to Compare Populations	

1 Thinking Proportionally

Pacing: 42 Sessions

Topic 1: Circles and Ratio

In this topic, students develop formulas for the circumference and area of circles and use them to solve problems. They begin the topic by reviewing the terminology of circles. Students write ratios of the measures of the distance around and across different circles, noting that this ratio is constant. They learn that the irrational number π is the ratio of a circle's circumference and diameter lengths. Students use this relationship to write a formula for the circumference of a circle. They then decompose a circle and rearrange the pieces to form a familiar shape to derive the formula for a circle's area.

Standards: 6.RP.1, 7.RP.1, 7.G.1, 7.G.4 **Pacing:** 11 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Pi: The Ultimate Ratio** Exploring the Ratio of Circle Circumference to Diameter	6.RP.1 7.G.4	2	Students use a ruler to make a collection of points equidistant from a given point to construct a circle and write its definition. They investigate the ratio between the circumference and diameter and conclude that the ratio is a constant that is about 3.14. Students use the relationship circumference _________ diameter 5 *p* to write the circumference formula. They calculate circumference using *p*, 3.14, and __ 227.	• A circle is a collection of points on the same plane equidistan from the same point. • The circumference of a circle is the distance around the circle. • In every circle, the ratio of the circumference to the diameter is the constant pi, or approximately 3.14. • The formula for the circumference of a circle is $C = \pi d$ or $C = 2\pi r$, where C is the circumference, d is the length of the diameter, and r is the length of the radius. • When you double the length of the radius of a circle, you double its circumference.
2	**That's a Spicy Pizza!** Area of Circles	7.RP.1 7.G.4	2	Students deconstruct a circle and reconstruct it into a parallelogram. They recognize the parallelogram's base and height as the circle's half circumference and radius, respectively, and use substitution to derive the area formula for a circle. Students solve real-world problems where they distinguish between circumference and area. They compare the measures of radii and areas of circles.	• The formula for calculating the area of a circle is $A = \pi r^2$, where A is the area, r is the length of the radius of the circle, and π is represented using the approximation 3.14. • When solving problems involving circles, use the circumference formula to determine the distance around a circle and the area formula to determine the space inside a circle. • When you double the length of a circle's radius, you quadruple its area. The area is quadrupled rather than doubled because in a circle's area formula, $A = \pi r^2$, the length of the radius is squared.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Circular Reasoning** Solving Area and Circumference Problems	7.G.4	2	Students recognize that when different figures have the same perimeter, the circle has the largest area. They use this concept to maximize area in a real-world situation, deriving a circle's area when given its circumference. Students calculate the area of composite figures and irregularly-shaped figures using addition or subtraction of areas.	• A circle has the largest area for a given perimeter. • To calculate the area of a composite figure, divide it into familiar shapes, calculate the area of each of those shapes, and then calculate the sum of those areas. • To calculate the area of an irregularly shaped region, view it as a larger familiar shape with a smaller familiar shape as a hole in it. • Calculate the difference of the areas of the two familiar shapes. • You can use the circumference and area of a circle formulas tsolve real-world problems.
4	**Pound for Pound, Inch for Inch** Scale and Scale Drawings	7.G.1	2	Students extend their understanding of ratios to make sense of scales. They use a scale factor to enlarge a circle and reproduce a scale drawing at a different scale. Students include similar figures in their discussion of scales and use proportions to calculate dimensions of similar figures. They solve problems using scales on a map and a blueprint.	• You can use a scale factor smaller than one to reduce a measurement and a scale factor larger than one to enlarge a measurement. • A scale drawing represents a real object or place in proportion to the real object or place it represents. Maps and blueprints are examples of scale drawings. • Similar figures have proportional dimensions. You can use proportions to solve for an unknown dimension of similar figures. • When you calculate area from a scale drawing, you must scale each dimension before applying the appropriate area formula
Learning Individually with MATHia or Skills Practice		7.G.1 7.G.4	3	**MATHia Unit:** Exploring the Ratio of Circle Circumference to Diameter **MATHia Workspace:** Investigating Circles **MATHia Unit:** Solving Area and Circumference Problems **MATHia Workspaces:** Developing the Area Formula for Circles / Calculating Circumference and Area of Circles **MATHia Unit:** Scale and Scale Drawings **MATHia Workspaces:** Critical Attributes of Similar Figures / Using Scale Drawings / Calculating Measurements Using a Scale	

Topic 2: Proportionality

In this topic, students begin by extending their work with rates to rates with fractional values. They then sort representations and classify relationships as linear or nonlinear and proportional or non-proportional. Students recall that in a proportional relationship, the graph is a straight line that passes through the origin and the table has constant ratios of corresponding values. They then use formal strategies to solve proportions. Students use inverse operations to solve for unknown values and discover that the product of the means equals the product of the extremes.

Standard: 7.RP.1, 7.RP.2a, 7.RP.2b, 7.RP.2c, 7.RP.2d, 7.RP.3, 7.G.1 **Pacing:** 18 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Poultry in Motion** Solving Problems with Ratios of Fractions	7.RP.1	3	Students review the multiple representations of ratios and unit rates. They write complex rates and rewrite them as unit rates and use ratio reasoning to convert units of measures. Students then write proportions and use rates to determine miles per hour.	• You can use part-to-part ratios, part-to-whole ratios, unit rates, and graphs representing ratio relationships to compare quantities in real-world problems. • A complex ratio is a ratio in which one or both of the quantities are fractions. When comparing complex ratios, rewrite them as unit rates. • Real-world situations often include complex ratios. When using a complex ratio to solve for an unknown in an equivalent ratio, first rewrite the complex ratio as a unit rate. • You can rewrite any rate as a unit rate. To convert a rate to a unit rate, you can divide the numerator by the denominator.
2	**How Does Your Garden Grow?** Defining Proportional Relationships	7.RP.2a	2	Students explore graphs and tables of proportional and non-proportional relationships. They determine that the graphs of proportional relationships are straight lines that pass through the origin. They also learn that tables of proportional relationships have a constant ratio of corresponding values of the quantities. Students learn the term *direct variation* and relate it to proportional relationships.	• Graphs of equivalent ratios form a straight line that passes through the origin. • Linear relationships are also proportional relationships when the ratio between corresponding values of the quantities is constant. • The graph of a proportional relationship is a straight line that passes through the origin. • A linear relationship represents a direct variation when the ratio between the output values and input values is constant. You can say that the quantities vary directly. • You can use multiple representations such as tables and graphs to show examples of proportional, or direct variation, relationships between two values within the context of real-world problems.
3	**Tagging Sharks** Solving Proportions Using Formal Strategies	7.RP.2c 7.RP.3 7.G.1	2	Students solve several proportions embedded in real-world situations. They use inverse operations to solve for unknown values in proportional relationships. Students learn that in a proportion written $a : b = c : d$, the product of the means (the two values in the middle) equals the product of the extremes (the two values on the outside).	• You can write different proportions to relate the same quantities. • In a proportion written as $a : b = c : d$, the product of the means (two values in the middle) equals the product of the extremes (two values on the outside). • When you solve a proportion, you determine the value that makes the proportion true. • To solve a proportion, you can rewrite it as the product of the means and extremes and apply inverse operations. • When you isolate the variable in an equation, you perform the operations to get the variable by itself on one side of the equals sign. • Multiplication and division are inverse operations. Inverse operations are operations that undo each other.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Complying with Title IX** Constant of Proportionality	7.RP.2b 7.RP.2c	2	Students explore equations of proportional relationships. They determine the constant of proportionality, the constant ratio of the outputs to the inputs in a proportional relationship. Students explore the reciprocal relationship of constants of proportionality in equations. They then use the constant of proportionality to write and solve equations.	• In a proportional relationship, a constant ratio $x : y$ or $y : x$ exists between the corresponding values of the two quantities. • The constant of proportionality, k, is the ratio $y : x$. • You can write an equation for a proportional relationship as $y = kx$. The constant of proportionality is the coefficient of the independent variable. • In a proportional relationship, you can write two different proportional equations depending on the input and output. The coefficients, or constants of proportionality, in the two equations are reciprocals.
5	**Fish-Inches** Identifying the Constant of Proportionality in Graphs	7.RP.2d	1	Students analyze real-world and mathematical situations, both proportional and non-proportional, represented on graphs and then identify the constant of proportionality when appropriate. Throughout the lesson, students interpret the meaning of points on graphs in terms of a proportional relationship, including the meaning of $(1, y)$ and $(0, 0)$.	• The graph of a proportional relationship is linear and passes through the origin. • Each point on the graph that represents a proportional relationship has the same $y : x$ ratio.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
6	**Minding Your Ps and Qs** Constant of Proportionality in Multiple Representations	7.RP.2a 7.RP.2b 7.RP.2c 7.RP.2d	2	Students recognize and represent proportional relationships between quantities. They determine and interpret the constant of proportionality for quantities that are proportional and represented in words, tables, equations, and graphs.	• The graph of two variables that are proportional is a line that passes through the origin, (0, 0). • When analyzing the table of two variables in proportion, the ratios of the *y*-value to the *x*-value for any pair are equivalent. • The equation used to represent a proportional relationship between two values is $y = kx$, where x is proportional to y, and k is the constant of proportionality. • You can create a table of equivalent ratios, a graph of a straight line through the origin, and an equation of the form $y = kx$ to represent a scenario describing quantities in a proportional relationship.
Learning Individually with MATHia or Skills Practice		7.RP.1 7.RP.2a 7.RP.2b 7.RP.2c	6	**MATHia Unit:** Ratio Representations **MATHia Workspaces:** Recognizing Proportional Relationships / Determining Characteristics of Graphs of Proportional Relationships **MATHia Unit:** Ratios of Fractions **MATHia Workspaces:** Fractional Rates / Determining and Comparing Fractional Rates / Solving Proportions Using Equivalent Ratios **MATHia Unit:** Defining Proportional Relationships **MATHia Workspace:** Exploring Proportions **MATHia Unit:** Solving Proportions Using Formal Strategies **MATHia Workspaces:** Rewriting Proportions as Products / Solving Proportions Using Means and Extremes **MATHia Unit:** Determining the Constant of Proportionality **MATHia Workspaces:** Writing Proportional Relationships with Equations / Converting Between Forms of Proportional Relationships **MATHia Unit:** Constant of Proportionality in Multiple Representations **MATHia Workspace:** Modeling the Constant of Proportionality	

Topic 3: Proportional Relationships

In this topic, students use their knowledge of proportionality to solve real-world problems about money. They solve various multistep ratio and percent problems, including tips, commissions, gratuities, simple interest, taxes, markups, and markdowns.

Standards: 7.RP.3, 7.EE.2, 7.G.6 **Pacing:** 13 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Markups and Markdowns** Introducing Proportions to Solve Percent Problems	7.RP.3	2	Students analyze a series of strategies to solve percent problems. They match percent situations to an appropriate bar model. Students use proportions to calculate the part, whole, or percent in real-world contexts. They also relate the percent as a constant of proportionality and write equations to solve percent problems.	• You can use the proportion: part whole = percent number 100 to solve for a part, whole, or percent number. • You can express a sale price in terms of the percent off or the reduced percent to pay. • When calculating a sale price, it is more efficient to use the percent required to pay rather than the percent discount. • You can use a proportion to solve a percent problem regardless of whether you are solving for the percent, the whole (original price), or the part (sale price or discount). • You can interpret a percent as a constant of proportionality to write an equation to solve percent problems.
2	**Perks of Work** Calculating Tips, Commission, and Simple Interest	7.RP.3	3	Students compare proportions and percent equations to solve for unknowns in multistep percent problems. They engage in contexts with tips and represent a commission situation with a table, graph, and equation. Students calculate interest, rate, or time using the simple interest formula. They use problem-solving strategies and percent relationships to identify a person's receipt and meal cost.	• A percent equation is the result of writing the proportion: $\frac{\text{part}}{\text{whole}} = \frac{\text{percent number}}{100}$ as percent × whole = part, where you express the percent as a decimal. • You can use proportions and percent equations to solve for the whole, a part, or a percent in a real-world situation. • Typical applications of percents include tips, commissions, and interest. • In percent equation, the percent is the constant of proportionality and the coefficient of the variable. • You can use percent relationships as a step in the process of solving more complicated math problems.
3	**No Taxation Without Calculation** Sales Tax and Fees	7.RP.3	1	Students compare the percent relationships in markups and markdowns, tips, discounts, commissions, fees, and taxes. They identify sales tax rates from a table, graph, and equation and represent a percent situation using these representations. They make sense of the formula t = (list price)(1 + tax rate) to calculate the total amount paid in percent situations.	• When calculating the total cost of a good or service that includes sales tax, x%, multiplying the list price by (1 + x%) is the same as adding the sales tax, x%, to the list price. • The relationship between an order amount and the total amount, including a percent fee, is proportional and represented by the equation $y = kx$. • Markups, tips, commissions, fees, and taxes represent increases over an original amount. • Discounts and markdowns represent decreases from an original amount.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**More Ups and Downs** Percent Increase and Percent Decrease	7.RP.3 7.G.6	2	Students apply the percent increase and percent decrease formulas and then investigate a situation with a percent increase, followed by the same percent decrease. They calculate a depreciation rate to a car's value over several years and recognize that the relationship is not proportional through their tabular and graphical models. Students also investigate percent increase in geometric contexts.	• The percent increase formula is $\frac{\text{amount of increase}}{\text{original amount}}$. • The percent decrease formula is $\frac{\text{amount of decrease}}{\text{original amount}}$. • You can apply percent increases and percent decreases to data sets and geometric measurements such as area, volume, and surface area.
Learning Individually with MATHia or Skills Practice		7.RP.3 7.EE.2	5	**MATHia Unit:** Analyzing Percent Models **MATHia Workspaces:** Fractional Percent Models / Converting with Fractional Percents **MATHia Unit:** Introducing Proportions to Solve Percent Problems **MATHia Workspaces:** Using Proportions to Solve Percent Problems / Solving Simple Percent Problems **MATHia Unit:** Calculating Sales Tax and Discounts **MATHia Workspaces:** Calculating Sales Tax or Discounts / Solving Problems with Both Sales Tax and Discounts / Analyzing Different Forms of Expressions **MATHia Unit:** Percent Increase and Percent Decrease **MATHia Workspaces:** Calculating Percent Change and Final Amounts / Using Percents and Percent Change	

2 Operating with Signed Numbers

Pacing: 19 Sessions

Topic 1: Adding and Subtracting Rational Numbers

In this topic, students use number lines and two-color counters to model addition and subtraction of integers before developing rules for the sum and difference of signed numbers. They begin by walking a number line to visualize adding integers. Students transition from physical movement to modeling the motion on number lines.

Standards: 7.NS.1, 7.NS.1a, 7.NS.1b, 7.NS.1c, 7.NS.1d, 7.NS.3 **Pacing:** 10 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Math Football** Using Models to Understand Integer Addition	7.NS.1 7.NS.1a 7.NS.1b 7.NS.1c 7.NS.1d	1	Students play Math Football, a game to model integer addition, using two colored number cubes to represent a positive and a negative integer. They write integer addition equations to describe given gameplays and create equations for the desired result. Students generalize possible game results that relate to the sum of a positive and negative integer.	• You can make sense of integer addition using a real-world situation. • You can write an equation to represent a situation with integers. • When combining numbers modeled by opposite directions, the sign of the resulting location corresponds to the sign of the cube showing the greater distance. • When combining numbers representing the same direction, the resulting direction is the same as the original direction.
2	**Walk the Line** Adding Integers, Part I	7.NS.1b	2	Students walk a number line to demonstrate the addition of integers as they interpret an integer's absolute value as the distance moved in either direction. They then use the number line to add integers or identify an unknown addend in a number sentence. Students recognize that they can apply their rules to rational numbers. They also write number sentences that sum to zero.	• When adding integers on a number line, the number's sign indicates which direction to move. The absolute value of the integer indicates how many spaces to move in that direction. • When adding a positive integer on a number line, move to the right on the number line. • When adding a negative integer, move to the left on the number line. • The sum of two positive numbers is positive. • The sum of two negative numbers is negative. • The sum of two numbers with different signs has the sign of the number with the greater absolute value.
3	**Two-Color Counters** Adding Integers, Part II	7.NS.1a 7.NS.1b	2	Students use the protons and neutrons of an atom, represented with two-color counters, to model positive and negative integers. They identify sums of zero in their model and learn the meaning of the term *additive inverse*. Students add integers using their model and compare this strategy to using a number line. They generalize their understanding to write rules to add integers.	• Two numbers with the sum of zero are additive inverses. • You can use two-color counters to add integers by pairing positive and negative charges to create zero and interpreting the remaining counters. • When you add two integers with the same sign, you add the integers and keep the sign of the integers. • When you add two integers with opposite signs, you subtract the integers' absolute values and use the sign of the integer with the greater absolute value.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**What's the Difference?** Subtracting Integers	7.NS.1c	3	Students walk the number line and use two-color counters to model subtraction of integers as they relate subtraction to the addition of an opposite. They generalize their understanding to write rules to subtract integers and use their strategy to solve problems. Students transfer their knowledge to calculate the distance between two numbers by taking the difference of their absolute values.	• You can use number lines and two-color counters to model the subtraction of integers. • The distance between two numbers on the number line is the absolute value of their difference. • You can think of subtraction as adding the opposite number. Therefore, you can rewrite the subtraction problem $a - b$ as the addition problem $a + (-b)$. • When subtracting integers, first rewrite the subtraction sentence as an addition sentence. Then follow the rules for adding integers.
5	**All Mixed Up** Adding and Subtracting Rational Numbers	7.NS.3	1	Students extend their understanding of the addition and subtraction of integers to all rational numbers. They solve problems with and without context, using number line models or rules. Students express mixed numbers and decimal expressions as the sum of two rational numbers on a number line to show how the sign applies to the whole number and the component less than one.	• You can incorporate the rules for adding and subtracting integers with the strategies you know for operating with fractions and decimals to add and subtract rational numbers. • To solve real-world problems, you may need to add or subtract rational numbers. • You can decompose a rational number into a whole number component and the part less than one. The sign of the rational number applies to both components.
Learning Individually with MATHia or Skills Practice		7.NS.1a 7.NS.1b	1	**MATHia Unit:** Using Models to Understand Integers **MATHia Workspace:** Understanding Opposites **MATHia Unit:** Adding and Subtracting Integers **MATHia Workspaces:** Adding and Subtracting Negative Integers / Using Number Lines to Add and Subtract Integers / Developing Algorithms for Adding and Subtracting Integers	

Topic 2: Multiplying and Dividing Rational Numbers

In this topic, students again use number lines and two-color counters to model the multiplication of integers before developing rules for the product of signed numbers.

Standard: 7.NS.1d, 7.NS.2a, 7.NS.2b, 7.NS.2c, 7.NS.2d, 7.NS.3, 7.RP.3 **Pacing:** 9 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Equal Groups** Multiplying and Dividing Integers	7.NS.2a 7.NS.3	2	Students represent the product of integers using counters and repeated addition on a number line. They use patterns to reason why the product of two negative numbers is positive and expand their understanding to cases with more factors. Once they derive their rules for the multiplication of integers, students create and analyze signed fact families to derive the rules for the division of integers.	• You can use repeated addition, two-color counters, number lines, and patterns to develop the rules for multiplication of integers. • You can use signed fact families to develop the rules for the division of integers. • To multiply and divide integers, perform the usual multiplication and division algorithms, and then apply the correct sign to the product or quotient. • When you multiply or divide two integers with the same sign, the result is positive. When you multiply or divide two integers with different signs, the result is negative. • The product of an odd number of negative integers is negative. The product of an even number of negative integers is positive.
2	**Be Rational!** Quotients of Integers	7.RP.3 7.NS.2b 7.NS.2d	2	Students divide integers and classify the quotients; they learn that the terminating and repeating decimal results are rational numbers. Students perform operations with positive and negative rational numbers as they calculate percent error and solve real-world problems. Students express rational numbers written as negative fractions in equivalent forms by changing the negative sign's position.	• Rational numbers are numbers you can write $\frac{as}{ab}$, where a and b are integers and b does not equal 0. You can express any rational number as a decimal. • Terminating and repeating decimals are rational numbers. Decimals that do not terminate or repeat are not rational numbers. • You can express a negative fraction with the negative sign in the numerator, denominator, or front of the fraction. • The formula for percent error is $\frac{\text{actual value - estimated value}}{\text{actual value.}}$

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Properties Schmoperties** Using Number Properties to Interpret Expressions with Signed Numbers	7.NS.1d 7.NS.2c 7.NS.3	2	Students evaluate numeric expressions with signed rational numbers and identify the properties to justify their results. They calculate the opposite of numeric expressions by distributing –1 and rewrite expressions by factoring out –1. Students liken these operations to reflecting a positive value across zero to determine its opposite.	• You can use reflections across zero on a number line to determine the opposite of an expression. • Taking the opposite of an expression is the same as using the Distributive Property to multiply the expression by −1. • Subtracting a number is the same as adding the opposite of that number. Rewriting subtraction as addition allows you to apply the Commutative Property. • You can apply the Commutative, Associative, and Distributive Properties to evaluate expressions with rational numbers more efficiently.
Learning Individually with MATHia or Skills Practice		7.NS.1d 7.NS.2b 7.NS.2d 7.NS.3	3	**MATHia Unit:** Multiplying and Dividing Integers **MATHia Workspace:** Integer Products and Quotients **MATHia Unit:** Quotients of Integers **MATHia Workspace:** Converting Rational Numbers to Decimals **MATHia Unit:** Rewriting Numeric Expressions **MATHia Workspaces:** Operating with Numeric Expressions / Evaluating Simple Numeric Expressions with Integers / Evaluating Numeric Expressions Involving Integers with Parentheses and Exponents / Evaluating Simple Numeric Expressions with Rational Numbers / Evaluating Complex Numeric Expressions with Rational Numbers **MATHia Unit:** Using Number Properties to Interpret Expressions with Signed Numbers **MATHia Workspaces:** Problem Solving with Rational Numbers / Operating with Signed Decimals / Operating with Signed Fractions	

3 Reasoning Algebraically

Pacing: 36 Sessions

Topic 1: Two-Step Expressions and Equations

In this topic, students explore algebraic expressions with rational coefficients, building on their prior work with positive coefficients. They represent variable expressions on a number line and make connections between variable and numeric expressions. Students use their previous knowledge of evaluating expressions to verify the equivalence of expressions.

Standards: 7.EE.1, 7.EE.3, 7.EE.4a **Pacing:** 16 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**No Substitute for Hard Work** Evaluating Algebraic Expressions	7.EE.1 7.EE.3	1	Students transfer their understanding of numeric expressions to algebraic expressions. They recognize the difference between linear and nonlinear expressions. Students combine like terms with rational coefficients and substitute values in the expressions to verify their equivalency. They then write algebraic expressions to represent problem situations and use substitution to solve them.	• The algebraic expression $-x$ means the opposite of x. It does not imply that the value is always negative. • Like terms are parts of an algebraic expression that have the same variable raised to the same power. The coefficients of the variable may be different. • Combining like terms is a strategy to solve problems with like terms more efficiently. • To evaluate an algebraic expression, substitute each variable in the expression with a number or numeric expression, and then perform all possible mathematical operations.
2	**Picture Algebra** Modeling Equations as Equal Expressions	7.EE.4a	2	Students use bar models to solve two-step equations. After analyzing and interpreting a given bar model, they create bar models for situations. Students make sense of the terms in the algebraic expression generated from their model, write an equation, use the model to solve the equation, and interpret the solution in terms of the context.	• You can use a bar model to represent a real-world situation, write an equation, and solve a problem. • An equation is a mathematical sentence created by placing an equal sign between two expressions. • The solution to an equation is a value for the unknown that makes the equation true. • You can solve equation with the same structure using the same process.
3	**Expressions That Play Together ...** Solving Equations on a Double Number Line	7.EE.4a	1	Students generate equivalent expressions on a double number line and then use these strategies to isolate the variable to solve two-step equations, including those with negative and fractional coefficients, on a double number line. They reverse the process and start with a solution to create an equivalent two-step equation and then compare the steps to generate and solve an equation.	• You can represent an equation as two equivalent expressions on a double number line. • You can use double number lines to solve for unknowns in equations. • When you multiply or divide by -1 on a double number line, you reflect the expressions across zero. • When you solve an equation, you reverse the operations used to create the original equation.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Formally Yours** Using Inverse Operations to Solve Equations	7.EE.4a	3	Students solve a two-step equation using a double number line and identify which Properties of Equality they applied. The properties provide the connection to use inverse operations to solve two-step equations. Students apply these methods to solve two-step equations, including a literal equation. They also use strategies to solve equations more efficiently by eliminating decimals and fractions.	• There are two specific structures of two-step equations, $ax + b = c$ and $a(x + b) = c$. You can solve these equations by applying the Properties of Equality. • When you apply the Properties of Equality to an equation, the equivalent equation has the same solution as the original equation. • When solving two-step equations, you can apply the Properties of Equality in any order. • You can apply the Properties of Equality to rewrite equations and eliminate decimals and fractions to solve them more efficiently. • Literal equations are equations in which the variables represent specific measures.
5	**Put It on the Plane** Representing Equations with Tables and Graphs	7.EE.4a	2	Students create a table, graph the data in the first quadrant, and write an equation to model a situation. They relate the quantities in the equation and graph and use the representations to solve problems. Students duplicate the process for another context; however, this time, the graph lies in Quadrants I and II. They identify the advantages and disadvantages of these different representations.	• You can express a real-world problem situation using multiple representations—a scenario, table, graph, and equation. Each representation has advantages and disadvantages. • A table provides specific values for a given problem situation. • A graph is a visual representation of the data related to a real-world situation. • An equation generalizes a problem situation. • You can use negative numbers to represent elapsed time or time in the past.
Learning Individually with MATHia or Skills Practice		7.EE.1 7.EE.4a	7	**MATHia Unit:** Rewriting Algebraic Expressions **MATHia Workspaces:** Rewriting Simple Algebraic Expressions with Integer Coefficients / Rewriting Algebraic Expressions Involving Integer Coefficients with Four Operations / Rewriting Algebraic Expressions Involving Integer Coefficients with Parentheses and Exponents / Rewriting Complex Algebraic Expressions Involving Integer Coefficients / Rewriting Algebraic Expressions Involving Integer Coefficients **MATHia Unit:** Modeling Equations as Equal Expressions **MATHia Workspaces:** Using Picture Algebra with Equations / Identifying Attributes of Linear Relationships / Analyzing Models of Two-Step Linear Relationships / Modeling Two-Step Expressions / Checking Solutions to Linear Equations **MATHia Unit:** Solving Equations on a Double Number Line **MATHia Workspaces:** Exploring Two-Step Equations with Double Number Lines / Using Double Number Lines to Solve Two-Step Equations (Type In) / Using Double Number Lines to Solve Two-Step Equations (No Type In) **MATHia Unit:** Using Inverse Operations to Solve Equations **MATHia Workspaces:** Exploring Two-Step Equations / Solving with Multiplication (No Type In) / Solving with Multiplication (Type In) / Solving with Division (No Type In) / Solving with Division (Type In) / Solving Two-Step Equations **MATHia Unit:** Representing Equations with Tables and Graphs **MATHia Workspaces:** Graphs of Equations / Using Graphs to Solve Equations	

Topic 2: Multiple Representations of Equations and Inequalities

In this topic, students apply the Distributive Property to rewrite equivalent expressions and solve equations by expanding and factoring them in various ways.

Standards: 7.EE.1, 7.EE.2, 7.EE.4a, 7.EE.4b, 7.EE.4, 7.EE.3 **Pacing:** 20 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Mathematics Gymnastics** Rewriting Expressions Using the Distributive Property	7.EE.1	3	Students focus on expressions written as the product of two factors. They combine like terms with a common variable expression and use the Distributive Property to rewrite complex expressions. Students rewrite linear expressions by factoring out the GCF or the coefficient of the variable. They apply the Distributive Property to solve equations.	• You can combine like terms that are a single variable or a variable expression. • You can use the Distributive Property with subtraction: $a(b - c) = ab - ac$. • Rewriting an algebraic expression in a different equivalent form, such as by factoring out a common term, may reveal the expression's structure. • You can rewrite algebraic expressions by factoring out the greatest common factor (GCF). • You can use the Distributive Property to solve equations.
2	**Stations, Stacks, and Structure** Structure of Linear Equations	7.EE.2 7.EE.4a	3	Students investigate situations where they apply one rate to the first item and a different rate to the remaining items. They write two equations, one in terms of $x - 1$ and the other in terms of x, to represent the situations. Students compare the two forms of equations and interpret the meaning of their values. They use their tables, equations, and graphs to solve problems.	• Rewriting an expression in different forms can shed light on the relationships between quantities in real-world situations. • You can use equivalent equations with the structures $y = c + d(x - 1)$ and $y = ax + b$ to represent the same situation. • To apply a constant rate to the independent quantity except for the first value, use the expression $(x - 1)$ for the independent quantity. • In the equation $y = ax + b$, a represents how the graph increases, and b is the value where the graph touches the y-axis.
3	**Be Greater Than** Solving Inequalities with Inverse Operations	7.EE.4b	3	Students develop the rules to solve inequalities in one-variable by adding, subtracting, multiplying, or dividing each side of the inequality by the same value and observing its impact on the inequality statement. They solve two-step inequalities and graph their solution sets. Students compare the steps to solve a two-step equation and a two-step inequality.	• An inequality is any mathematical sentence that has an inequality symbol. • The solution set of an inequality is all values that make the inequality statement true. • The graph of an inequality in one variable is the set of all points on a number line that make the inequality true. • The Properties of Inequalities allow you to solve inequalities involving any numbers. • To solve an inequality, apply the same inverse operation steps as you would to solve an equation; however, reverse the inequality symbol when you multiply or divide both sides by a negative value.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Deep Dive** Using Equations and Inequalities to Solve Problems	7.EE.4b	2	Students engage in contexts with inequalities in two variables. They write an equation, create a table, graph the line, and interpret its unit rate of change. Given an inequality statement regarding an interval of dependent values, students use the graph to estimate the corresponding interval of independent values. They then write and solve inequalities to model the same situation.	• The solution set to an inequality informs but is not necessarily the same as the solution set to the problem situation. • The unit rate of change describes how the dependent variable changes for every one unit that the independent variable changes. • You need to write and solve an inequality for an exact answer to a problem situation. You can use graphs and tables to make estimations.
5	**Texas Tea** Using Multiple Representations to Solve Problems	7.EE.4	2	Students move among multiple representations. They write an equation from a real-world situation, use their equation to solve problems, and then create a table and graph. Next, students write an equation from a table and then use their equation to construct a graph. Lastly, students create a table from a graph, write an equation for the table, and create a context for the models.	• You can represent a situation using a scenario, table, equation, and graph. • Starting from any one of these models, you can create the other three representations.
Learning Individually with MATHia or Skills Practice		7.EE.1 7.EE.4a 7.EE.4b 7.EE.4 7.EE.3	7	**MATHia Unit:** Rewriting Variable Expressions Using the Distributive Property **MATHia Workspaces:** Factoring Linear Expressions / Solving with the Distributive Property Over Multiplication / Solving with the Distributive Property Over Division **MATHia Unit:** Analyzing Linear Equations Involving the Distributive Property **MATHia Workspaces:** Analyzing Models of Linear Relationships Involving the Distributive Property / Modeling Integer Rates of Change / Modeling Fractional Rates of Change / Modeling Using the Distributive Property Over Division **MATHia Unit:** Solving Inequalities with Inverse Operations **MATHia Workspaces:** Graphing Inequalities with Rational Numbers / Solving One-Step Linear Inequalities / Solving Two-Step Linear Inequalities **MATHia Unit:** Using Equations and Inequalities to Solve Problems **MATHia Workspaces:** Determining the Value of an Independent Variable / Writing Linear Equations and Inequalities from a Scenario / Using Linear Equations and Inequalities **MATHia Unit:** Using Multiple Representations to Solve Problems **MATHia Workspaces:** Solving Problems with Integers / Solving Problems with Decimals and Fractions	

4 Analyzing Populations and Probabilities

Pacing: 25 Sessions

Topic 1: Introduction to Probability

In this topic, students conduct simple experiments and determine theoretical and experimental probabilities of simple events. They use number cubes, coins, spinners, and marbles in a bag to calculate probabilities and learn the terminology of probability, including outcome, experiment, sample space, event, simple event, probability, complementary events, and equally likely.

Standards: 7.RP.3, 7.SP.7a, 7.SP.7b, 7.SP.5, 7.SP.6, 7.SP.7, 7.SP.8c **Pacing:** 9 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Rolling, Rolling, Rolling...** Defining and Representing Probability	7.SP.5	2	Students learn vocabulary related to probability. They calculate the probability of simple events and their complements and express the results as fractions, decimals, and percents. Students also estimate probabilities using the benchmarks 0, $\frac{1}{2}$, and 1 on a number line. They realize that the sum of the probabilities for all outcomes of any experiment will always be 1.	• Probability is a measure of the likelihood that an event will occur. • The probability of an event is the ratio $\frac{\text{number of times an event can occur}}{\text{number of possible outcomes}}$. • Complementary events are events that together contain all of the outcomes in the sample space. The sum of their probabilities is 1. • You can calculate the probability of a complementary event, *P*(not event), using the probability formula or calculating 1 − *P*(event). • The sum of the probabilities for all outcomes of any experiment is always 1. • Probability values range from 0 to 1. The probability of an impossible event is 0. The probability of an event certain to happen is 1.
2	**Give the Models a Chance** Using Probability Models	7.SP.7	1	Students analyze an experiment where the outcomes are not equally likely. They calculate each outcome's probability and the sum of the probabilities. Students construct uniform and non-uniform probability models and use the models to determine probabilities. They calculate probabilities based on the areas formed by a square and an inscribed circle.	• A probability model is a list of each possible outcome along with its probability. The sum of all the probabilities for the outcomes is always 1. • A uniform probability model is a model in which all of the probabilities are equally likely to occur. A non-uniform probability model is a model in which all probabilities are not equally likely to occur. • You can use a probability model to determine the probability of an event. • You can determine probabilities in geometric situations that involve areas.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Toss the Cup** Determining Experimental Probability of Simple Events	7.RP.3 7.SP.6 7.SP.7b	2	Students flip a coin, toss a cup, and spin a spinner to calculate experimental probabilities. They learn that as the number of trials increases, the experimental probability approaches the theoretical probability. Students use percent error to compare the probabilities. They use proportional reasoning to predict the number of times that an event will occur throughout an experiment.	• Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed: $P_E = \frac{\text{number of times the event occurs}}{\text{total number of trials performed}}$ • Theoretical probability is the ratio of the number of desired outcomes to the total possible outcomes: $P_T = \frac{\text{number of desired outcomes}}{\text{total possible outcomes.}}$ • Percent error, $\frac{P_E - P_T}{P_T \cdot 100}$, is one way to measure the difference between experimental and theoretical probabilities. • As the number of trials increases, the experimental probability approaches the theoretical probability, and the percent error between the two probabilities decreases. • You cannot calculate the experimental or theoretical probability for some situations.
4	**A Simulating Conversation** Simulating Simple Experiments	7.SP.6 7.SP.7a	2	Students learn the meaning of the term *simulation*. They use spinners, coins, a random number generator, and tools of their choice to simulate situations such as taking a multiple-choice test or predicting the percent of females born in a hospital. Students display their simulation results in a table and as dot plots, and they interpret the results in the context of the situation.	• A simulation is an experiment that models a real-life situation. • When conducting a simulation, you must choose a model that has the same probability as the event. • As the number of trials in an experiment increases, the experimental probability approaches the theoretical probability. • You can use technology to generate random numbers to perform a simulation.
Learning Individually with MATHia or Skills Practice		7.SP.5 7.SP.6 7.SP.7a 7.SP.8c	2	**MATHia Unit:** Introduction to Probability **MATHia Workspaces:** Determining Probabilities / Modeling Simple Events / Comparing Experimental and Theoretical Probability / Simulating Simple Events	

Topic 2: Compound Probability

In this topic, students build on their understanding of probability concepts from Introduction to Probability. They use arrays and lists to organize the possible outcomes of an experiment that includes two simple events. Students calculate experimental and theoretical probabilities of events and use proportional reasoning to determine percent error and to make predictions of expected numbers of outcomes.

Standards: 7.SP.6, 7.SP.7, 7.SP.7b, 7.SP.8a, 7.SP.8b, 7.SP.8c **Pacing:** 7 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Evens or Odds?** Using Arrays to Organize Outcomes	7.SP.6 7.SP.7b 7.SP.8a 7.SP.8b	2	Students make an organized list of all the possible outcomes when rolling two six-sided number cubes. They then create a sample space for all the possible sums. Students calculate the experimental and theoretical probabilities and the percent error between them for even and odd sums. They also calculate probabilities for flipping two coins and spinning a spinner twice.	• You can use an array to organize the possible outcomes of two events. • A sample space is not always the same as the list of possible outcomes. Sometimes you base the sample space on two possible outcomes. • To calculate the theoretical probability of two events, create an array of the possible outcomes, list the sample space, create a probability model, and base your response on the model. • You can use the same strategy to solve probability problems with similar structures.
2	**Puppy Love** Using Tree Diagrams	7.SP.7 7.SP.8b	1	Students use coin tosses to simulate a dog having a litter of 3 puppies in which all three puppies are female and summarize the results in a probability model. Using the same context, they construct a tree diagram to list all outcomes and calculate the theoretical probabilities. Students use tree diagrams to create probability models for spinning a spinner twice and guessing on a 3-question test.	• You can simulate situations that involve more than one event to calculate experimental probabilities. • You can use a tree diagram to create an organized list and determine the possible outcomes when a situation includes more than one event. • At the end of each branch of a tree diagram, identify how it relates to the probability question to create a list of the possible outcomes. • To calculate probability, you can create a tree diagram, list the outcomes, then construct and use a probability model to respond to the question.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
3	**Pet Probability** Determining Compound Probability	7.SP.7b 7.SP.8a 7.SP.8b	1	Students learn that a compound event combines two or more events using the word and or or. They identify the outcomes that make up an and or or event and calculate the probability. Students rewrite a probability question with an or statement as the sum of each event's probability without counting an outcome twice when it pertains to both events. They apply their understanding in a game context.	• A compound event combines two or more events, using the word *and* or the word *or*. • An and statement means that both outcomes must occur. An or statement means that that one, the other, or both outcomes must occur. • The probability of a compound event with the word and is the probability of two or more simultaneous events. • The probability of a compound event with the word or is the probability of one or more simple events occurring. • When solving a probability question with or, be sure to list outcomes that apply to both events only one time.
4	**On a Hot Streak** Determining Compound Probability		1	Students learn that a compound event combines two or more events using the word and or or. They identify the outcomes that make up an and or or event and calculate the probability. Students rewrite a probability question with an or statement as the sum of each event's probability without counting an outcome twice when it pertains to both events. They apply their understanding in a game context.	• A compound event combines two or more events, using the word *and* or the word *or*. • An and statement means that both outcomes must occur. An or statement means that that one, the other, or both outcomes must occur. • The probability of a compound event with the word and is the probability of two or more simultaneous events. • The probability of a compound event with the word or is the probability of one or more simple events occurring. • When solving a probability question with or, be sure to list outcomes that apply to both events only one time.
Learning Individually with MATHia or Skills Practice		7.SP.8a 7.SP.8b 7.SP.8c	1	**MATHia Unit:** Compound Probability **MATHia Workspaces:** Introduction to Compound Events / Determining Compound Probabilities Using Data Tables / Calculating Compound Probabilities / Determining Compound Probabilities Using Tree Diagrams / Simulating Compound Events	

Topic 3: Drawing Inferences

In this topic, students explore the second component of the statistical process: data collection. They learn about samples, populations, censuses, parameters, and statistics. Students discuss the importance of representative samples, including random samples, to generalize the samples' populations. Students collect random and non-random samples using hands-on tools and simulation strategies and then use proportional reasoning to estimate the parameter of interest. They compute percent error and conclude that samples' statistics are more likely to represent the population's parameter when the sample is random.

Standards: 7.SP.1, 7.SP.2, 7.SP.3, 7.SP.4 **Pacing:** 9 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**We Want to Hear From You!** Collecting Random Samples	7.SP.1	2	Students summarize the components of the statistical process and formulate a statistical question. They learn that they can use a statistic from a sample to predict a parameter for a population. They see the importance of having a random sample and identify whether given methods create a random sample. They select a random sample and explain how to use a random number generator to create one.	• A population is the entire set of items from which you can select data. A census is the data collected from a population. • A sample is a selection from a population. A statistic is the data collected from the sample. • A parameter is a characteristic used to describe a population. You can use a statistic from a sample to predict a parameter for a population. • A sample should represent the population. • In a random sample, every member of the population has the same chance of being selected. • You can use a random number generator to select a random sample.
2	**Tiles, Gumballs, and Pumpkins** Using Random Samples to Draw Inferences	7.SP.1 7.SP.2	2	Students use sampling to solve real-world problems. They calculate the area of sample squares in a diagram, display the class results in a dot plot, and scale their sample data for the population. Students consider the variation in the approximated areas for multiple samples. They then design and implement a strategy to predict the number of pumpkins in a field.	• Compared to any other sampling method, random sampling is more likely to generate statistics that accurately predict a population's parameter. • You can use statistics from multiple random samples and data analysis, including proportional reasoning, to estimate a population's parameter.
3	**Spicy or Dark?** Comparing Two Populations	7.SP.3	2	Students compare two sets of data in real-world contexts. They display the data in dot plots and calculate the measures of center and spread. For symmetric data, students calculate mean and MAD, and for skewed data, they calculate the median and IQR. Students make a recommendation based on the results.	• You can compare data sets represented as dot plots. • When comparing data sets, consider the measures of center and spread. The measures of center are the mean and median. The measures of spread are the MAD and IQR. • When you compare two symmetric data sets, use their means and MADs. • When you compare two skewed data sets, use their medians and IQRs. • A smaller MAD or IQR identifies more consistent data.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Finding Your Spot to Live** Using Random Samples from Two Populations to Draw Conclusions	7.SP.3 7.SP.4	2	Students analyze and interpret data to make decisions or validate claims. They collect a random sample from provided data, create a dot plot, calculate the measures of center and spread, and interpret the data regarding radio listenership. Students also consider data related to cities' temperatures, interpret histograms and box-and-whisker plots related to gallery visits, and analyze rainfall data.	• For some real-world problems, you need to compare the center and spread of two data sets. • To compare the center and spread of data sets, you can calculate the mean, median, MAD, and IQR. • You can identify information about the center and spread of data in dot plots, histograms, and box-and-whisker plots without making any calculations. • When two data sets have close centers and large spreads, the data sets are likely to overlap, reducing the confidence in a decision based on the data.
Learning Individually with MATHia or Skills Practice		7.SP.1 7.SP.3 7.SP.4	1	**MATHia Unit:** Drawing Inferences **MATHia Workspace:** Using Statistics to Draw Inferences About a Population **MATHia Unit:** Comparing Two Populations **MATHia Workspaces:** Comparing Characteristics of Data Displays / Comparing Populations Using Data Displays / Using Random Samples to Compare Populations	

CARNEGIE LEARNING

5 Constructing and Measuring

Pacing: 20 Sessions

Topic 1: Angles and Triangles

In this topic, students learn about formal constructions. They use patty paper and a compass and straightedge to duplicate segments and angles.

Standards: 7.G.2, 7.G.5 **Pacing:** 9 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Here's Lookin' at Euclid** Geometric Constructions	7.G.2	2	Students distinguish measuring tools from construction tools as they differentiate the concepts of sketch and construct. They learn how to correctly draw, sketch, and name each of the essential building blocks of geometry. Students use a compass to construct circles and arcs. They then duplicate line segments and angles using only construction tools.	• Geometric figures created by sketching are less exact than figures created by construction with a compass and straightedge. • A point, line, and plane are the essential building blocks of geometry and the foundation to create all other geometric figures. • An arc is a set of points that are the same distance from a circle's center. You can use arcs to duplicate line segments and angles.
2	**Special Delivery** Special Angle Relationships	7.G.5	2	Students use protractors and patty paper to explore special angle pairs formed when two lines intersect. They calculate the supplement and complement of angles and classify adjacent angles, linear pairs, and vertical angles. Students then use these special angle pairs in multi-step problems to write and solve equations for unknown angles.	• Two angles are supplementary when the sum of their angle measures is equal to 180°. • Two angles are complementary when the sum of their angle measures is equal to 90°. • Two lines, line segments, or rays are perpendicular when they intersect to form 90° angles. • Adjacent angles are two angles that share a common vertex and share a common side. • A linear pair of angles is composed of two adjacent angles whose non-common sides form a line. Linear pairs are supplementary. • Vertical angles are two non-adjacent angles formed by two intersecting lines. Vertical angles are congruent.
3	**Consider Every Side** Constructing Triangles Given Sides	7.G.2	2	Students use patty paper, pasta, and construction tools to investigate triangles. They determine the conditions needed to produce unique triangles, more than one triangle, or no triangles from two or three given side lengths. Students discover the Triangle Inequality Theorem.	• Constructing a triangle given the length of two sides does not result in a unique triangle. • Constructing a triangle given the length of three segments, such that the sum of two segment lengths is greater than the third length, results in a unique triangle. • The sum of the lengths of any two sides of a triangle is greater than the third side's length.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**Unique or Not?** Constructing Triangles Given Angles	7.G.2	2	Students use patty paper and construction tools to investigate triangles. They determine the conditions needed to produce unique triangles, more than one triangle, or no triangles when given at least one angle.	• Constructing a triangle given the measure of three angles does not result in a unique triangle. • Constructing a triangle given the measure of two angles and one side length does not result in a unique triangle. • An included angle is the angle whose sides consist of the specified sides of a triangle. • An included side is the side between two specified angles of a triangle. • Constructing a triangle given the length of two sides and the measure of the included angle results in a unique triangle. • Constructing a triangle given the measure of two angles and the included side length results in a unique triangle.
Learning Individually with MATHia or Skills Practice		7.G.5	1	**MATHia Unit:** Special Angle Relationships **MATHia Workspaces:** Calculating Angles / Exploring Angle Relationships / Solving for Angle Measures	

Topic 2: Three-Dimensional Figures

In this topic, using clay, students explore cross-sections of right rectangular prisms and pyramids. These activities build their spatial sense and visualization abilities and help their see the additional connections between two- and three-dimensional objects. Next, they use nets of familiar right rectangular prisms and pyramids to discover that the volume of a pyramid is one-third the volume of the prism with the same base and height.

Standards: 7.G.3, 7.G.6 **Pacing:** 11 Sessions

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
1	**Solid Surgery** Cross-Sections of Rectangular Prisms and Pyramids	7.G.3	3	Students model and describe two-dimensional cross-sections that result from slicing a cube, a right rectangular pyramid, and a rectangular prism that is not a cube. They compare the two-dimensional cross-sections of right rectangular pyramids and right rectangular prisms.	• The two-dimensional figure formed by intersecting a plane and a solid is a cross-section of the solid. • A right rectangular prism has bases and lateral faces that are rectangles. • A right rectangular pyramid has a rectangular base and four triangular lateral faces, with the height perpendicular to the base. • The possible cross-sections formed by slicing any right rectangular prism with a plane are a square, a rectangle that is not a square, a triangle, a pentagon, a hexagon, and a parallelogram that is not a rectangle. • The possible cross-sections formed by slicing any right rectangular pyramid by a plane are a triangle, a rectangle that is not a square, and a trapezoid.
2	**Hey, Mister, Got Some Bird Seed?** Volume of Pyramids	7.G.6	2	Students determine the volume of pyramids. They determine how volume is affected when dimensions are doubled or tripled. Students then solve mathematical and real-world problems involving volumes of pyramids and objects composed of solid figures.	• A pyramid is a polyhedron with one base. It has the same number of triangular faces as there are sides of the base. • You call the triangular faces of a pyramid the lateral faces. • A rectangular pyramid is a pyramid that has a rectangle as its base. • A triangular pyramid is a pyramid that has a triangle as its base.
3	**The Sound of Surface Area** Surface Area of Pyramids	7.G.6	2	Students compare and contrast the surface areas of geometric solids. They apply volume and surface area concepts to solve real-world and mathematical problems involving surface area of pyramids and composite solids.	• A prism is a polyhedron with two parallel and congruent faces called bases. The lateral faces are parallelograms. • A triangular prism is a prism that has triangles as its bases. • A pyramid is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The lateral faces are triangles. • A rectangular pyramid has a rectangle as its base. • A triangular pyramid has a triangle as its base.

Lesson	Title / Subtitle	Standards	Pacing*	Lesson Summary	Essential Ideas
4	**More Than Four Sides of the Story** Volume and Surface Area of Prisms and Pyramids	7.G.6	2	Students calculate the volumes and surface areas of prisms and pyramids with regular polygon bases. They develop a strategy and calculate the areas of regular polygons. Students then extend their understanding to calculate the surface areas and volumes of composite solids. They solve real-world and mathematical problems involving volume and surface area of two- and three-dimensional objects.	• The formula for the volume of any prism is $V = Bh$, where B is the base area and h is the height. • The formula for the volume of any pyramid is $V = \frac{1}{3}Bh$, where B is the base area and h is the height. • The surface area of any geometric solid is the sum of the areas of the surfaces of the solid. • A regular polygon has congruent sides and congruent angles. • You can decompose a regular n-gon into n congruent triangles. • To calculate the area of the n-gon, determine the area of one of the n congruent triangles of a regular n-gon and multiply by n.
	Learning Individually with MATHia or Skills Practice	7.G.3 7.G.6	2	**MATHia Unit:** Cross-Sections **MATHia Workspace:** Visualizing Cross-Sections of Three-Dimensional Shapes **MATHia Unit:** Volume of Prisms and Pyramids **MATHia Workspaces:** Calculating Volume of Right Prisms / Understanding Volume Formulas for Right Prisms / Using Volume of Right Prisms / Relating Volumes of Prisms and Pyramids / Calculating Volume of Pyramids **MATHia Unit:** Surface Area of Pyramids and Prisms **MATHia Workspaces:** Volume and Surface Area of Prisms and Pyramids	

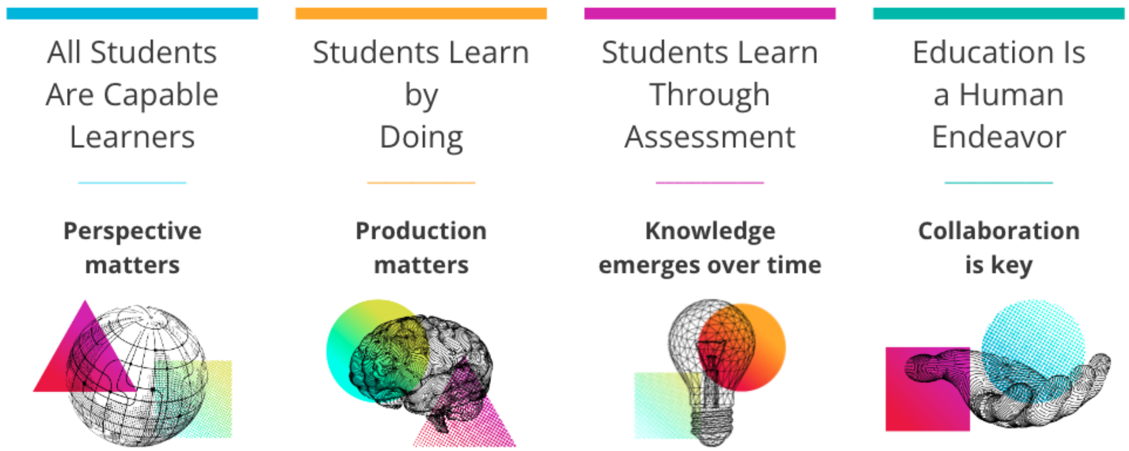
Total Sessions: 142

Learning Together: 103
Learning Individually : 39

Supports of Diversity, Equity and Inclusion

Please provide any information relative to supporting culturally responsive instruction, multi-language learners, and students with disabilities

At Carnegie Learning, we aim to make math accessible to every student, regardless of background, by delivering culturally responsive and racially diverse instructional materials. Resources follow best practices around equitable teaching and learning, classroom discourse, building relationships, collaborative learning, and more. Our guiding principles support development in these equitable practices.



Problems within Carnegie Learning's Math Solutions are written to reflect multiculturalism and include real-world scenarios and locations. Using proper names that reflect diverse cultures and situations found throughout rural and urban United States reduces linguistic and cultural bias. We regularly evaluate our resources through external partnerships for various forms of bias, microaggression, etc. All student characters within the instructional resources represent intelligent, curious learners with various interests.

Throughout the Teacher's Implementation Guide, interleaved notes on lesson pages provide teachers with point-of-use reminders to support language development, productive skills, and student interactions. The teacher materials include Additional Facilitation Notes consisting of differentiation strategies, common student misconceptions, and suggestions to extend certain activities.

Student Look-Fors

Appreciating the perspective of others and empathizing with their ideas are key elements of social awareness.

Continually encourage students to appreciate diversity in perspectives, backgrounds, and cultures as they work together during the year.