## Eureka Math ${ }^{2}$ Level 4 Correlation to <br> Connecticut Model Curriculum

| Level 4: Fractional Units |  |  |  |
| :---: | :---: | :---: | :---: |
| Model Unit Name | Model Unit Standards | Lessons | Pacing <br> Lessons that address concepts in more than one unit are only counted once. |
| Understanding and Using Place Value to Multiply and Divide | 4.NBT.A. 1 | Module 1: Place Value Concepts for Addition and Subtraction <br> Topic B: Place Value and Comparison Within 1,000,000 <br> Lesson 6: Demonstrate that a digit represents 10 times the value of what it represents in the place to its right. | 45 days |
|  | 4.NBT.A. 2 | Module 1: Place Value Concepts for Addition and Subtraction <br> Topic B: Place Value and Comparison Within 1,000,000 <br> Lesson 5: Organize, count, and represent a collection of objects. <br> Lesson 7: Write numbers to $1,000,000$ in unit form and expanded form by using place value structure. <br> Lesson 8: Write numbers to 1,000,000 in standard form and word form. <br> Lesson 9: Compare numbers within 1,000,000 by using >, $=$, and <. <br> Module 1: Place Value Concepts for Addition and Subtraction <br> Topic C: Rounding Multi-Digit Whole Numbers <br> Lesson 10: Name numbers by using place value understanding. <br> Lesson 11: Find 1, 10, and 100 thousand more than and less than a given number. |  |




| Understanding and Using Place Value to Multiply and Divide (cont.) |  | Module 3: Multiplication and Division of Multi-Digit Numbers <br> Topic F: Remainders, Estimating, and Problem Solving <br> Lesson 21: Find whole-number quotients and remainders. <br> Lesson 22: Represent, estimate, and solve division word problems. |  |
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| Factors and Multiples | 4.OA.A. 1 | Module 1: Place Value Concepts for Addition and Subtraction <br> Topic A: Multiplication as Multiplicative Comparison <br> Lesson 1: Interpret multiplication as multiplicative comparison. <br> Lesson 2: Solve multiplicative comparison problems with unknowns in various positions. <br> Lesson 3: Describe relationships between measurements by using multiplicative comparison. <br> Lesson 4: Represent the composition of larger units of money by using multiplicative comparison. <br> Module 1: Place Value Concepts for Addition and Subtraction <br> Topic B: Place Value and Comparison Within 1,000,000 <br> Lesson 6: Demonstrate that a digit represents 10 times the value of what it represents in the place to its right. | 10 days |
|  | 4.OA.B. 4 | Module 2: Place Value Concepts for Multiplication and Division <br> Topic E: Factors and Multiples <br> Lesson 21: Find factor pairs for numbers up to 100 and use factors to identify numbers as prime or composite. <br> Lesson 22: Use division and the associative property of multiplication to find factors. <br> Lesson 23: Determine whether a whole number is a multiple of another number. <br> Lesson 24: Recognize that a number is a multiple of each of its factors. <br> Lesson 25: Explore properties of prime and composite numbers up to 100 by using multiples. |  |
|  | 4.OA.C. 5 | Module 2: Place Value Concepts for Multiplication and Division <br> Topic E: Factors and Multiples <br> Lesson 26: Use relationships within a pattern to find an unknown term in the sequence. |  |


| Multi-Digit Whole Number Computation | 4.NBT.B. 4 | Module 1: Place Value Concepts for Addition and Subtraction <br> Topic D: Multi-Digit Whole Number Addition and Subtraction <br> Lesson 16: Add by using the standard algorithm. <br> Lesson 17: Solve multi-step addition word problems by using the standard algorithm. <br> Lesson 18: Subtract by using the standard algorithm, decomposing larger units once. <br> Lesson 19: Subtract by using the standard algorithm, decomposing larger units up to 3 times. <br> Lesson 20: Subtract by using the standard algorithm, decomposing larger units multiple times. <br> Lesson 21: Solve two-step word problems by using addition and subtraction. <br> Lesson 22: Solve multi-step word problems by using addition and subtraction. | 12 days |
| :---: | :---: | :---: | :---: |
|  | 4.OA.A. 2 | Module 1: Place Value Concepts for Addition and Subtraction <br> Topic A: Multiplication as Multiplicative Comparison <br> Lesson 1: Interpret multiplication as multiplicative comparison. <br> Lesson 2: Solve multiplicative comparison problems with unknowns in various positions. <br> Lesson 3: Describe relationships between measurements by using multiplicative comparison. <br> Lesson 4: Represent the composition of larger units of money by using multiplicative comparison. |  |
|  |  | Module 2: Place Value Concepts for Multiplication and Division <br> Topic B: Multiplication of Tens and Ones by One-Digit Numbers Lesson 9: Solve multiplication word problems. <br> Module 2: Place Value Concepts for Multiplication and Division <br> Topic D: Problem Solving with Measurement Lesson 20: Solve word problems involving additive and multiplicative comparisons. |  |
|  | 4.OA.A. 3 | Module 1: Place Value Concepts for Addition and Subtraction <br> Topic D: Multi-Digit Whole Number Addition and Subtraction <br> Lesson 16: Add by using the standard algorithm. <br> Lesson 17: Solve multi-step addition word problems by using the standard algorithm. <br> Lesson 21: Solve two-step word problems by using addition and subtraction. <br> Lesson 22: Solve multi-step word problems by using addition and subtraction. |  |


| Multi-Digit Whole Number Computation (cont.) |  | Module 1: Place Value Concepts for Addition and Subtraction <br> Topic E: Metric Measurement Conversion Tables <br> Lesson 23: Express metric measurements of length in terms of smaller units. <br> Lesson 24: Express metric measurements of mass and liquid volume in terms of smaller units. <br> Module 3: Multiplication and Division of Multi-Digit Numbers <br> Topic F: Remainders, Estimating, and Problem Solving <br> Lesson 21: Find whole-number quotients and remainders. <br> Lesson 22: Represent, estimate, and solve division word problems. <br> Lesson 23: Solve multi-step word problems and interpret remainders. <br> Lesson 24: Solve multi-step word problems and assess the reasonableness of solutions. |  |
| :---: | :---: | :---: | :---: |
| Comparing Fractions and Understanding Decimal Notation | 4.NF.A. 1 | Module 4: Foundations for Fraction Operations <br> Topic B: Equivalent Fractions <br> Lesson 8: Generate equivalent fractions with smaller units for unit fractions. <br> Lesson 9: Generate equivalent fractions with smaller units for non-unit fractions. <br> Lesson 10: Generate equivalent fractions with larger units. <br> Lesson 11: Represent equivalent fractions by using tape diagrams, number lines, and multiplication or division. <br> Lesson 12: Generate equivalent fractions for fractions greater than 1 and generate equivalent mixed numbers. | 24 days |
|  | 4.NF.A. 2 | Module 4: Foundations for Fraction Operations <br> Topic C: Compare Fractions <br> Lesson 13: Compare fractions by using the benchmarks $0, \frac{1}{2}$, and 1 . <br> Lesson 14: Compare fractions with related denominators. <br> Lesson 15: Compare fractions with related numerators. <br> Lesson 16: Generate a common numerator or denominator to compare fractions. <br> Lesson 17: Apply fraction comparison strategies to compare fractions greater than 1. |  |
|  | 4.NF.C. 5 | Module 5: Place Value Concepts for Decimal Fractions <br> Topic B: Tenths and Hundredths <br> Lesson 5: Decompose 1 one and express hundredths in fraction form and decimal form. <br> Lesson 6: Represent hundredths as a place value unit. <br> Lesson 7: Write mixed numbers in decimal form with hundredths. <br> Lesson 8: Represent decimal numbers in expanded form. |  |



| Building Understanding of Addition, Subtraction and Multiplication of Fractions (cont.) |  | Module 4: Foundations for Fraction Operations <br> Topic D: Add and Subtract Fractions <br> Lesson 18: Estimate sums and differences of fractions by using benchmarks. <br> Lesson 19: Add and subtract fractions with like units. <br> Lesson 20: Subtract a fraction from a whole number. <br> Lesson 21: Solve addition and subtraction word problems and estimate the reasonableness of the answers. <br> Lesson 22: Add two fractions with related units. (Optional) <br> Module 4: Foundations for Fraction Operations <br> Topic E: Add and Subtract Mixed Numbers <br> Lesson 23: Add a fraction to a mixed number. <br> Lesson 24: Add a mixed number to a mixed number. <br> Lesson 25: Subtract a fraction from a mixed number, part 1. <br> Lesson 26: Subtract a fraction from a mixed number, part 2. <br> Lesson 27: Subtract a mixed number from a mixed number. <br> Lesson 28: Represent and solve word problems with mixed numbers by using drawings and equations. |  |
| :---: | :---: | :---: | :---: |
|  | 4.NF.B. 4 | Module 4: Foundations for Fraction Operations <br> Topic F: Repeated Addition of Fractions as Multiplication <br> Lesson 31: Decompose non-unit fractions into a product of a whole number and a unit fraction. <br> Lesson 32: Multiply a fraction by a whole number by using the associative property. <br> Lesson 33: Solve word problems involving multiplication of a fraction by a whole number. <br> Lesson 34: Multiply a mixed number by a whole number by using the distributive property. |  |
| Solving Problems Involving Measurement and Data | $\begin{aligned} & \text { 4.MD.A. } 1 \\ & \text { 4.MD.A. } 2 \end{aligned}$ | Module 1: Place Value Concepts for Addition and Subtraction <br> Topic E: Metric Measurement Conversion Tables <br> Lesson 23: Express metric measurements of length in terms of smaller units. <br> Lesson 24: Express metric measurements of mass and liquid volume in terms of smaller units. | 11 days |



| Solving Problems Involving Measurement and Data (cont.) |  | Module 2: Place Value Concepts for Multiplication and Division <br> Topic B: Multiplication of Tens and Ones by One-Digit Numbers <br> Lesson 7: Multiply by using an area model and the distributive property. <br> Module 2: Place Value Concepts for Multiplication and Division <br> Topic D: Problem Solving with Measurement <br> Lesson 18: Investigate and use formulas for the perimeter of a rectangle. <br> Lesson 19: Apply area and perimeter formulas to solve problems. <br> Lesson 20: Solve word problems involving additive and multiplicative comparisons. |  |
| :---: | :---: | :---: | :---: |
|  | 4.MD.B. 4 | Module 4: Foundations for Fraction Operations <br> Topic E: Add and Subtract Mixed Numbers Lesson 29: Solve problems by using data from a line plot. Lesson 30: Represent data on a line plot. |  |
| Exploring Angles and Angle Measurement | 4.MD.C. 5 | Module 6: Angle Measurements and Plane Figures <br> Topic B: Angle Measurement <br> Lesson 7: Explore angles as fractional turns through a circle. <br> Lesson 8: Use a circular protractor to recognize a $1^{\circ}$ angle as a turn through $\frac{1}{360}$ of a circle. <br> Lesson 9: Identify and measure angles as turns and recognize them in various contexts. <br> Lesson 10: Use $180^{\circ}$ protractors to measure angles. <br> Lesson 11: Estimate and measure angles with a $180^{\circ}$ protractor. | 10 days |
|  | 4.MD.C. 6 | Module 6: Angle Measurements and Plane Figures <br> Topic B: Angle Measurement <br> Lesson 8: Use a circular protractor to recognize a $1^{\circ}$ angle as a turn through $\frac{1}{360}$ of a circle. <br> Lesson 10: Use $180^{\circ}$ protractors to measure angles. <br> Lesson 11: Estimate and measure angles with a $180^{\circ}$ protractor. <br> Lesson 12: Use a protractor to draw angles up to $180^{\circ}$. |  |
|  | 4.MD.C. 7 | Module 6: Angle Measurements and Plane Figures <br> Topic C: Determine Unknown Angle Measures <br> Lesson 13: Decompose angles by using pattern blocks. <br> Lesson 14: Find unknown angle measures within right and straight angles. <br> Lesson 15: Find unknown angle measures within a decomposed angle of up to $180^{\circ}$. <br> Lesson 16: Find unknown angle measures around a point. |  |
| $\checkmark$ |  | $\checkmark$ | $\checkmark$ |


| Understanding Properties of Two-Dimensional Figures | $\begin{array}{\|l\|} \hline \text { 4.G.A. } 1 \\ \text { 4.G.A. } \end{array}$ | Module 6: Angle Measurements and Plane Figures <br> Topic A: Lines and Angles <br> Lesson 1: Identify and draw points, lines, line segments, rays, and angles. <br> Lesson 2: Identify right, acute, obtuse, and straight angles. <br> Lesson 3: Draw right, acute, obtuse, and straight angles. <br> Lesson 4: Identify, define, and draw perpendicular lines. <br> Lesson 5: Identify, define, and draw parallel lines. <br> Lesson 6: Relate geometric figures to a real-world context. <br> Module 6: Angle Measurements and Plane Figures <br> Topic B: Angle Measurement <br> Lesson 10: Use $180^{\circ}$ protractors to measure angles. <br> Lesson 11: Estimate and measure angles with a $180^{\circ}$ protractor. <br> Lesson 12: Use a protractor to draw angles up to $180^{\circ}$. <br> Module 6: Angle Measurements and Plane Figures <br> Topic D: Two-Dimensional Figures and Symmetry <br> Lesson 18: Analyze and classify triangles based on side length, angle measures, or both. <br> Lesson 19: Construct and classify triangles based on given attributes. <br> Lesson 20: Sort polygons based on a given rule. | 10 days |
| :---: | :---: | :---: | :---: |
|  | 4.G.A. 3 | Module 6: Angle Measurements and Plane Figures <br> Topic D: Two-Dimensional Figures and Symmetry Lesson 17: Recognize, identify, and draw lines of symmetry. |  |

## Eureka Math ${ }^{2}$ Scope and Sequence: Year at a Glance <br> Level 4: Fractional Units

If a district uses this resource to implement the state model curriculum for grade 4, the following scope and sequence should be followed to ensure alignment and attention to the progressions of mathematics.

| Module 1 <br> Place Value Concepts for Addition and Subtraction | Module 2 <br> Place Value Concepts for Multiplication and Division | Module 3 <br> Multiplication and Division of Multi-Digit Numbers | Module 4 <br> Foundations for Fraction Operations | Module 5 <br> Place Value Concepts for Decimal Fractions | Module 6 <br> Angle Measurements and Plane Figures |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Topic A: Multiplication as Multiplicative Comparison <br> Lesson 1: Interpret multiplication as multiplicative comparison. <br> 4.OA.A.1, 4.OA.A.2, MP7, <br> 4.Mod1.AD1, 4.Mod1.AD2, <br> 4.Mod1.AD3 <br> Lesson 2: Solve multiplicative comparison problems with unknowns in various positions. <br> 4.OA.A.1, 4.OA.A.2, MP7, <br> 4.Mod1.AD1, 4.Mod1.AD2, <br> 4.Mod1.AD3 <br> Lesson 3: Describe relationships between measurements by using multiplicative comparison. <br> 4.OA.A.1, 4.OA.A.2, MP2, <br> 4.Mod1.AD1, 4.Mod1.AD2, <br> 4.Mod1.AD3 | Topic A: Compose and Decompose Units of Ten <br> Lesson 1: Multiply multiples of 10 by one-digit numbers by using the associative property of multiplication. <br> 4.NBT.B.5, MP7, 4.Mod2.AD6 <br> Lesson 2: Divide two- and threedigit multiples of 10 by one-digit numbers. <br> 4.NBT.B.6, MP2, 4.Mod2.AD7 <br> Lesson 3: Investigate and use a formula for the area of a rectangle. <br> 4.MD.A.3, MP1, 4.Mod2.AD11 <br> Topic B: Multiplication of Tens and Ones by One-Digit Numbers <br> Lesson 4: Multiply by using familiar strategies. <br> 4.NBT.B.5, MP5, 4.Mod2.AD6 | Topic A: Multiplication and Division of Multiples of Tens, Hundreds, and Thousands <br> Lesson 1: Divide multiples of 100 and 1000 . <br> 4.NBT.B.6, MP8, 4.Mod3.AD3 <br> Lesson 2: Multiply by multiples of 100 and 1000. <br> 4.NBT.B.5, MP7, 4.Mod3.AD2 <br> Lesson 3: Multiply a two-digit multiple of 10 by a two-digit multiple of 10 . <br> 4.NBT.B.5, MP3, 4.Mod3.AD2 <br> Topic B: Division of Thousands, Hundreds, Tens, and Ones <br> Lesson 4: Apply place value strategies to divide hundreds, tens, and ones. <br> 4.NBT.B.6, MP1, 4.Mod3.AD3 | Topic A: Fraction Decomposition and Equivalence <br> Lesson 1: Decompose whole numbers into a sum of unit fractions. <br> NF.B.3.a, 4.NF.B.3.b, MP7, 4.Mod4.AD4 <br> Lesson 2: Decompose fractions into a sum of unit fractions. <br> 4.NF.B.3.a, 4.NF.B.3.b, MP2, <br> 4.Mod4.AD4 <br> Lesson 3: Decompose fractions into a sum of fractions. <br> 4.NF.B.3.a, 4.NF.B.3.b, MP6, 4.Mod4.AD4 <br> Lesson 4: Represent fractions by using various fraction models. 4.NF.B.3.a, 4.NF.B.3.b, MP4, 4.Mod4.AD4 | Topic A: Exploration of Tenths <br> Lesson 1: Organize, count, and represent a collection of money. 4.NF.C.6, MP5, 4.Mod5.AD3 <br> Lesson 2: Decompose 1 one and express tenths in fraction form and decimal form. <br> 4.NF.C.6, MP8, 4.Mod5.AD3 <br> Lesson 3: Represent tenths as a place value unit. <br> 4.NF.C.6, MP7, 4.Mod5.AD3 <br> Lesson 4: Write mixed numbers in decimal form with tenths. <br> 4.NF.C.6, MP6, 4.Mod5.AD3 | Topic A: Lines and Angles <br> Lesson 1: Identify and draw points, lines, line segments, rays, and angles. <br> 4.G.A.1, MP6, 4.Mod6.AD4, 4.Mod6.AD5 <br> Lesson 2: Identify right, acute, obtuse, and straight angles. <br> 4.G.A.1, MP7, 4.Mod6.AD4, <br> 4.Mod6.AD5 <br> Lesson 3: Draw right, acute, obtuse, and straight angles. <br> 4.G.A.1, MP6, 4.Mod6.AD4, <br> 4.Mod6.AD5 <br> Lesson 4: Identify, define, and draw perpendicular lines. <br> 4.G.A.1, MP6, 4.Mod6.AD4, <br> 4.Mod6.AD5 <br> Lesson 5: Identify, define, and draw parallel lines. <br> 4.G.A.1, MP6, 4.Mod6.AD4, <br> 4.Mod6.AD5 |

## Lesson 4: Represent the

 composition of larger units of money by using multiplicative comparison.4.OA.A.1, 4.OA.A.2, MP7
4.Mod1.AD1, 4.Mod1.AD2, 4.Mod1.AD3

Topic B: Place Value and Comparison within 1,000, 000

Lesson 5: Organize, count, and represent a collection of objects. 4.NBT.A.2, MP5, 4.Mod1.AD7

Lesson 6: Demonstrate that a digit represents 10 times the value of what it represents in the place to its right.
4.OA.A.1, 4.NBT.A.1, MP8, 4.Mod1.AD1, 4.Mod1.AD2, 4.Mod1.AD6

Lesson 7: Write numbers to 1,000,000 in unit form and expanded form by using place value structure.
4.NBT.A.2, MP7, 4.Mod1.AD7

Lesson 8: Write numbers to 1,000,000 in standard form and word form.
4.NBT.A.2, MP3, 4.Mod1.AD7

Lesson 9: Compare numbers within $1,000,000$ by using $>$, $=$, and $<$ 4.NBT.A.2, MP6, 4.Mod1.AD8

Lesson 5: Multiply by using place value strategies and the distributive property.
4.NBT.B.5, MP7, 4.Mod2.AD6

Lesson 6: Multiply with regrouping by using place value strategies and the distributive property. 4.NBT.B.5, MP6, 4.Mod2.AD6

Lesson 7: Multiply by using an area model and the distributive property. 4.NBT.B.5, 4.MD.A.3, MP7
4.Mod2.AD6, 4.Mod2.AD1

Lesson 8: Multiply by applying the distributive property and write equations.
4.NBT.B.5, MP3, 4.Mod2.AD6

Lesson 9: Solve multiplication word problems.
4.OA.A.2, 4.NBT.B.5, MP5,
4.Mod2.AD1, 4.Mod2.AD6

Lesson 10: Multiply by applying simplifying strategies. (Optional) 4.NBT.B.5, MP5, 4.Mod2.AD6

Topic C: Division of Tens and Ones by One-Digit Numbers

Lesson 11: Divide by using familiar strategies.
4.NBT.B.6, MP2, 4.Mod2.AD7

Lesson 12: Divide two-digit numbers by one-digit numbers by using an area model.
4.NBT.B.6, MP7, 4.Mod2.AD7

Lesson 5: Apply place value strategies to divide thousands, hundreds, tens, and ones 4.NBT.B.6, MP5, 4.Mod3.AD3

Lesson 6: Connect pictorial representations of division to long division.
4.NBT.B.6, MP6, 4.Mod3.AD3

Lesson 7: Represent division by using partial quotients 4.NBT.B.6, MP8, 4.Mod3.AD3

Lesson 8: Choose and apply a method to divide multi-digit numbers.
4.NBT.B.6, MP6, 4.Mod3.AD3

Topic C: Multiplication of up to Four-Digit Numbers by One-Digit Numbers

Lesson 9: Apply place value strategies to multiply three-digit numbers by one-digit numbers. 4.NBT.B.5, MP5, 4.Mod3.AD2

Lesson 10: Apply place value strategies to multiply four-digit numbers by one-digit numbers 4.NBT.B.5, MP7, 4.Mod3.AD2

Lesson 11: Represent multiplication by using partial products 4.NBT.B.5, MP8, 4.Mod3.AD2

Lesson 12: Multiply by using various recording methods in vertical form.
4.NBT.B.5, MP6, 4.Mod3.AD2

Lesson 5: Rename fractions greater than 1 as mixed numbers. 4.NF.B.3.a, 4.NF.B.3.b, MP7, 4.Mod4.AD4

Lesson 6: Rename mixed numbers as fractions greater than 1. 4.NF.B.3.a, 4.NF.B.3.b, MP5, 4.Mod4.AD4

Topic B: Equivalent Fractions

Lesson 7: Rename fractions as a sum of equivalent smaller unit fractions.
4.NF.B.3.a, 4.NF.B.3.b, MP2, 4.Mod4.AD4

Lesson 8: Generate equivalent fractions with smaller units for unit fractions.
4.NF.A.1, MP8, 4.Mod4.AD1,

## 4.Mod4.AD2

Lesson 9: Generate equivalent fractions with smaller units for nonunit fractions.
4.NF.A.1, MP7, 4.Mod4.AD1, 4.Mod4.AD2

Lesson 10: Generate equivalent fractions with larger units. 4.NF.A.1, MP6, 4.Mod4.AD1, 4.Mod4.AD2

Lesson 11: Represent equivalent fractions by using tape diagrams, number lines, and multiplication or division.
4.NF.A.1, MP8, 4.Mod4.AD1,
4.Mod4.AD2

Topic B: Tenths and Hundredths

Lesson 5: Decompose 1 one and express hundredths in fraction form and decimal form.
4.NF.C.5, 4.NF.C.6, MP2,
4.Mod5.AD1, 4.Mod5.AD

Lesson 6: Represent hundredths as a place value unit.
4.NF.C.5, 4.NF.C.6, MP3, 4.Mod5.AD1, 4.Mod5.AD3

Lesson 7: Write mixed numbers in decimal form with hundredths. 4.NF.C.5, 4.NF.C.6, MP7, 4.Mod5.AD1, 4.Mod5.AD3

Lesson 8: Represent decimal numbers in expanded form 4.NF.C.5, 4.NF.C.6, MP2, 4.Mod5.AD1, 4.Mod5.AD3

Topic C: Comparison of Decimal Numbers

Lesson 9: Compare measurements expressed as decimal numbers. 4.NF.C.7, MP2, 4.Mod5.AD4

Lesson 10: Use pictorial representations to compare decimal numbers.
4.NF.C.7, MP5, 4.Mod5.AD4

Lesson 11: Compare and order decimal numbers. 4.NF.C.7, MP3, 4.Mod5.AD4

Lesson 6: Relate geometric figures to a real-world context. 4.G.A.1, MP2, 4.Mod6.AD4, 4.Mod6.AD5

Topic B: Angle Measurement
Lesson 7: Explore angles as fractional turns through a circle. 4.MD.C.5.a, MP7, 4.Mod6.AD1

Lesson 8: Use a circular protractor to recognize a $1^{\circ}$ angle as a turn through $\frac{1}{360}$ of a circle.
4.MD.C.5, 4.MD.C.6, MP8,
4.Mod6.AD1, 4.Mod6.AD2

Lesson 9: Identify and measure angles as turns and recognize them in various contexts.
4.MD.C.5, 4.MD.C.5.a, 4.MD.C.5.b, MP2, 4.Mod6.AD1

Lesson 10: Use $180^{\circ}$ protractors to measure angles.
4.MD.C.5, 4.MD.C.5.a,
4.MD.C.5.b, 4.MD.C.6, 4.G.A.1, MP6, 4.Mod6.AD1,
4.Mod6.AD2, 4.Mod6.AD4

Lesson 11: Estimate and measure angles with a $180^{\circ}$ protractor 4.MD.C.5, 4.MD.C.5.a,
4.MD.C.5.b, 4.MD.C.6, 4.G.A.1, MP6, 4.Mod6.AD1 4.Mod6.AD2, 4.Mod6.AD4

Lesson 12: Use a protractor to draw angles up to $180^{\circ}$.
4.MD.C.6, 4.G.A.1, MP6,
4.Mod6.AD2, 4.Mod6.AD4

## Topic C: Rounding Multi <br> Digit Whole Numbers

Lesson 10: Name numbers by using place value understanding.

## 4.NBT.A.2, MP8, 4.Mod1.AD7

Lesson 11: Find 1,10 , and 100 thousand more than and less than a given number.
4.NBT.A.2, MP1, 4.Mod1.AD7

Lesson 12: Round to the nearest thousand.
4.NBT.A.3, MP6, 4.Mod1.AD9

Lesson 13: Round to the nearest ten thousand and hundred thousand.
4.NBT.A.3, MP6, 4.Mod1.AD9

Lesson 14: Round multi-digit numbers to any place. 4.NBT.A.3, MP8, 4.Mod1.AD9

Lesson 15: Apply estimation to real-world situations by using rounding.
4.OA.A.3, 4.NBT.A.3, MP3,
4.Mod1.AD4, 4.Mod1.AD9

Topic D: Multi-Digit Whole Number Addition and Subtraction

Lesson 16: Add by using the standard algorithm.
4.OA.A.3, 4.NBT.B.4, MP4,
4.Mod1.AD4, 4.Mod1.AD10

Lesson 13: Divide three-digit numbers by one-digit numbers by using an area model.
4.NBT.B.6, MP3, 4.Mod2.AD7

Lesson 14: Divide two-digit numbers by one-digit numbers by using place value strategies. 4.NBT.B.6, MP6, 4.Mod2.AD7

Lesson 15: Divide three-digit numbers by one-digit numbers by using place value strategies. 4.NBT.B.6, MP7, 4.Mod2.AD7

Lesson 16: Divide by using the break apart and distribute strategy. 4.NBT.B.6, MP1, 4.Mod2.AD7

Topic D: Problem Solving with Measurement

Lesson 17: Express measurements of length in terms of smaller units. 4.MD.A.1, 4.MD.A.2, MP8, 4.Mod2.AD8, 4.Mod2.AD9, 4.Mod2.AD10

Lesson 18: Investigate and use formulas for the perimeter of a rectangle.
4.MD.A.3, MP4, 4.Mod2.AD11

Lesson 19: Apply area and perimeter formulas to solve problems.
4.MD.A.3, MP2, 4.Mod2.AD11

Lesson 20: Solve word problems involving additive and multiplicative comparisons. 4.OA.A.2, 4.MD.A.2, 4.MD.A.3, MP1, 4.Mod2.AD1, 4.Mod2.AD9, 4.Mod2.AD10, 4 Mod2.AD11

Topic D: Multiplication of Two-Digit Numbers by TwoDigit Numbers

Lesson 13: Multiply two-digit numbers by two-digit multiples of 10. 4.NBT.B.5, MP5, 4.Mod3.AD2 Lesson 14: Apply place value strategies to multiply two-digit numbers by two-digit numbers. 4.NBT.B.5, MP2, 4.Mod3.AD2

Lesson 15: Multiply with four partial products. 4.NBT.B.5, MP6, 4.Mod3.AD2

Lesson 16: Multiply with two partia products.
4.NBT.B.5, MP7, 4.Mod3.AD2

Lesson 17: Apply the distributive property to multiply. 4.NBT.B.5, MP2, 4.Mod3.AD2

Topic E: Problem Solving with Measurement

Lesson 18: Express units of time in terms of smaller units. 4.MD.A.1, 4.MD.A.2, MP1, 4.Mod3.AD4, 4.Mod3.AD5

Lesson 19: Express customary measurements of weight in terms of smaller units.
4.MD.A.1, 4.MD.A.2, MP4, 4.Mod3.AD4, 4.Mod3.AD5

Lesson 12: Generate equivalent fractions for fractions greater than 1 and generate equivalent mixed numbers.
4.NF.A.1, MP3, 4.Mod4.AD1, 4.Mod4.AD2

Topic C: Compare Fractions
Lesson 13: Compare fractions by using the benchmarks $0, \frac{1}{2}$, and 1 . 4.NF.A.2, MP3, 4.Mod4.AD3

Lesson 14: Compare fractions with related denominators.
4.NF.A.2, MP5, 4.Mod4.AD3

Lesson 15: Compare fractions with related numerators
4.NF.A.2, MP5, 4.Mod4.AD3

Lesson 16: Generate a common numerator or denominator to compare fractions. 4.NF.A.2, MP1, 4.Mod4.AD3

Lesson 17: Apply fraction comparison strategies to compare fractions greater than 1 . 4.NF.A.2, MP7, 4.Mod4.AD3

Topic D: Add and Subtract Fractions

Lesson 18: Estimate sums and differences of fractions by using benchmarks.
4.NF.B.3.a, 4.NF.B.3.b,
4.NF.B.3.d, MP3, 4.Mod4.AD4, 4.Mod4.AD7

## Topic D: Addition of Tenths

 and HundredthsLesson 12: Apply fraction equivalence to add tenths and hundredths.
4.NF.C.5, MP1, 4.Mod5.AD2 Lesson 13: Apply fraction equivalence to add mixed numbers with tenths and hundredths. 4.NF.C.5, MP7, 4.Mod5.AD2

Lesson 14: Solve word problems with tenths and hundredths. 4.NF.C.5, 4.MD.A.2, MP4, 4.Mod5.AD2, 4.Mod5.AD5

## Topic C: Determine

 Unknown Angle MeasuresLesson 13: Decompose angles by using pattern blocks. 4.MD.C.7, MP2, 4.Mod6.AD3

Lesson 14: Find unknown angle measures within right and straight angles. 4.MD.C.7, MP7, 4.Mod6.AD3

Lesson 15: Find unknown angle measures within a decomposed angle of up to $180^{\circ}$
4.MD.C.7, MP5, 4.Mod6.AD3

Lesson 16: Find unknown angle measures around a point. 4.MD.C.7, MP1, 4.Mod6.AD3

Topic D: Two-Dimensiona Figures and Symmetry

Lesson 17: Recognize, identify, and draw lines of symmetry. 4.G.A.3, MP7, 4.Mod6.AD7

Lesson 18: Analyze and classify triangles based on side length, angle measures, or both
4.G.A.1, 4.G.A.2, MP3,
4.Mod6.AD5, 4.Mod6.AD6

Lesson 19: Construct and classify triangles based on given attributes. 4.G.A.1, 4.G.A.2, MP6, 4.Mod6.AD5, 4.Mod6.AD6

## Lesson 17: Solve multi-step

 addition word problems by using the standard algorithm
## 4.OA.A.3, 4.NBT.B.4, MP2,

## 4.Mod1.AD5, 4.Mod1.AD10

Lesson 18: Subtract by using the standard algorithm, decomposing larger units once.
4.NBT.B.4, MP6, 4.Mod1.AD10

Lesson 19: Subtract by using the standard algorithm, decomposing larger units up to 3 times.
4.NBT.B.4, MP1, 4.Mod1.AD10

Lesson 20: Subtract by using the standard algorithm, decomposing larger units multiple times.
4.NBT.B.4, MP7, 4.Mod1.AD10

Lesson 21: Solve two-step word problems by using addition and subtraction.
4.OA.A.3, 4.NBT.B.4, MP4, 4.Mod1.AD5, 4.Mod1.AD10

Lesson 22: Solve multi-step word problems by using addition and subtraction.
4.OA.A.3, 4.NBT.B.4, MP2, 4.OA.A.3, 4.NBT.B.4, MP2,
4.Mod1.AD5, 4.Mod1.AD10

Topic E: Factors and Multiples

Lesson 21: Find factor pairs for numbers up to 100 and use factors to identify numbers as prime or composite.
4.OA.B.4, MP6, 4.Mod2.AD2, 4.Mod2.AD4

Lesson 22: Use division and the associative property of multiplication to find factors. 4.OA.B.4, MP3, 4.Mod2.AD2, 4.Mod2.AD4

Lesson 23: Determine whether a whole number is a multiple of another number. 4.OA.B.4, MP7, 4.Mod2.AD3

Lesson 24: Recognize that a number is a multiple of each of its factors.
4.OA.B.4, MP8, 4.Mod2.AD3

Lesson 25: Explore properties of prime and composite numbers up to 100 by using multiples. 4.OA.B.4, MP3, 4.Mod2.AD3, 4.Mod2.AD4

Lesson 26: Use relationships within a pattern to find an unknown term in the sequence. 4.OA.C.5, MP6, 4.Mod2.AD5

Lesson 20: Express customary measurements of liquid volume in terms of smaller units. 4.MD.A.1, 4.MD.A.2, MP5, 4.Mod3.AD4, 4.Mod3.AD5

## Topic F: Remainders,

 Estimating, and Problem SolvingLesson 21: Find whole-number quotients and remainders. 4.OA.A.3, 4.NBT.B.6, MP2, 4.Mod3.AD1, 4.Mod3.AD3

Lesson 22: Represent, estimate, and solve division word problems. 4.OA.A.3, 4.NBT.B.6, MP1, 4.Mod3.AD1, 4.Mod3.AD3

Lesson 23: Solve multi-step word problems and interpret remainders. 4.OA.A.3, MP4, 4.Mod3.AD1

Lesson 24: Solve multi-step word problems and assess the reasonableness of solutions. 4.OA.A.3, MP3, 4.Mod3.AD1

Lesson 19: Add and subtract fractions with like units. 4.NF.B.3.a, 4.NF.B.3.b, MP4 4.Mod4.AD4

Lesson 20: Subtract a fraction from a whole number.
4.NF.B.3.a, 4.NF.B.3.b,
4.NF.B.3.d, 4.MD.A.2, MP1, 4.Mod4.AD4, 4.Mod4.AD7

Lesson 21: Solve addition and subtraction word problems and estimate the reasonableness of the answers.
4.NF.B.3.a, 4.NF.B.3.b, 4.NF.B.3.d, 4.MD.A.2, MP1, 4.Mod4.AD4, 4.Mod4.AD7

Lesson 22: Add two fractions with related units. (Optional) 4.NF.B.3.a, 4.NF.B.3.b, MP5, 4.Mod4.AD4

Topic E: Add and Subtract Mixed Numbers

Lesson 23: Add a fraction to a mixed number. 4.NF.B.3.c, MP7, 4.Mod4.AD5

Lesson 24: Add a mixed number to a mixed number.
4.NF.B.3.c, 4.NF.B.3.d, MP7,
4.Mod4.AD5, 4.Mod4.AD7

Lesson 25: Subtract a fraction from a mixed number, part 1.
4.NF.B.3.c, MP6, 4.Mod4.AD6

Lesson 20: Sort polygons based on a given rule.
4.G.A.1, 4.G.A.2, MP1,
4.Mod6.AD5, 4.Mod6.AD6
Topic E: Metric
Measurement Conversion
Tables

Tables
Lesson 23: Express metric
measurements of length in terms of smaller units.
4.MD.A.1, 4.MD.A.2, MP7,
4.Mod1.AD11, 4.Mod1.AD12

Lesson 24: Express metric measurements of mass and liquid volume in terms of smaller units. 4.MD.A.1, 4.MD.A.2, MP5, 4.Mod1.AD11, 4.Mod1.AD12

Lesson 26: Subtract a fraction from
a mixed number, part 2.
4.NF.B.3.c, MP7, 4.Mod4.AD6

Lesson 27: Subtract a mixed
number from a mixed number
4.NF.B.3.c, 4.NF.B.3.d, 4.MD.A.2

MP2, 4.Mod4.AD6, 4.Mod4.AD7
Lesson 28: Represent and solve
word problems with mixed
numbers by using drawings and equations.
4.NF.B.3.d, 4.MD.A.2, MP4, 4.Mod4.AD7

Lesson 29: Solve problems by
using data from a line plot. 4.MD.B.4, MP1, 4.Mod4.AD11

Lesson 30: Represent data on a line plot.
4.MD.B.4, MP6, 4.Mod4.AD11

Topic F: Repeated Addition of Fractions as Multiplication

Lesson 31: Decompose non-unit fractions into a product of a whole number and a unit fraction. 4.NF.B.4.a, MP7, 4.Mod4.AD8

Lesson 32: Multiply a fraction by a whole number by using the
associative property.
4.NF.B.4.b, MP8, 4.Mod4.AD9

Lesson 33: Solve word problems involving multiplication of a fraction by a whole number 4.NF.B.4.b, 4.NF.B.4.c, 4.MD.A.2, MP2, 4.Mod4.AD9, 4.Mod4.AD10


## Year-Long Curriculum Overview: Levels 3-5 \| STORY OF UNITS

Trimester and quarter indicators are provided as a guide for pacing A few optional lessons in each grade level are included in total number of lessons. About 30 additional days are allotted at each level for assessment and responsive teaching.


## Supports of Diversity, Equity, and Inclusion

## Providing Culturally Responsive Instruction

Eureka Math ${ }^{2}$ values the funds of knowledge that students bring into the classroom and acknowledges that deep learning happens when all students are able to leverage their diverse life experiences while learning mathematics.


One of the ways Eureka Math ${ }^{2}$ invites students into mathematics and celebrates the diversity present in every classroom is by highlighting for teachers those specific lesson moments that can be tailored to bring students' experiences from their home and communities into the classroom. For example, a strategically placed Universal Design for Learning (UDL) margin note in grade 4 module 1 lesson 4 suggests that teachers leverage life experiences by using real pennies instead of images of pennies while working with a place value chart.


Adjusting questions to make them more meaningful to students provides options for recruiting interest by personalizing and contextualizing the content to learners' lives. In grade 3 module 4 lesson 13, students reason about, represent, and solve a two-step area word problem. A Universal Design for Learning Engagement margin note encourages the teacher to adjust the existing problem context by identifying items in the school or community with the dimensions and quantity in the problem, making the context more relevant and meaningful to students.

Students' experiences from their home and communities are also leveraged through Family Math. Family Math is a letter to families that describes the major concepts in the current topic. Each letter uses words and phrases that should be familiar to the student from the lessons in the topic. It includes visual supports that students can use to explain the concepts or strategies to their family or that can help adults at home understand or unpack a concept. Family Math also includes simple and practical at-home activities to extend learning and help students see mathematics in their world.

Students are diverse, and any one classroom can have students from either an individualist frame of reference or a collectivist frame of reference. The teacher-writers of Eureka Math ${ }^{2}$ considered both frames of reference in intentionally balancing activities that build off individualism as well as collectivism.

In her book Culturally Responsive Teaching and the Brain, Zaretta Hammond references collectivism as emphasizing relationships, interdependence within a community, and cooperative learning (page 25). In Eureka Math ${ }^{2}$, a collectivist approach to learning mathematics is present in the embedded cooperative learning structures in open-middle and open-ended tasks. Specifically, the instructional routines Numbered Heads and Co-construction are rooted in students working cooperatively in groups to deepen their mathematical conceptual understanding. See grade 3 module 6 lesson 6, in which students work with a partner to create a context that could apply to an elapsed time situation in the routine Co-construction. Also, grade 5 module 3 lesson 3 features the routine Numbered Heads, in which students work in groups of three to describe how to solve one multiplication problem.

Beyond the instructional routines, Eureka Math ${ }^{2}$ leverages the power of student relationships and interdependence through frequent partner and group work. For any partner or group work referenced in the instructional materials, teachers may make use of strategic, flexible groupings that build off students' strengths, including home language. A Language Support margin note in the first lesson of every module serves to remind teachers to leverage students' cultural perspectives when strategically placing students in partners.

Hammond references individualism as emphasizing individual achievement and independence (page 25). In Eureka Math ${ }^{2}$, an individualist approach to learning mathematics may be seen in the embedded systems for independent practice in every lesson, such as Exit Tickets and Practice Sets. Additionally, the instructional routines Critique a Flawed Response and Take a Stand both start with students working on a math problem individually before engaging in student discourse. See grade 4 module 3 lesson 3, in which students engage with the Critique a Flawed Response routine by first identifying an error individually before a whole-class discussion. Also, grade 3 module 3 lesson 6 invites students to engage with the Take a Stand routine, in which students first determine how they would decompose a number before engaging in a whole-class discussion about different ways to decompose numbers.

Beyond balancing individualism and collectivism, Eureka Math ${ }^{2}$ activities and problems provide students with mirrors in which to see their own cultural perspectives reflected, as well as windows through which to view others' cultural perspectives.

Eureka Math ${ }^{2}$ is an inclusive mathematics curriculum that represents diverse doers of math. The curriculum's images, fine art, and pictures of people represent diversity through problems and exercises related to real-life experiences, perspectives, and contributions of people from various cultures, ethnicities, and gender identities. These representations affirm student identities while rejecting the stereotypes and biases that have excluded many students from mathematical learning in favor of a more robust and inclusive perspective. Representing a diverse array of doers of mathematics in the curriculum inspires all students to think of themselves as mathematicians.


For example, Eureka Math ${ }^{2}$ includes various mathematical activities that involve counting on hands or simulating a number line with one's fingers. In images throughout the curriculum, care was taken to include a variety of body types and skin tones.

The names used in word problems and for sample students in the lesson vignettes are intentionally diverse to represent the wide variety of students who use the curriculum. The names in studentfacing word problems are also designed for readability to ensure that they are not a barrier to accessing the math.

## Story of Units ${ }^{\text {® }}$

Tam, Kit, Zan, Ren, Mac, Jon, Baz, Liv, Jade, Ling, Sal, Deepa, Oka, Mia, Gabe, Pablo, Shea, Jayla, Shen, Lacy, Sasha, Yuna, Leo, Adesh, Toby

According to CAST, "individuals are engaged by information and activities that are relevant and valuable to their interests and goals." (UDL Guidelines, Engagement, Checkpoint 7.2) Eureka Math ${ }^{2}$ also leverages students' experiences, goals, and interests through Math Pasts (described below), art connections, and wordless context videos.

To honor the diverse contributions to the development of the field of mathematics, to build knowledge about our shared math history, and to empower every child to see themselves as able to do mathematics, nearly every module in Eureka Math ${ }^{2}$ includes a feature called Math Past. Each Math Past tells the history of some big ideas in the module, recounting the story of the mathematics through artifacts, discoveries, and other contributions from cultures around the world. Math Past also provides ideas about how to engage students in the history of mathematics.

Math Past counters the traditional Eurocentric perspective and celebrates the many contributions of Black, Indigenous, and People of Color communities to the history of mathematics.

For example, students explore large numbers in expanded form by first engaging with Egyptian hieroglyphics in Launch of grade 4 module 1 lesson 8. The Math Past Teacher Resource highlights the specific hieroglyphic numerals and their connection to familiar objects for Egyptians 4,000 years ago. Studying the mathematics used by people thousands of years ago helps students view mathematics as a worthwhile and useful subject that stands the test of time.

In a similar vein, Eureka Math ${ }^{2}$ connects works of fine art to the standards of each grade level. Each Teach book opens with a stunning work of fine art that has a connection to the math learned in the grade. There is also a wide variety of additional pieces of art embedded in each grade's lessons. For example:

- Land (the culminating section of each day's lesson) in grade 3 module 1 lesson 1 relates Diego Rivera's painting Flower Vendor to making equal groups to count the total number of objects.
- Land in grade 4 module 2 lesson 19 connects the painting Composition with Large Red Plane, Yellow, Black, Gray and Blue by Piet Mondrian to the formulas for area and perimeter of rectangles.


Land (10)

you wonder?
Guide students to think about the painting in terms of their experience with the counting
collection. Tell the class that the children in the painting are making bundles of flowers for the woman to carry.

- Land in grade 5 module 3 lesson 7 connects the painting Thirteen Rectangles by Wassily Kandinsky to multiplying fractions less than one by unit fractions.

Additionally, Eureka Math ${ }^{2}$ lessons include more than 190 videos. These wordless context-building videos highlight how we use math to solve problems in our everyday lives and make sense of the world around us. Three types of highly engaging videos may be found in the curriculum: character animation, collage animation, and live action.

Students can identify with the diverse set of actors and characters in the videos, which helps them visualize how math is part of their own lives. Through these videos, students will more readily realize that math surrounds

Land 80
Dobrief 5 min
Objective: Multiply tractions less than 1 by unit fractions pictorially-

 them and that they, too, can engage in mathematical pursuits. The videos allow students to see themselves in the math problems they encounter, which lowers the barrier to engagement and makes the math classroom a more welcoming place.

Wordless videos in lessons serve many other purposes as well, such as the following:

- They make the context for a given problem come alive, putting all students on the same footing by activating or building the requisite background knowledge.
- They remove any language and reading barriers to the written word problem.
- They raise the accessibility of mathematics through accurate and inclusive representation.
- They show the many ways in which we interact with math in the world around us and how these interactions spark curiosity and joy.
- They help students see the delight and wonder associated with being a mathematician.
- They create excitement and buzz in the classroom about the content of the new word problem
- They invite students to tell the story of the math problem, to notice, to wonder, and to drive the discussion.


## Examples include:

- Grade 3 module 6 lesson 12: Real World Perimeters
-Grade 4 module 1 lesson 23: Running Meters and Kilometers
- Grade 5 module 3 lesson 12: Julie's Birdhouse
- Grade 5 module 5 lesson 20: Cubes in a Cylinder

Specific instructional prompts, engaging word problems, accessible and engaging tasks, art connections, Math Past connections, and context videos throughout Eureka Math ${ }^{2}$ work together to create a powerful curriculum that welcomes all students and invites them to become doers of mathematics.

## Addressing Learner Variance

To ensure success of all learners, every Eureka Math ${ }^{2}$ lesson includes Universal Design for Learning (UDL) strategies and scaffolds that address learner variance. These suggestions promote flexibility with engagement, representation, and action and expression, the three UDL principles described by CAST. These strategies and scaffolds are complements to the curriculum's overall alignment with the UDL Guidelines and were designed to support educators in effectively teaching students who experience difficulty in mathematics. The strategies are based on research specific to mathematics instruction.

According to Teaching Mathematics Meaningfully: Solutions for Reaching Struggling Learners, Second Edition, (page 71) "Students who have learning difficulties that affect their ability to do well in mathematics come from a variety of backgrounds and experiences. Although each of these students is individual and unique, students often demonstrate one or more of the nine learning characteristics..." The nine learning characteristics described include: learned helplessness, passive learning, knowledge and skills gaps, math anxiety, memory disabilities, attention disabilities, metacognitive thinking disabilities, processing disabilities, and reading disabilities. Some of these characteristics can affect all students who may be struggling in math regardless of whether they have learning-related disabilities (learned helplessness, passive learning, knowledge and skills gaps, math anxiety). Other characteristics result from learning-related disabilities (memory disabilities, attention disabilities, metacognitive thinking disabilities, processing disabilities, and reading disabilities). These learning characteristics as well as curriculum factors can result in common mathematics performance traits of students who struggle in mathematics.

UDL: Action \& Expression

Consider comparing the correct solution for problem 3 with an incorrect work sample. Present a chart that shows the correct work in sample A and the incorrect work in sample B to emphasize how the placement of parenthes affects the value of the expression. Ask in sample $A$ to the work in sample $B$ ? Why is it incorrect to evaluate problem 3 the way it is shown in sample B?" Post the chart for the remainder of the topic as an example of why parentheses are used and the importance of their placement. Use color coding and annotation to highlight these features, such as in the following example:


According to Allsopp et. al (2018), "Mathematics visuals appear to be most effective when used in conjunction with other effective instructional practices. An example of this is the use of explicit instruction techniques in conjunction with visuals. (page 192)." "Explicit cueing techniques can be utilized with visuals in ways that help students attend to the visual's most important features and its representation of the mathematical idea. Simple techniques, such as color-coding, using directional arrows, and highlighting, can help students focus on what is most relevant." An example of this is found in grade 7 module 5 lesson 7.

A variety of other strategies suggested in the literature are the foundation of all UDL margin notes found in Eureka Math². Each margin note is aligned to a strategy found to minimize the impacts of one of the nine learning characteristics listed above. Strategies include, but are not limited to:

- Break down tasks into manageable chunks.
- Demonstrate the belief that students can be successful.
- Visually organize to cue student to important aspects of concept.
- Teach students to change their frame of thinking.
- Embed math in relevant contexts.
- Help students make connections to prior knowledge
- Engage students by addressing interests.
- Celebrate progress and success.
- Cultivate a growth mindset.
- Relate math to students' lives.

UDL: Representation

To support students in transitioning from the array to the tape diagram, consider using interlocking cubes. Model the array vertically as 6 nines by using 5 cubes of one color and 1 cube of another color. Then rotate the array horizontally to show that it looks like the tape diagram.


- Use concrete materials.
- Associate content with meaningful context.
- Use a variety of strategies (visual, auditory, tactile, or kinesthetic).
- Provide visual organizers.
- Provide think alouds.
- Use novel learning contexts.
- Help students focus on what is important rather than on things that are irrelevant.

Eureka Math ${ }^{2}$ embeds differentiation through the simple-to-complex sequencing of lesson and Practice problems. This logical sequence gradually reduces scaffolds and builds in complexity, allowing teachers to differentiate assignments for either individual or small-group work. For all students, including those working above grade level, the gradual reduction of support and increase in complexity builds independent thinking and encourages productive struggle. Problems toward the end of the Problem Set (a lesson's daily independent practice) are often open-ended, at Depth of Knowledge (DOK) levels 2 and 3, and integrate two or more standards and/or Standards for Mathematical Practice. Teachers can assign problems of different complexities to students according to their needs or allow students to select problems in the 10-minute (approximate) timeframe. Lessons provide differentiation suggestions at the point of instruction to support a wide variety of learners. Differentiation margin notes found in the Teach book offer guidance for adapting instruction so that all students can successfully access grade-level content. There are two types of Differentiation margin notes: Support and Challenge. Challenge boxes suggest ways to keep students working at a more advanced level engaged by providing opportunities for extension while Support boxes offer specific, lesson-based scaffolds for helping students access content.

In this example from grade 5 module 6 lesson 2, the Differentiation margin note offers a suggestion for students to interact with the lesson objective of describing the location of points in the coordinate plane at a deeper level of complexity by describing a point's location relative to another point rather than providing an ordered pair.

This Support box from grade 4 module 4 lesson 2 encourages the use of concrete models to bolster understanding of fractions greater than 1 and their decomposition into both unit and nonunit fractions.

## Supporting Multilanguage Learners

Eureka Math ${ }^{2}$ writers relied on language development research to outline and build in the language support needed for multilanguage learners to engage with the language-rich lessons. With the goal of supporting the clear, concise, and precise use of reading, writing, speaking, and listening in English, Eureka Math ${ }^{2}$

| Differentiation: Challenge |
| :--- |
| Consider challenging students by describing |
| a point's location relative to another point, |
| as opposed to providing a ordered pair. |
| For example, ask students to plot a point |
| that is 3 units to the left and 4 units up from |
| point $P$ in problem 1 . Then have students |
| name the ordered pair for the point |
| they plotted. |
| Differentiation: Support |
| Consider providing a concrete example of a |
| fraction greater than 1 before presenting $\frac{6}{5}$. |
| Pair students and ask one partner to shade |
| all 4 fourths of their fraction strip. Have |
| partners lay the shaded $\frac{4}{4}$ and $\frac{2}{4}$ side by side. |
| Ask students what fraction is shaded and |
| have them write an equation to show $\frac{6}{4}$ as the |
| sum of $\frac{4}{4}$ and $\frac{2}{4}$. |

supports multilanguage learners through each lesson's instructional design. It does this by including instructional best practices, support for mathematical discourse, and support for the different tiers of terminology. Additionally, Language Support margin notes provide just-in-time, targeted instructional recommendations to support multilanguage learners.

## Instructional Best Practices

The following table outlines the instructional best practices included in Eureka Math².

| Practice | Eureka Math ${ }^{2}$ |
| :--- | :--- |
| Activate prior knowledge <br> (mathematics content, terminology, contexts) | The daily Fluency and Launch lesson components activate prior <br> knowledge to prepare students for new learning. Context videos <br> demonstrate math concepts in a concrete or real-world context. |
| Provide multiple entry points to the mathematics | Recurring Notice and Wonder routines and frequent open-middle and <br> open-ended tasks provide multiple points of entry for students to <br> participate. The inclusion of fine art and Math Past history components <br> engages students with math in the real world. |
| Use clear, concise student-facing language | Readability guidelines ensure that words are never an obstacle to math <br> learning. |
| Provide strategic active processing time | Frequent mathematical discourse, core instructional routines, and the <br> 10/2 principle expand dipportunities for students to synthesize and <br> process new information. |
| Illustrate multiple modes and formats | Varied physical and visual models, such as digital interactives, context <br> videos, and graphic organizers, help students make connections and <br> deepen understanding. |
| Provide opportunities for strategic review | Daily fluency activities, distributed practice Remember problems, Exit <br> Tickets, and comprehensive assessments provide frequent <br> opportunities for strategic review. |

## Mathematical Discourse

To support all learners, lessons provide ample authentic and engaging opportunities for students to read, write, speak, and listen. Eureka Math ${ }^{2}$ supports teachers in creating language-rich classrooms by modeling teacher-student discourse and by providing suggestions for supported student-tostudent discourse. Because curricula in general have an abundance of receptive language experiences (reading and listening), Eureka Math ${ }^{2}$ focuses specific supports on language production (speaking and writing) in mathematics.

The instructional routines that promote discourse are aligned with Stanford's Language Design Principles of supporting sense-making, optimizing output, cultivating conversation, and maximizing linguistic and cognitive meta-awareness.

Eureka Math ${ }^{2}$ periodically includes Language Support notes that suggest specific sentence frames and sentence starters to support multilanguage learners in student-tostudent discussions, such as those used in instructional routines. General sentence frames and sentence starters are provided in the Talking Tool which is referenced often during times of student-to-student discourse.

| Talking Tool |  |
| :---: | :---: |
| Share Your Thinking $\varepsilon$ | thow. <br> I did it thin wor becouse The aniwerlis__becouse My drowing thows |
| Agree or Dingorce | I agree because .... <br> That is true becoute ... <br> I disogree becouse. . <br> That is not true becouse. . . . <br> Do you ogree or divogree wh $\qquad$ 7 Why |
| Ask for Reasening | whydarow? <br> Connuevextion...? what cos we co innt? <br> Hown- mistad to - |
| Say it Acain e | I heord you say.... _resold.... Another way to vory that is What does that mean? |

## Terminology

Eureka Math ${ }^{2}$ lessons give students experience with a new mathematical concept before naming it with a precise mathematical term. Students may see a mathematical concept come to life in a digital interactive, manipulate counters in groups, or use an instructional routine to engage in mathematical discourse before the teacher gives that concept a name. In addition, teachers are provided with educative guidance, either in the body of the lesson or in a Language Support margin note, to support students in pairing the written term with a visual representation. Eureka Math ${ }^{2}$ highlights domain-specific terms from previous lessons in the current lesson, along with instructional recommendations for supporting those terms. These instructional recommendations focus on previewing the meaning of the terms before students are expected to interact with them
in the mathematics of the lesson. Additionally, domain-specific terms from previous lessons are also supported by pairing the written term with a visual representation. For each grade, the academic verbs needed to engage with the mathematics were considered. Each grade in Eureka Math ${ }^{2}$ offers a carefully curated list of targeted academic verbs that appear in the lessons for students to preview before they are expected to understand and use the language. For example, before students are asked to classify in grade 4 module 6 (page 39), lessons preview the meaning of the academic verb, supporting the meaning of the term in a class discussion and through a Language Support box, shown to the right.

Multiple-meaning terms encompass homophones like whole and hole, homographs like scale and scale, and other pronunciation-based challenges, like the difference between estimate (as a noun, as in, What is your estimate?) and estimate (as a verb, as in, Estimate the sum.). Lessons call out multiple-meaning terms that could affect emergent bilingual learners' understanding of the mathematics. Lessons also include Language Support notes to preview the meaning of the term in the lesson. These previews include pairing the term with a visual, with real items, or with a video to highlight the different meanings of the term and emphasize the specific meaning used in the lesson.

## Language Support Boxes

A Language Support margin note appears in the first lesson of every module to prompt teachers to consider using strategic, flexible grouping in each activity of the module to support multilanguage learners. These grouping suggestions invite teachers to use students' knowledge and home language by pairing students in different ways. Each of these different ways of pairing students has specific benefits for multilanguage learners. The Language Support margin notes also highlight discourse, language or terminology supports.

## Language Support

To further develop students' understanding of the word classify, explain that we classify things regularly. Consider using an example such as animals. Provide students with the following categories: fish, bird, insect. Name some animals and invite students to classify them. Invite students to describe what makes each category distinct from the other categories.

To learn more, please visit the Great Minds MLL blog: https://gm.greatminds.org/how-to-support-multilingual-learners-in-engaging-in-math-conversations-in-the-classroom

## Works Cited

Allsopp, David H., LouAnn H. Lovin, and Sarah van Ingen. Teaching Mathematics Meaningfully: Solutions for Reaching Struggling Learners.2nd ed. Baltimore: Brookes, 2018.

Beck, Isabel L., Margaret G. McKeown, and Linda Kucan. Bringing Words to Life: Robust Vocabulary Instruction.2nd ed. New York: The Guilford Press, 2013.

CAST. "Optimize relevance, value, and authenticity." Accessed June 3, 2022.
https://udlguidelines.cast.org/engagement/recruiting-interest/relevance-value-authenticity.
CAST. "Support planning and strategy development." Accessed June 3, 2022.
https://udlguidelines.cast.org/action-expression/executive-functions/strategy-
development/strategy-development-research.
Great Minds. Eureka Math ${ }^{2 T M}$. Washington, DC: Great Minds, 2021. https://greatminds.org/math
Hammond, Zaretta. Culturally Responsive Teaching and the Brain. Thousand Oaks: Corwin, 2014.
Stanford University Graduate School of Education. Principles for the Design of Mathematics
Curricula: Promoting Language and Content Development.
Retrieved from https://ul.stanford.edu/resource/principles-design-mathematics-curricula

