Connecticut Mathematics Model Curricula Alignment

Resource Name:

AF Math powered by Leap Educational Consulting

Alignment Grade 4			
Model Unit Name	**Model Unit Standards**	**Resource Unit(s) Number and Resources Lessons**	**Pacing**
This is the title of the unit in the model curricula	*These are the standards addressed in the unit*	*This is the unit(s) that aligns with the model unit from the resource* *These are the lessons from the identified units that align to the standards within the model unit*	*This is the expected number of days for instruction*
Understanding and Using Place Value to Multiply and Divide	4.NBT.A.1, 4.NBT.A.2, 4.NBT.A.3, 4.NBT.B.5, 4.NBT.B.6	**Unit 2: Place Value:** ● Lesson 1: Express numbers in a variety of forms using 1,000 book- 4.NBT.A.2 ● Lesson 2: Manipulate and express numbers in a variety of forms using 1,000 book- 4.NBT.A.2 ● Lesson 3: Express numbers up to ten-thousand place in standard, expanded, and written form- 4.NBT.A.2 ● Lesson 4: Express numbers up to hundred-thousand place in standard, expanded, and written form- 4.NBT.A.2 ● Lesson 5: Non-standard partitioning in the thousands- 4.NBT.A.1, 4.NBT.A.2 ● Lesson 6: Non-standard partitioning in the ten- thousands- 4.NBT.A.1, 4.NBT.A.2 ● Lesson 7: Non-standard partitioning in the hundred-thousands- 4.NBT.A.1, 4.NBT.A.2 ● Lesson 8: Place Value Relationships: Ten Times Greater- 4.NBT.A.1 ● Lesson 9: Apply Ten Times Greater Place Value Relationship to Problems- 4.NBT.A.1 ● Lesson 10: Apply Ten Times Greater Place Value Relationship to Problems Day 2- 4.NBT.A.1 ● Lesson 11: Compare and Order Numbers through the Hundred-Thousands Place- 4.NBT.A.2 ● Lesson 12: Compare and Order Numbers through the Hundred-Thousands Place Day 2- 4.NBT.A.2	31 days

		• Lesson 13: Round Numbers to the Thousands Place- 4.NBT.A.3 • Lesson 14: Round Numbers to the Ten-Thousands Place- 4.NBT.A.3 • Lesson 15: Round Numbers to the Hundred-Thousands Place (Extension of Days 1 & 2)- 4.NBT.A.3 • Lesson 16: Use rounding to estimate sums and differences- 4.NBT.A.3 **Unit 4: Multiplication and Division 2:** • Lesson 7: Multiply by multiples of 10 and 100- 4.NBT.B.5 • Lesson 8: Multiply 1-digit by 2 & 3-digit factors: Concrete- 4.NBT.B.5 • Lesson 9: Multiply 1-digit by 2 & 3-digit factors: Pictorial & Area Model- 4.NBT.B.5 • Lesson 10: Multiply 2-digit by 2-digit factors: Concrete & Pictorial- 4.NBT.B.5 • Lesson 11: Multiply 2-digit by 2-digit factors: Grids Drawn to Scale (Beginning of Area Model)- 4.NBT.B.5 • Lesson 12: Multiply 2-digit by 2-digit factors: Area Model- 4.NBT.B.5 • Lesson 13: Multiply 1-digit by up to 4-digit and 2-digit by 2-digit factors using the Area Model- 4.NBT.B.5 • Lesson 14: Divide by multiples of 10/100/1000- 4.NBT.B.6 • Lesson 15: Divide up to 4-digit dividends by 1-digit divisors: Concrete - 4.NBT.B.6 • Lesson 16: Divide up to 4-digit dividends by 1-digit divisors: Pictorial- 4.NBT.B.6 • Lesson 17: Divide up to 4-digit dividends by 1-digit divisors: Abstract- 4.NBT.B.6 • Lesson 18: Divide up to 4-digit dividends by 1-digit divisors: Abstract Day 2- 4.NBT.B.6 • Lesson 19: Divide up to 2-digit dividends by 1-digit divisors: Area Model- 4.NBT.B.6 • Lesson 20: Divide up to 3-digit dividends by 1-digit divisors: Area Model- 4.NBT.B.6 • Lesson 21: Divide up to 4-digit dividends by 1-digit divisors: Area Model- 4.NBT.B.6	
Factors and Multiples	4.OA.A.1, 4.OA.B.4, 4.OA.C.5	**Unit 1: Multiplication and Division 1:** • Lesson 1: Multiples- 4.OA.B.4 • Lesson 2: Factors and multiples- 4.OA.B.4 • Lesson 3: Factor using rainbows- 4.OA.B.4 • Lesson 4: Factor Pairs/Determining Prime or Composite- 4.OA.B.4 • Lesson 5: Multiplicative Compare with Multiplier Unknown- 4.OA.A.1* • Lesson 6: Multiplicative Compare with Larger Unknowns- 4.OA.A.1* • Lesson 7: Multiplicative Compare with Larger Unknowns (Day 2)- 4.OA.A.1* • Lesson 8: Multiplicative Compare with Smaller Unknowns- 4.OA.A.1* • Lesson 9: Multiplicative Compare with Smaller Unknowns Day 2- 4.OA.A.1* • Lesson 10: Multiplicative Compare with all problem types and symbols for unknowns- 4.OA.A.1* • Lesson 11: Multiplicative Compare with all problem types and symbols for unknowns- 4.OA.A.1* • Lesson 12: Record, Identify, and Extend Patterns and Relationships- 4.OA.C.5 • Lesson 13: Extend and Record observations of patterns with a focus on Geometric Patterns- 4.OA.C.5	14 lessons

		*These lessons also address standard 4.OA.A.2.	
Multi-Digit Whole Number Computation	4.NBT.B.4, 4.OA.A.2, 4.OA.A.3	**Unit 1: Multiplication and Division 1:** ● Lesson 5: Multiplicative Compare with Multiplier Unknown- 4.OA.A.2* ● Lesson 6: Multiplicative Compare with Larger Unknowns- 4.OA.A.2* ● Lesson 7: Multiplicative Compare with Larger Unknowns (Day 2)- 4.OA.A.2* ● Lesson 8: Multiplicative Compare with Smaller Unknowns- 4.OA.A.2* ● Lesson 9: Multiplicative Compare with Smaller Unknowns Day 2- 4.OA.A.2* ● Lesson 10: Multiplicative Compare with all problem types and symbols for unknowns- 4.OA.A.2* ● Lesson 11: Multiplicative Compare with all problem types and symbols for unknowns- 4.OA.A.2* **Unit 3: Addition and Subtraction:** ● Lesson 2: Standard Algorithm for Addition- 4.NBT.B.4 ● Lesson 3: Standard Algorithm for Addition- 3 or more Addends- 4.NBT.B.4 ● Lesson 4: Subtract Expanded Form and Visual Models- 4.NBT.B.4 ● Lesson 5: Standard Algorithm for Subtraction- 4.NBT.B.4 ● Lesson 6: Standard Algorithm for Subtraction Across Zeros- 4.NBT.B.4 ● Lesson 7: Standard Algorithm for Subtraction Across Zeros- 4.NBT.B.4 ● Lesson 8: Mixed Addition & Subtraction Practice (Error Analysis)- 4.NBT.B.4 ● Lesson 9: Mixed Addition & Subtraction Practice (Error Analysis)- 4.NBT.B.4 **Unit 5: Story Problems:** ● Lesson 1: Interpret remainders- 4.OA.A.3 ● Lesson 2: Interpret remainders: Multi-Digit Dividends- 4.OA.A.3 ● Lesson 3: Interpret remainders: Multi-Digit Dividends Day 2- 4.OA.A.3 ● Lesson 4: Represent Multi-Step Word Problems Involving all 4 Operations- 4.OA.A.3 ● Lesson 5: Solve Two-Step Word Problems Using Estimation to Check: Addition & Subtraction- 4.OA.A.3 ● Lesson 6: Solve Two-Step Word Problems Using Estimation to Check: Multiplication & Division- 4.OA.A.3 ● Lesson 7: Solve Two-Step Word Problems Using Estimation to Check- 4.OA.A.3 ● Lesson 8: Create and Solve 2-step Problems- 4.OA.A.3 ● Lesson 9: Multi-step word problems, all 4 operations- 4.OA.A.3 ● Lesson 10: Solve Multi-Step Word Problems- 4.OA.A.3 *These lessons also address standard 4.OA.A.1.	25 lessons
Comparing Fractions and Understanding Decimal Notation	4.NF.A.1, 4.NF.A.2, 4.NF.C.5, 4.NF.C.6, 4.NF.C.7	**Unit 6: Fractions:** ● Lesson 3: Equivalent Fractions with Larger Denominators Multiplication- 4.NF.A.1 ● Lesson 4: Equivalent fractions with the identity property- 4.NF.A.1	20 lessons

		• Lesson 5: Equivalent Fractions with Smaller Denominators Visual Models and Division- 4.NF.A.1 • Lesson 6: Equivalent Fractions with Tape Diagrams and Number Lines- 4.NF.A.1 • Lesson 7: Equivalent Fractions: All Strategies- 4.NF.A.1 • Lesson 8: Compare Fractions with ½ as a Benchmark and Number Lines- 4.NF.A.2 • Lesson 9: Compare with Identity Property Like Denominators- 4.NF.A.2 • Lesson 10: Compare with Identity Property Like Numerators- 4.NF.A.2 • Lesson 11: Compare Fractions using all Strategies- 4.NF.A.2 **Unit 7: Decimals:** • Lesson 1: Represent Tenths as Fractions Visuals Words and Decimals- 4.NF.C.6 • Lesson 2: Represent Hundredths Fractions Visuals Words and Decimals- 4.NF.C.6 • Lesson 3: Represent Decimals as Money Amounts- 4.NF.C.6 • Lesson 4: Model Equivalencies of Tenths & Hundredths- 4.NF.C.5, 4.NF.C.6 • Lesson 5: Write Fractions & Decimals in Word & Expanded Form- 4.NF.C.6 • Lesson 6: Compare Decimals Visual Models- 4.NF.C.7 • Lesson 7: Compare and order decimals- 4.NF.C.7 • Lesson 8: Compare and order decimals Day 2- 4.NF.C.7 • Lesson 9: Add Tenths & Hundredths using Fractional Model- 4.NF.C.5 • Lesson 10: Add Tenths & Hundredths using Fractional Models Day 2- 4.NF.C.5 • Lesson 11: Solve Word Problems With Decimals- 4.NF.C.5	
Building Understanding of Addition, Subtraction and Multiplication of Fractions	4.NF.B.3, 4.NF.B.4	**Unit 6: Fractions:** • Lesson 1: Decompose Fractions Using Visuals and Addition- 4.NF.B.3 • Lesson 2: Decompose Fractions Using Visuals, Addition and multiplication- 4.NF.B.4 • Lesson 12: Add and Subtract Fractions with Like Denominators- 4.NF.B.3 • Lesson 13: Multiply Fractions and Whole Numbers- 4.NF.B.4 • Lesson 14: Converting Mixed Numbers and Fractions Greater than One- 4.NF.B.3 • Lesson 15: Add Mixed Numbers Concrete and Pictorial- 4.NF.B.3 • Lesson 16: Add Mixed Numbers Pictorial and Abstract- 4.NF.B.3 • Lesson 17: Add Mixed Numbers Abstract Day 2- 4.NF.B.3 • Lesson 18: Subtract Mixed Numbers from Wholes Concrete and Pictorial- 4.NF.B.3 • Lesson 19: Subtract Mixed Numbers Concrete and Pictorial- 4.NF.B.3 • Lesson 20: Subtract Mixed Numbers Pictorial- 4.NF.B.3 • Lesson 21: Subtract Mixed Numbers Pictorial and Abstract- 4.NF.B.3 • Lesson 22: Add and Subtract Mixed Numbers: Mixed practice- 4.NF.B.3 • Lesson 23: Multiply Fractions and Whole Numbers: Word Problems- 4.NF.B.4 • Lesson 25: Interpret and Solve Word Problems Based on Line Plots- 4.NF.B.3, 4.NF.B.4* *This lesson also addresses standard 4.MD.B.4.	15 lessons

Solving Problems Involving Measurement and Data	4.MD.A.1, 4.MD.A.2, 4.MD.A.3	**Unit 4: Multiplication and Division 2:** ● Lesson 1: Determine Perimeter using Formulas- 4.MD.A.3 ● Lesson 2: Determine Area using Formulas- 4.MD.A.3 ● Lesson 3: Determine Perimeter by Decomposing Rectilinear Figures into rectangles- 4.MD.A.3 ● Lesson 4: Determine Area by Decomposing Rectilinear Figures into rectangles- 4.MD.A.3 ● Lesson 5: Determine Unknown Side Length Using Area Formula- 4.MD.A.3 ● Lesson 6: Determine Unknown Side Length Using Perimeter Formula- 4.MD.A.3 ● Lesson 22: Customary and metric length- 4.MD.A.1 ● Lesson 23: Customary and metric capacity- 4.MD.A.1 ● Lesson 24: Customary and metric weight/mass- 4.MD.A.1 ● Lesson 25: Conversion tables length, weight, capacity- 4.MD.A.1 ● Lesson 26: Conversion tables time- 4.MD.A.1 **Unit 8: Story Problems 2:** ● Lesson 1: Add & Subtract Metric Conversions-Length, Mass, Capacity- 4.MD.A.2 ● Lesson 2: Conversions and Problem-Solving with Customary Measurement- 4.MD.A.2 ● Lesson 3: Multi-Step Word Problems All 4 Operations Metric & Customary Units- 4.MD.A.2 ● Lesson 4: Decimal place value with money- 4.MD.A.2 ● Lesson 5: Determine change by counting up- 4.MD.A.2 ● Lesson 6: Determine unknown time (start, elapsed, or end)- 4.MD.A.2 ● Lesson 7: Multi-step Measurement Word Problems Fractions of Units- 4.MD.A.2 ● Lesson 8: Multi-step Measurement Word Problems Fractions of Units- 4.MD.A.2	19 lessons
Exploring Angles and Angle Measurement	4.MD.C.5, 4.MD.C.6, 4.MD.C.7	**Unit 10: Angle Measurement:** ● Lesson 1: Find Angle Measures as Quarter Turns and Rotations in a Circle- 4.MD.C.5, ● Lesson 2: Find Angle Measure with Fractions of a Circle- 4.MD.C.5 ● Lesson 3: Find Angle Measure Using Pattern Blocks- 4.MD.C.5 ● Lesson 4: Use a protractor to measure angles- 4.MD.C.6 ● Lesson 5: Use a protractor to draw angles- 4.MD.C.6 ● Lesson 6: Find Angle Measure with Additive Property Complementary and Supplementary Angles- 4.MD.C.7 ● Lesson 7: Find Angle Measure with Additive Property Full Circle Angles- 4.MD.C.7	7 lessons
Understanding Properties of Two-Dimensional Figures	4.G.A.1, 4.G.A.2, 4.G.A.3	**Unit 9: Geometry:** ● Lesson 1: Identify and draw points, lines, line segments, and rays- 4.G.A.1 ● Lesson 2: Identify and draw parallel and perpendicular lines- 4.G.A.1 ● Lesson 3: Identify and draw right, acute, and obtuse angles- 4.G.A.1 ● Lesson 4: Classify triangles- 4.G.A.2 ● Lesson 5: Classify and draw quadrilaterals- 4.G.A.2 ● Lesson 6: Identify and Draw Lines of symmetry- 4.G.A.3	6 lessons

Scope and Sequence

If a district uses this resource to implement the state model curriculum for grade 4, the following scope and sequence should be followed to ensure alignment and attention to the progressions of mathematics.

Unit One: Multiplication & Division 1

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Multiples** SWBAT define and identify multiples of certain numbers by skip-counting and using multiplication facts.	4.OA.B.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
2	**Factors and multiples** SWBAT define and identify factors of certain numbers by thinking of multiplication facts and arrays.	
3	**Factor using rainbows** SWBAT find all factors of a given number by creating factor rainbows.	
4	**Factor Pairs/Determining Prime or Composite** SWBAT identify numbers as prime and composite by listing factor pairs and using factor rainbows, and determining the number of factor pairs a given number has.	
5	**Multiplicative Compare with Multiplier Unknown** SWBAT understand the meaning of multiplicative language and represent and solve multiplicative compare problems involving multiplicative relationships by annotating and labeling word problems and creating visual representations (MC-MU).	4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
6	**Multiplicative Compare with Larger Unknowns** SWBAT understand the meaning of multiplicative language and represent and solve multiplicative compare problems involving multiplication by annotating and labeling word problems and creating visual representations (MC-LU).	
7	**Multiplicative Compare with Larger Unknowns (Day 2)** SWBAT understand the meaning of multiplicative language and represent and solve multiplicative compare problems involving multiplication by annotating and labeling word problems and creating visual representations (MC-LU).	
8	**Multiplicative Compare with Smaller Unknowns**	

	SWBAT interpret, represent and solve multiplicative compare problems involving division by annotating and labeling word problems and creating visual representations (MC-SU).	
9	**Multiplicative Compare with Smaller Unknowns Day 2** SWBAT interpret, represent and solve multiplicative compare problems involving division by annotating and labeling word problems and creating visual representations (MC-SU).	
10	**Multiplicative Compare with all problem types and symbols for unknowns** SWBAT represent and solve all types of multiplicative compare problem types by annotating as needed and labeling word problems and creating visual representations (MC-LU, MC-SU, and MC-MU). SWBAT write equations with variables to represent all types of multiplicative compare problems.	
11	**Multiplicative Compare with all problem types and symbols for unknowns** SWBAT interpret multiplication equations as multiplicative compare statements by replacing symbols with words.	
12	**Record, Identify, and Extend Patterns and Relationships** SWBAT determine the rules and apply that rule to extend the pattern by finding the repeated relationship between consecutive terms in the pattern. SWBAT find the rules and fill in tables based on the rule by finding the repeated relationship between pairs of numbers in tables. SWBAT record mathematical observations about patterns by using knowledge of number patterns and asking themselves, "how do these numbers relate to one another?"	4.OA.C.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
13	**Extend and Record observations of patterns with a focus on Geometric Patterns** SWBAT identify and extend various types of patterns including geometry patterns by noticing consistencies and repetitions in their observations and recording using an organized method (tables, charts, etc.). SWBAT describe observations and rules about patterns by using mathematical vocabulary and explaining the relationship between terms in the patterns.	
14	**Unit Assessment**	

Unit Two: Place Value

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Express numbers in a variety of forms using 1,000 book** SWBAT count by hundreds up to 1,000 and identify hundreds that come between by creating a 1,000 book. SWBAT write numbers through 9,999 in standard, expanded and written form by thinking about place value and saying numbers out loud.	4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each

2	**Manipulate and express numbers in a variety of forms using 1,000 book** SWBAT identify a result of changing the amount in a given place value of a number by using place value strategies rather than addition/subtraction strategies.	place, using >, =, and < symbols to record the results of comparisons.
3	**Express numbers up to ten-thousand place in standard, expanded, and written form** SWBAT write numbers through 99,999 in standard, expanded and written form by using place value blocks, drawings, place value charts, and saying numbers out loud. SWBAT convert between numbers through 99,999 written in standard, written, and expanded form as well as in place value charts and with non-standard partitioning by thinking about place value.	
4	**Express numbers up to hundred-thousand place in standard, expanded, and written form** SWBAT write numbers through 999,999 in standard, expanded and written form by using place value blocks, drawings, place value charts, and saying numbers out loud. SWBAT convert between numbers through 999,999 written in standard, written, and expanded form as well as in place value charts and with non-standard partitioning by thinking about place value.	
5	**Non-standard partitioning in the thousands** SWBAT convert between non-standard partitioning and expanded, standard and written form of numbers by thinking about exchanging groups of 10 for the next largest place value and decomposing numbers.	4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* 4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
6	**Non-standard partitioning in the ten- thousands** SWBAT convert between non-standard partitioning and expanded, standard and written form of numbers by thinking about exchanging groups of 10 for the next largest place value and decomposing numbers.	
7	**Non-standard partitioning in the hundred-thousands** SWBAT convert between non-standard partitioning and expanded, standard and written form of numbers by thinking about exchanging groups of 10 for the next largest place value and decomposing numbers.	
8	**Place Value Relationships: Ten Times Greater** SWBAT describe the relationship between place values by using multiplicative compare language and thinking about how many of a smaller place value they need to create 1 new larger place value.	4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
9	**Apply Ten Times Greater Place Value Relationship to Problems** SWBAT solve equations by applying their understanding of the ten times relationship between place values.	
10	**Apply Ten Times Greater Place Value Relationship to Problems Day 2**	

	SWBAT solve equations and word problems by applying their understanding of the ten times relationship between place values.	
11	**Compare and Order Numbers through the Hundred-Thousands Place** SWBAT compare and order numbers through 999,999 by thinking about the value of the digit in each place.	4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
12	**Compare and Order Numbers through the Hundred-Thousands Place Day 2** SWBAT compare and order numbers through 999,999 by thinking about the value of the digit in each place.	
13	**Round Numbers to the Thousands Place** SWBAT round numbers through 999,999 to the nearest ten, hundred or thousand by using number lines.	4.NBT.A.3 Use place value understanding to round multi-digit whole numbers to any place.
14	**Round Numbers to the Ten-Thousands Place** SWBAT round numbers through 999,999 to the nearest ten, hundred, thousand or ten-thousand by using number lines.	
15	**Round Numbers to the Hundred-Thousands Place (Extension of Days 1 & 2)** SWBAT round numbers through 999,999 to the nearest ten, hundred, thousand, ten thousand or hundred thousand by using number lines and considering the two numbers their number to round is between and rounding up or down.	
16	**Use rounding to estimate sums and differences** SWBAT fluently round numbers through 999,999 to place values through the hundred thousands and explain how they rounded by using number lines and thinking about place value. SWBAT estimate sums and differences to equations with numbers in the hundred-thousands by rounding and then adding/subtracting using a variety of strategies.	
17	**Unit Assessment**	

Unit Three: Addition & Subtraction

Lesson Number and Daily Aim		**Standard(s) in Lesson**
1	**Add Numbers with Expanded Form** SWBAT calculate sums with addends in the hundred thousands by using expanded notation addition.	3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
2	**Standard Algorithm for Addition**	4.NBT.B.4

	SWBAT calculate sums with addends through the hundred-thousands by using the standard algorithm.	Fluently add and subtract multi-digit whole numbers using the standard algorithm.
3	**Standard Algorithm for Addition- 3 or more Addends** SWBAT calculate sums with three or more addends through the hundred-thousands by using the standard algorithm.	
4	**Subtract Expanded Form and Visual Models** SWBAT calculate differences with addends in the hundred thousands by using expanded notation subtraction and relating it to pictures of place value blocks.	
5	**Standard Algorithm for Subtraction** SWBAT calculate differences through the hundred-thousands by using the standard algorithm for subtraction.	
6	**Standard Algorithm for Subtraction Across Zeros** SWBAT calculate differences when required to regroup across several zeros by using the standard algorithm.	
7	**Standard Algorithm for Subtraction Across Zeros** SWBAT calculate differences when required to regroup across several zeros by using the standard algorithm.	
8	**Mixed Addition & Subtraction Practice (Error Analysis)** SWBAT accurately and fluently solve addition and subtraction equations with numbers through the hundred-thousands using the standard algorithm.	
9	**Mixed Addition & Subtraction Practice (Error Analysis)** SWBAT accurately and fluently solve addition and subtraction equations with numbers through the hundred-thousands using the standard algorithm.	
10	**Unit Assessment**	

Unit Four: Multiplication & Division 2

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Determine Perimeter using Formulas** SWBAT solve for the perimeter of rectangles using formulas when problems explicitly and implicitly ask for perimeter with 2 sides lengths given.	4.MD.A.3 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
2	**Determine Area using Formulas**	

	SWBAT solve for the area of rectangles using formulas when problems explicitly and implicitly ask for area with 2 sides lengths given.	
3	**Determine Perimeter by Decomposing Rectilinear Figures into rectangles** SWBAT solve for the perimeter of rectilinear figures by decomposing them into smaller rectangles and adding all the sides.	
4	**Determine Area by Decomposing Rectilinear Figures into rectangles** Students will find the area of rectilinear figures by decomposing them into rectangles and then using the formula for area for each rectangle and adding them together. This might require using given sides to find the measurements of unknown side lengths.	
5	**Determine Unknown Side Length Using Area Formula** SWBAT find the unknown side length of a rectangle with a given area and additional side length by using the formula for area of rectangles and division or unknown factor multiplication.	
6	**Determine Unknown Side Length Using Perimeter Formula** SWBAT find the unknown side length of a rectangle with a given perimeter and additional side length by using the formula for perimeter of rectangles, subtraction, and division or unknown factor multiplication.	
7	**Multiply by multiples of 10 and 100** SWBAT multiply by multiples of ten by using place value knowledge, changing the place values of digits, and recognizing patterns and rules in the amount of zeroes added to products.	4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
8	**Multiply 1-digit by 2 & 3-digit factors: Concrete** SWBAT solve 2x1 and 3x1 multiplication problems by using place value blocks (arrays) and pictures of place value blocks to build arrays and then adding up the partial products.	
9	**Multiply 1-digit by 2 & 3-digit factors: Pictorial & Area Model** SWBAT solve 2x1, 3x1, and 4x1 multiplication problems by using pictures of place value blocks (array) and the area model to represent the problem and then adding up the partial products.	
10	**Multiply 2-digit by 2-digit factors: Concrete & Pictorial** SWBAT solve 2x2 multiplication problems by using pictures of place value blocks (array) to represent the problem and then adding up the partial products.	
11	**Multiply 2-digit by 2-digit factors: Grids Drawn to Scale (Beginning of Area Model)** Students will solve 2-digit x 2-digit multiplication problems by creating a grid drawn to scale to represent the area and then adding up all the partial products (areas of each section of the grid).	
12	**Multiply 2-digit by 2-digit factors: Area Model** SWBAT solve 2x2 multiplication problems by using the area model to represent the problem and then adding up the partial products.	

13	**Multiply 1-digit by up to 4-digit and 2-digit by 2-digit factors using the Area Model** SWBAT solve 2/3/4x1 and 2x2 multiplication problems by using the area model to represent the problem and then adding up the partial products.	
14	**Divide by multiples of 10/100/1000** SWBAT divide by multiples of 10 when the basic fact divides evenly by using place value knowledge and the changes in place values.	4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
15	**Divide up to 4-digit dividends by 1-digit divisors: Concrete** SWBAT divide 2- 3- and 4-digit dividends by 1 digit divisors using place value blocks and sharing to represent the problem.	
16	**Divide up to 4-digit dividends by 1-digit divisors: Pictorial** SWBAT divide 2 and 3-digit dividends by 1 digit divisors using pictorial or abstract sharing division by relating the place value blocks to the pictures to the values/words.	
17	**Divide up to 4-digit dividends by 1-digit divisors: Abstract** SWBAT divide 2, 3 and 4-digit dividends by 1 digit divisors using pictorial or abstract sharing division by relating the place value blocks to the pictures/values/words (4.NBT.6).	
18	**Divide up to 4-digit dividends by 1-digit divisors: Abstract Day 2** SWBAT divide 2, 3 and 4-digit dividends by 1 digit divisors using pictorial or abstract sharing division by relating the place value blocks to the pictures/values/words (4.NBT.6).	
19	**Divide up to 2-digit dividends by 1-digit divisors: Area Model** SWBAT divide 2- digit dividends by 1-digit divisors using the area model for division (as finding side length) by relating it to the area model for multiplication and thinking about missing factors.	
20	**Divide up to 3-digit dividends by 1-digit divisors: Area Model** SWBAT divide 3- digit dividends by 1-digit divisors using the area model for division (as finding side length) by relating it to the area model for multiplication and thinking about missing factors.	
21	**Divide up to 4-digit dividends by 1-digit divisors: Area Model** SWBAT divide 4- digit dividends by 1-digit divisors using the area model for division (as finding side length) by relating it to the area model for multiplication and thinking about missing factors.	
22	**Customary and metric length** SWBAT articulate the customary and metric units of length and use benchmarks to describe their relative size and ratio.	4.MD.A.1 Know relative sizes of measurement units within one system of units, including: ft, in; km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for*
23	**Customary and metric capacity** SWBAT articulate the customary and metric units of capacity and use benchmarks to describe their relative size and ratio.	

24	**Customary and metric weight/mass** SWBAT articulate the customary and metric units of weight/mass and use benchmarks to describe their relative size and ratio.	*feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …*
25	**Conversion tables length, weight, capacity** SWBAT use ratios and tables to convert between units of length, weight, and capacity, within a given system, to solve conversion problems. SWBAT solve mixed measurement conversion problems by using tables and thinking about the ratios.	
26	**Conversion tables time** SWBAT articulate the units of time and use benchmarks to describe their relative size and ratio. SWBAT use ratios and tables to convert between units of time to solve conversion problems.	
27	**Unit Assessment**	

Unit Five: Story Problems 1

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Interpret remainders** SWBAT interpret remainders of problems with basic division facts by visualizing, representing, labeling all numbers, and using the context of the question.	4.OA.A.3 Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
2	**Interpret remainders: Multi-Digit Dividends** SWBAT interpret remainders of problems with multi-digit dividends by visualizing, representing, labeling all numbers, and using the context of the question.	
3	**Interpret remainders: Multi-Digit Dividends Day 2** SWBAT interpret remainders of problems with multi-digit dividends by visualizing, representing, labeling all numbers, and using the context of the question.	
4	**Represent Multi-Step Word Problems Involving all 4 Operations.** SWBAT annotate and represent multi-step word problems by thinking about the big problem and the smaller problems that need to be solved.	
5	**Solve Two-Step Word Problems Using Estimation to Check: Addition & Subtraction** SWBAT solve 2-step word problems involving addition and subtraction by visualizing, representing, estimating, and using a variety of strategies to calculate. SWBAT estimate solutions to 2-step word problems involving addition and subtraction in order to assess reasonableness.	
6	**Solve Two-Step Word Problems Using Estimation to Check: Multiplication & Division**	

	SWBAT solve 2-step word problems involving multiplication and division by visualizing, representing, estimating, and using a variety of strategies to calculate. SWBAT estimate solutions to 2-step word problems involving multiplication and division in order to assess reasonableness.	
7	**Solve Two-Step Word Problems Using Estimation to Check** SWBAT solve 2-step word problems involving all four operations by visualizing, representing, estimating, and using a variety of strategies to calculate.	
8	**Create and Solve 2-step Problems** SWBAT create word problems to match given representations and solve the word problems by interpreting and labeling the representation and using a variety of strategies to solve.	
9	**Multi-step word problems, all 4 operations** SWBAT solve multi-step word problems involving all four operations by visualizing, representing, estimating, and using variety of strategies to calculate. SWBAT estimate solutions to multi-step word problems involving addition and subtraction in order to assess reasonableness by rounding numbers first, then doing quick mental math or calculations.	
10	**Solve Multi-Step Word Problems** SWBAT solve multi-step problems that use all four operations by visualizing, representing, estimating to check, and using a variety of strategies to calculate.	
11	**Unit Assessment**	

Unit Six: Fractions

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Decompose Fractions Using Visuals and Addition** SWBAT decompose fractions into unit fractions and other combinations of smaller fractions by using visuals and addition.	4.NF.B.3.a-b Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. *Example: 3/4 = 1/4 + 1/4 + 1/4*. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
2	**Decompose Fractions Using Visuals, Addition and multiplication**	4.NF.B.4.a Multiply a fraction by a whole number.

	SWBAT decompose fractions into unit fractions and other combinations of smaller fractions by using visuals, addition and multiplication.	a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
3	**Equivalent Fractions with Larger Denominators Multiplication** SWBAT create equivalent fractions by partitioning fractions into smaller parts using visual models and writing multiplication equations.	4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
4	**Equivalent fractions with the identity property** SWBAT use the identify property (with multiplication) to find and identify equivalent fractions.	
5	**Equivalent Fractions with Smaller Denominators Visual Models and Division** SWBAT find equivalent fractions by grouping unit fraction to create larger fractions and using the identify property (with division).	
6	**Equivalent Fractions with Tape Diagrams and Number Lines** SWBAT plot and identify equivalent fractions on number lines by partitioning intervals equally into smaller intervals.	
7	**Equivalent Fractions: All Strategies** SWBAT identify equivalent fractions using a variety of strategies.	
8	**Compare Fractions with ½ as a Benchmark and Number Lines** SWBAT compare fractions using the ½ benchmark.	4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
9	**Compare with Identity Property Like Denominators** SWABT compare fractions by using the identity and changing each of them to equivalent fractions with like denominators.	
10	**Compare with Identity Property Like Numerators** SWBAT compare fractions by using the identity property and changing each of them to equivalent fractions with like numerators.	
11	**Compare Fractions using all Strategies** SWBAT compare fractions using a variety of strategies.	
12	**Add and Subtract Fractions with Like Denominators** SWABT add and subtract fractions with like denominators.	4.NF.B.3.a Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$ a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. *Example: 3/4 = 1/4 + 1/4 + 1/4*.
13	**Multiply Fractions and Whole Numbers**	4.NF.B.4.b

	SWBAT multiply fractions and whole numbers.	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
14	**Converting Mixed Numbers and Fractions Greater than One** SWBAT understand the meaning of fractions greater than one and mixed numbers and convert between them.	4.NF.B.3.c Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
15	**Add Mixed Numbers Concrete and Pictorial** SWBAT add mixed numbers using fraction tiles and pictures.	
16	**Add Mixed Numbers Pictorial and Abstract** SWBAT add mixed numbers using pictures and abstract strategies.	
17	**Add Mixed Numbers Abstract Day 2** SWBAT add mixed numbers using abstract strategies.	
18	**Subtract Mixed Numbers from Wholes Concrete and Pictorial** SWBAT subtract mixed numbers and fractions from whole numbers using fraction tiles and pictures.	
19	**Subtract Mixed Numbers Concrete and Pictorial** SWBAT subtract mixed numbers from mixed numbers using fraction tiles and pictures.	
20	**Subtract Mixed Numbers Pictorial** SWBAT subtract mixed numbers from mixed numbers using pictures.	
21	**Subtract Mixed Numbers Pictorial and Abstract** SWABT subtract mixed numbers using abstract strategies.	
22	**Add and Subtract Mixed Numbers: Mixed practice** SWABT add and subtract mixed numbers using a variety of strategies	
23	**Multiply Fractions and Whole Numbers: Word Problems** SWBAT multiply fractions and whole numbers in the context of word problems.	4.NF.B.4.b-c Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5)*

		as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.) c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
24	**Create and Interpret Line Plots** SWBAT to create and interpret line plots.	4.MD.B.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
25	**Interpret and Solve Word Problems Based on Line Plots** SWBAT solve word problems with fractions based on interpreting line plots. SWBAT interpret line plots in order to solve word problems that involve adding and subtracting fractions and mixed numbers, comparing fractions, and multiplying fractions and whole numbers.	4.MD.B.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* 4.NF.B.4.b-c Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

		4.NF.B.3.d Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
26	**Unit Assessment**	

Unit Seven: Decimals

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Represent Tenths as Fractions Visuals Words and Decimals** SWBAT understand the meaning of decimals to the tenths place by showing them as fractions, visual models and in word form.	4.NF.C.6* Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*
2	**Represent Hundredths Fractions Visuals Words and Decimals** SWBAT understand the meaning of decimals in the hundredths place by showing them as fractions, visual models, and in word form.	
3	**Represent Decimals as Money Amounts** SWBAT understand the meaning of decimals by relating them to money amounts.	
4	**Model Equivalencies of Tenths & Hundredths** SWBAT identify equivalent values of tenths and hundredths written as decimals using place value understanding, visual models and equivalent fractions.	4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* 4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*
5	**Write Fractions & Decimals in Word & Expanded Form** SWBAT write decimals in word form and expanded form.	4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*
6	**Compare Decimals Visual Models**	4.NF.C.7

	SWBAT compare decimals by showing them on number lines and with visual models.	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
7	**Compare and order decimals** SWBAT compare decimals by comparing digits in corresponding place values.	
8	**Compare and order decimals Day 2** SWBAT compare decimals by comparing digits in corresponding place values.	
9	**Add Tenths & Hundredths using Fractional Models** SWBAT add fractions with denominators of 10 and 100 and express the sum as a fraction or decimal.	4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
10	**Add Tenths & Hundredths using Fractional Models Day 2** SWBAT add fractions with denominators of 10 and 100 and express the sum as a fraction or decimal.	
11	**Solve Word Problems With Decimals** SWBAT apply skills of adding tenths and hundredths to complex word problems.	
12	**Unit Assessment**	

Unit Eight: Story Problems 2

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Add & Subtract Metric Conversions-Length, Mass, Capacity** SWBAT add and subtract mixed metric units of length, mass, and capacity by using conversion ratios and a variety of strategies including counting on and the standard algorithm.	4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
2	**Conversions and Problem-Solving with Customary Measurement** SWBAT add and subtract mixed customary units of length, weight, and capacity by using conversion ratios and a variety of strategies including counting on and the standard algorithm.	
3	**Multi-Step Word Problems All 4 Operations Metric & Customary Units** SWBAT solve multi-step word problems with all four operations using mixed metric and customary units by visualizing, representing, and choosing from a variety of strategies to solve.	
4	**Decimal place value with money** SWBAT express money amounts as decimals (as fractions of dollars).	
5	**Determine change by counting up** SWBAT solve word problems involving adding money amounts and determining change.	

6	**Determine unknown time (start, elapsed, or end)** SWBAT determine start times, end times or elapsed time when two of these three are given.	
7	**Multi-step Measurement Word Problems Fractions of Units** SWBAT represent and solve multi-step word problems with fractions and decimals amounts of units, all four operations, and metric and customary units of length, capacity and weight/mass, and money and time.	
8	**Multi-step Measurement Word Problems Fractions of Units** SWBAT represent and solve multi-step word problems with fractions and decimals amounts of units, all four operations, and metric and customary units of length, capacity and weight/mass, and money and time.	
9	**Unit Assessment**	

Unit Nine: Geometry

Lesson Number and Daily Aim		**Standard(s) in Lesson**
1	**Identify and draw points, lines, line segments, and rays** SWBAT identify and draw lines and points using points to name them.	4.G.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2	**Identify and draw parallel and perpendicular lines** SWBAT identify, name and draw parallel and perpendicular lines.	
3	**Identify and draw right, acute, and obtuse angles** SWBAT identify and sketch right, obtuse and acute angle by determining if they look greater than, equal to, or smaller than a square corner.	
4	**Classify triangles** SWBAT classify triangles as right, acute, or obtuse and equilateral, isosceles or scalene based on their side lengths and angle types.	4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
5	**Classify and draw quadrilaterals** SWBAT classify quadrilaterals as trapezoids, parallelograms, rhombuses, rectangles, and/or squares based on their angles and sides. SWBAT draw specific types of quadrilaterals with given attributes.	
6	**Identify and Draw Lines of symmetry** SWBAT identify and draw all lines of symmetry on two-dimensional figures.	4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

7	**Unit Assessment**	

Unit Ten: Angle Measurement

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Find Angle Measures as Quarter Turns and Rotations in a Circle** SWBAT identify reflex angles and determine angle measurements by determining their fraction of a full circle, 360 degrees. SWBAT identify the measurement of rotations as multiples of 90 degree turns.	4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
2	**Find Angle Measure with Fractions of a Circle** SWBAT identify a degree as 1/360 of a circle and identify other angle measurements as fractions of a circle/360 degrees. SWBAT identify reflex angles and determine angle measurements by determining their fraction of a full circle, 360 degrees.	
3	**Find Angle Measure Using Pattern Blocks** SWBAT determine angle measurements using angles of shapes that combine to form benchmark angles.	
4	**Use a protractor to measure angles** SWBAT measure angles using protractors.	4.MD.C.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
5	**Use a protractor to draw angles** SWBAT draw angles with given measurements using protractors.	
6	**Find Angle Measure with Additive Property Complementary and Supplementary Angles** SWBAT find angle measurements using the additive property of angles with right and straight angles.	4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
7	**Find Angle Measure with Additive Property Full Circle Angles** SWBAT find angle measurements using the additive property of angles and benchmark angles up to 360 degrees.	
8	**Unit Assessment**	

Supports of Diversity, Equity and Inclusion

Please provide any information relative to supporting culturally responsive instruction, multi-language learners, and students with disabilities

A unique aspect of the AF Math powered by Leap Educational Consulting Program is that we do not simply offer a curriculum. We know that rigorous materials are only one component of meeting the needs of all students. In most cases, the work is not around adopting a curriculum, but rather executing the content with excellence. In order to support this excellence and to support schools and districts in the change management work of a new curriculum, Leap Educational Consulting offers a range of comprehensive coaching services.

Our Vision: We envision a world in which all students experience an education that empowers them to lead lives of choice and opportunity.

Our Mission: We work in deep partnership with educators committed to dismantling systemic racism to accelerate their impact. We do this by providing comprehensive supports that build their capacity to drive equitable and sustainable academic outcomes for all students.

Our Approach: At Leap, we believe that all students can thrive in math. The formula for success is simple – access to grade level math curriculum coupled with excellent instruction will equal equitable outcomes for kids. Our curriculum was rated amongst the highest on EdReports.

Our Theory of Change: If Leap provides effective, best-in-class support to school leaders…Then school leaders will lead with equity, have a high bar for what students can do, be equipped with the technical and adaptive skills to effectively develop teachers quickly, and create systems that will ensure strong student math outcomes...Then teachers will provide more rigorous, standards-aligned, inquiry-based, engaging math instruction to all students...Then students will have access to grade-level math content and an educational experience where they are deeply engaged, powerful doers and critical thinkers of mathematics and are well-positioned for college and beyond.

Our Results:
From our most recent test results in elementary school (ES) Math:

- 44% achieved at least 10 points growth in state results in one year
- 78% achieved at least 5 points growth in state results in one year

From our most recent test results in middle school (MS) Math:

- 53% achieved at least 10 points growth in state results in one year
- 93% achieved at least 5 points growth in state results in one year

What this looks like in action: Schools and districts partner with Leap Educational Consulting for support in their adoption and implementation of the AF Math powered by Leap Educational Consulting Program. This support is highly customizable. We recommend new schools and districts elect the "full partnership," which includes:

- **Weekly personalized coaching of the school leader** that is leading the implementation of the AF Math powered by Leap Educational Consulting Program: We believe that by building the skillset of the instructional leader, this will lead to rapid progress with teachers and students. This personalized coaching includes skills such as effective observations, teacher coaching meetings, leading intellectual preparation with teachers, leading looking at student work protocols with teachers, facilitating professional development, and leading change. The leader receives support in leveraging these skills to ensure strong implementation of the AF Math powered by Leap Educational Consulting Program.
- **In person and virtual site visits throughout the school year** to do a "deep dive" on progress towards strong implementation of the AF Math powered by Leap Educational Consulting Program, review of student data, and action planning based on progress to lead to rapid growth.
- **Access to resources,** including all curriculum and assessments, a bank of pre-designed professional development sessions for teachers, videos of lessons in execution by teachers across the country, a network of other leaders implementing the AF Math powered by Leap Educational Consulting Program, and more.

- **Professional development** for the school leader in a virtual setting of peers across the country. This development includes an initial summer launch training as well as ongoing sessions throughout the school year. Some of the professional developments include materials that can be then used in teacher professional development.
- **Differentiated support,** including support preparing for the state exam.

More information is available at https://www.leapeducationalconsulting.org/.

Program Overview: Differentiation and Working with Special Populations

AF Math powered by Leap Educational Consulting is built on the belief that all learners, especially students with disabilities or special needs and Multilingual and English language Learners (MLLs/ ELLs), deserve great curriculum. Therefore, our curriculum is strategically built to align to some of the best practices for supporting ALL learners.

One tenet of the AF Math powered by Leap Educational Consulting program is that students are grouped heterogeneously for instruction. This follows the recommendations of the California Mathematics Framework (2021) and is based on research that has shown that students of all levels and backgrounds, including minority students, students of low socioeconomic status, and students at all initial achievement levels (including high achievers), perform better in mixed-level classrooms (Burris et al., 2006). While we strongly recommend that students are heterogeneously grouped for core instruction, we recognize the need for interventions and extensions for students in order to meet their learning needs, and our framework for supporting special populations are detailed below.

Supporting Students with Disabilities

Without strong support, students with disabilities can easily struggle with learning mathematics and feel unsuccessful. Therefore, it is critical that strong curricular materials are designed to provide support for all student learners, especially those with diagnosed disabilities (Hott et al., 2014). The AF Math powered by Leap Educational Consulting Program was designed with this in mind and is based on several bodies of research about best practices for the instruction of students with math disabilities, including the work of the What Works Clearinghouse (an investment of the Institute of Education Sciences within the U.S. Department of Education) and the Council for Learning Disabilities (an international organization composed of professionals who represent diverse disciplines).

Unit Overviews and lesson level materials include guidance around working with students with disabilities, including daily suggested interventions in the Workshop Section of the daily lesson plan. Teachers should reference these materials in conjunction with the information that follows in this Implementation Guide when planning instruction in order to best support all students.

The What Works Clearinghouse makes several recommendations for students struggling with mathematics in elementary and middle school, several of which are incorporated into the AF Math powered by Leap Educational Consulting Program and all of which are recommended for use by teachers and schools. The Council for Learning Disabilities (2014) also recommends 5 elements of “explicit and systematic instruction” (several of which overlap with the recommendations of the WWC) to be utilized in core mathematics instruction. These two sets of recommendations and guidance about how to incorporate them into instruction using the AF Math powered by Leap Educational Consulting Program are outlined below:

Research-Based Recommendations from the What Works Clearinghouse (2009)	
Recommendations	**Guidance for Incorporating this Recommendation into Instruction**
Recommendation 1: Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	Each Unit Overview includes a pre-assessment. We recommend that teachers use this as a screening tool for all students prior to each unit. Teachers should then use data from the pre-assessment to inform tier 1 instruction and tier 2 and 3 interventions. We also recommend that in combination with data from pre-assessments, teachers use state assessment data to inform instruction and interventions for grades at which data is available.
Recommendation 2: Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	In alignment with this recommendation, the core focus of instruction in the elementary grades is on building understanding of whole numbers, with an average of over 80% of AF Math powered by Leap Educational Consulting Program K-5 instructional materials, including intervention materials, focused on the major content of the grade level. When students are engaged in a unit focused on additional or supporting standards, teachers may choose to prioritize major content for interventions, even if the interventions are then not fully aligned to the unit being taught. Developing students' foundational proficiencies, paying special attention to counting, place value, and addition, subtraction, multiplication, and division, is essential, and thus, alignment with the core curriculum is not necessary. (In cases where intervention is not aligned to the core curriculum, that intervention needs to be supplemental, however, not in place of core instruction.)
Recommendation 3: Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	According to the National Mathematics Advisory Panel (2008), explicit instruction includes "teachers providing clear models for solving a particular problem type using an array of examples," students receiving extensive practice, including many opportunities to think aloud or verbalize their strategies as they work, and students being provided with extensive feedback." General instruction using the AF Math powered by Leap Educational Consulting Program rarely involves the teacher explicitly modeling how to solve a problem type, but does encourage teachers to guide students to explicitly model problem-solving approaches for one another and to explicitly model techniques themselves when students do not naturally discover/ apply them. Many of the suggested interventions included in lesson plans for small group interventions direct teachers to explicitly model strategies more frequently; this is in alignment with the recommendations of the National Mathematics Advisory Panel, which specifies that not all mathematics instruction should be explicit, but rather that struggling students receive some explicit instruction regularly. Teachers should feel comfortable explicitly modeling strategies for students as needed, so long as students are able to articulate how and why the strategies work. (This understanding should be built by introducing and relating strategies along the CPA continuum as discussed above.)

	General instruction for all students involves many rounds of practice, opportunities to discuss thinking through turn and talks, and feedback; students in small group interventions naturally receive more opportunities for feedback given the lowered teacher to student ratio. We also recommend that teachers incorporate additional opportunities for students to get “meta” about their strategies through extra turn and talks during intervention and/or through conferencing one-on-one with teachers. Checks for understanding that ask students to articulate their thinking and explain their strategies are included in lesson plans to help teachers think of questions they may ask.
Recommendation 4: Interventions should include instruction on solving word problems that is based on common underlying structures.	The Math Stories Guide should be used as a resource for interventions for students struggling with math stories. Experts recommend teaching students who are struggling with math stories to recognize problem types/ structures and to use their understanding of those to determine appropriate solutions (Yan Ping Xin et al., 2005). The math stories protocol outlined in the guide is designed based on the recognition that the breakdown for students is often in understanding the problem. By strategically separating the operating (reading and identifying the operation used to solve) from the calculating doing the math with the numbers to get an answer) into separate steps of the protocol, teachers are able to better support struggling students. The steps of visualizing and representing the story problem support reading and listening comprehension as a bridge to then behind able to identify structures and choose a calculation strategy. While there are opportunities to discuss computation and number sense within the protocol, the overarching intention of math stories is to understand why decontextualized computation *makes sense* given a context through representation. Each story problem type in the Math Stories Guide is coded by problem type to support teachers in understanding the various structures and helping students to do so as well. A guide to each problem type is also included for teachers. The representation step of the math stories protocol encourages visual models; using visual representations to model story problems helps students recognize the structure of various problem types and to relate various problem types and structures so that they can transfer known solution methods from familiar to unfamiliar problems. When students are struggling with story problems, we encourage teachers to focus more heavily on concrete and pictorial representations to build understanding. Additionally, to support student understanding of different modeling and solution strategies in different contexts, several sample discussion questions and key points are provided in the Math Stories Guide.
Recommendation 5. Intervention materials should include opportunities for students to work	As noted above, the AF Math powered by Leap Educational Consulting Program is designed to move students along the concrete-pictorial-abstract (CPA) continuum as they develop conceptual

with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	understanding of grade level mathematics. This is 100% in alignment with this recommendation from the WWC. All unit overviews and the Math Stories Guide include an outline of concrete-pictorial-abstract strategies that students may use to solve problems. Teachers first move when intervening should be to move students back along the continuum. If a student is struggling with abstract application of a concept, the intervention should move back to pictorial models and solution strategies and then use questioning to relate the pictorial to abstract. If a student is struggling with the pictorial, teachers should move back to concrete. Beyond supporting students with learning disabilities, use of the CPA method of instruction has been shown to reduce mathematical anxiety of students in primary school (Putri et al., 2020b) and to increase students' sense of self-efficacy (Yuliyanto et al., 2019) and thus is recommended for use in general instruction as well.
Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	10-15 minutes of Calendar/ Practice is a recommended component of the Program. Teachers may opt to use some or all of this time to pull small groups for targeted practice to build fluency. Additionally, every grade level includes a practice workbook that is largely focused on fluency. Additional recommendations for building fluency from WWC include explicitly introducing efficient strategies for counting to add and subtract for grades K-2. Strategies such as counting on, up, and back, along with efficient strategies to add and subtract based on derived facts are central in many lessons in lower elementary. As teachers provide interventions in these lessons, they should be sure to encourage students to work toward the most efficient strategies in order to help students develop fluency.
Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Exit tickets and unit assessments are provided to help teachers monitor the progress of students receiving intervention. Teachers should feel free to re-use exit tickets and/or to re-administer unit pre- and post-assessments as well.
Recommendation 8. Include motivational strategies in tier 2 and tier 3 interventions.	Motivational strategies are not explicitly built into the curriculum, but the AF Math powered by Leap Educational Consulting Program recognizes the importance and impact of such strategies on student achievement and encourages teachers to incorporate these into their instruction for all students, especially those receiving tier 2 and 3 interventions. Some strategies we have seen teachers use successfully and recommend are (in order of ease of implementation/ level of intervention): 1. Basic Verbal and Nonverbal Praise: Things as simple as high fives, smiles, and positive narration can have a positive impact on student motivation. We recommend that these moments of praise are linked to engagement, focus, and effort. 2. A Point System: Students may earn points either individually or as a small group for focus and effort. These points can be prizes unto themselves or may be linked to a reward. Rewards

	might be items, phone calls home, or a group activity like extra time with pattern blocks or a dance party. 3. Student Goal Setting and Monitoring: Students can set their own academic and behavioral goals at the start of a year, unit, or term. Each day or every several days, they reflect on their progress toward that goal. This is most effective for older students, and we recommend pairing it with one of the above recommendations.
5 Elements of Explicit and Systematic Instruction Recommended by The Council for Learning Disabilities (Hott et al., 2014)	
Recommended Element	**Guidance for Incorporating this Recommendation into Instruction**
Element 1: Specific and Clear Teacher Models	As outlined above (WWC Recommendation 3), specific and clear teacher models are rarely built into materials for general instruction, as the curriculum is intended to be problem-based and student-driven; nonetheless, the AF Math powered by Leap Educational Consulting Program does encourage the use of specific and clear student models that are shared whole group for the purpose of clearly modeling mathematical strategies and thinking for students who may be struggling. (In some instances, teachers also are directed to explicitly model if a particular focal strategy is not used by students.) Lesson plans include guidance for teachers, including questions to ask, to help students clearly articulate their thinking and give examples so that their peers may make sense of their models and apply them. Similarly, the lesson plans include questions teachers may ask to ensure student understanding of the models after they are shared; these are often in the form of a turn and talk after the strategy/ modeling share and/or the consolidate the learning question at the end of the introduction. Additionally, as noted above, many of the suggested interventions included in lesson plans for small group interventions direct teachers to explicitly model strategies more frequently; this is in alignment with the recommendations of the National Mathematics Advisory Panel, which specifies that not all mathematics instruction should be explicit, but rather that struggling students receive some explicit instruction regularly. Teachers should feel comfortable explicitly modeling strategies for students as needed, so long as students are able to articulate how and why the strategies work.
Element 2: Examples that are Sequenced in Level of Difficulty	Examples and problems introduced in whole and small groups are strategically sequenced along the concrete-pictorial-abstract continuum to build conceptual understanding. Sometimes the level of difficulty builds over the course of the year, other times over the course of a unit or several days, and sometimes it builds within a single lesson. As described above (WWC Recommendation 5), teachers should always feel comfortable moving flexibly along the continuum based on student data. Most materials can be accessed anywhere along the continuum. (For example, a problem asking students to combine 5 and 6 can be tackled concretely using cubes, pictorially drawing pictures, or abstractly by counting on or making a ten.)

	Within student work, examples are also categorized by level of difficulty. In Game Intro Lessons (K-2 only), this takes the form of adjusting the game; suggestions for how to adjust the game to increase or decrease difficulty level within the standard and without increasing/ decreasing workload are provided in the workshop section of the plan under suggested interventions and extensions. In Exercise-based lessons (2-4 only), problems within the independent work are strategically sequenced to help students build on their mastery of the concept from basic to more advanced problems; the problems are also labeled by difficulty. By clearly marking these problems in terms of difficulty, teachers can easily differentiate for student needs and allow all students to complete the same number of problems aligned to the same standards and objectives, but at the appropriate difficulty level.
Element 3: Scaffolding	All lesson plans include a teacher script included scaffolded questions to be used as needed based on student understanding. These are often in the forms of sub-bullets or are sometimes labeled "back-pocket questions" in the plan. These additional questions are designed to provide scaffolding to the original question in the script to be used only if needed if students are struggling. These questions may be useful for differentiation for students with disabilities and could be used for co-teaching models or additional one-on-one support for students. Additionally, nearly every question in the lesson plan includes an exemplar student response. These exemplar responses are included so that teachers can recognize when students are not articulating the understanding required by the lesson and can plan to scaffold up. As a part of the lesson internalization process, teachers should anticipate student responses to the key questions in the lesson plan and backwards plan any additional scaffolds they might use to move students from the anticipated response to the exemplar one. The common misconceptions sections of unit overviews are a useful resource for this internalization process.
Element 4: Consistent Feedback	A key tenet of all lesson types is student discussion and feedback. Throughout each lesson, students have multiple opportunities to work independently, with partners, and engage in class discussion and analysis of student work. One purpose of this is to ensure all students, but especially those with disabilities or who struggle in mathematics, have multiple opportunities to revise and check their own mastery. Additionally, the lesson plans include guidance to teachers about circulating to provide feedback. In the Workshop section of the plan, there are suggestions to teachers about what to focus each "lap" of feedback on as they circulate the room. The first lap often focuses on procedural skills, while additional laps focus on conceptual skills. (The lesson plans suggest a focus for 2 laps, but teachers may circulate as many times as they can within a block.) The suggestions for the focus of feedback can be applied to both workshop and independent practice, and as teachers circulate, they should have pen-in-hand in

	order to give immediate and efficient feedback. Guidance for what teachers should look for and questions they can ask to ensure understanding as they circulate are provided within this section of the plan.
Element 5: Frequent Opportunity for Cumulative Review	Cumulative Review is a component of the AF Math powered by Leap Educational Consulting Program. It takes the form of calendar math/ practice in lower elementary and practice and cumulative review in upper elementary. Students are provided time for skills fluency and mixed practice to ensure mastery and fluency is developed with previously mastered content and opportunities are strategically built in for teachers to include re-teaches and/or practice with previously unmastered content.

Supporting Advanced Students

Part of supporting all learners is ensuring that advanced students also have opportunities to learn and grow by engaging with the grade level content at higher levels of complexity.

A problem-based approach is naturally differentiated as students choose the strategies they use to model and solve the problem. Teachers highlight particular strategies for the class, but they always affirm any strategy that works, regardless of its level of complexity. In a classroom implementing the AF Math powered by Leap Educational Consulting Program, students are expected to work with a variety of tools and strategies even as they work through the same set of problems; this allows advanced students to engage with the content at higher levels of complexity.

Daily lessons also provide suggested extension activities for students in the Workshop Section of the lesson plan so that teachers can encourage students to engage with the content at a higher level of complexity if they are not doing so naturally but are ready to. These extension suggestions include variations of the game that encourage more sophisticated strategies in Game Intro Lessons (K-2) and variations of the tasks or suggested strategies or tools students may use in Exercise Based Lesson (2-4). The independent practice for grades Exercise Based Lessons also include problems labeled by difficulty. Teachers should differentiate for student needs by assigning the most challenging problems to advanced students while allowing them to skip some of the simpler ones, so that they can engage with the same number of problems, but at the appropriate difficulty level.

Additionally, the Discussion section of the daily lesson plans always include a potential whole class extension/ application problem. These are often additional problems or tasks that ask students to apply the mathematical concepts taught that day, and like the focal problem of the day, students should be encouraged to use the strategy that makes sense to them in order to solve, once again allowing students to engage with the grade level content at a level that is appropriate to them.

Unit Overviews also provide aligned tasks. If teachers opt to use these tasks to supplement instruction, all students should engage in the task, but similar to the focal problems in the introduction, students can use their own strategies to solve, which allows for natural differentiation.

Supporting Multilingual and English Language Learners[1]

The AF Math powered by Leap Educational Consulting Program appreciates the importance of creating a classroom environment in which Multilingual and English language learners (MLLs/ ELLs) can thrive socially, emotionally, and academically. MLLs/ ELLs have the double-task of learning mathematics while continuing to build their language mastery. Therefore, additional support and thoughtful curriculum is often needed to ensure their mastery and support in learning. Our materials are designed to help teachers recognize and serve the unique educational needs of MLLs/ ELLs while also celebrating the assets they bring to the learning environment, both culturally and linguistically.

In designing the materials to ensure they adequately support teachers of MLLs/ ELLs, we relied heavily on the math guidelines and resources provided by the English Learners Success Forum.[2] Both the Game Introduction Lessons in lower elementary and the Exercise Based Lessons in upper elementary along with the Math Stories Protocols used in Math Stories at all grade levels build on the four design principles for promoting mathematical language use and development in curriculum and instructions outlined by Stanford's Graduate School of Education (Zwiers et al., 2017), Understanding Language/SCALE and recommended by the English Language Success Forum, and we have strategically included several mathematical language routines (MLRs) to support the language and content development of MLLs/ ELLs in all lesson plans and called them out explicitly for teachers in a third of lesson plans . We encourage teachers to incorporate more of these language routines into daily lessons (using the lessons that explicitly identify them already as guides) and the Math Stories block as student data indicates the need for more. These design principles and the MLRs to support MLLs/ ELLs are outlined below, along with guidance about how to implement each routine more regularly into the curriculum if needed:

Overview of Design Principles and Recommended Mathematical Language Routines (MLRs) [3]	
Design Principle 1: Support sense-making	
Principle in Action	**Overview of Recommended MLRs** (see table below for guidance for implementation)
Daily lesson plan scripts and the math stories protocols intentionally amplify rather than simplify student language by anticipating where students may have difficulty accessing the concepts and language and providing multiple ways for them to come to understanding. Every	• **MLR 2: Collect and Display:** The teacher captures student thinking and/or strategies visually and leads the class in a discussion. *Purpose: To capture students' oral words and phrases into a stable, collective reference.*

[1] We use both the terms Multilingual Learner (MLL) and English Language Learner (ELL) to refer to students who are learning English in addition to any other language(s) they speak or hear at home. The AF Math powered by Leap Educational Consulting Program appreciates the importance of taking an asset-based approach with students who are developing their at-home language or languages in addition to English. Rather than choose a single preferred term on behalf of the diverse group of learners who are developing additional languages, we encourage schools to choose which term they wish to use based on the preferences of their student populations.

[2] "Guidelines for Improving Math Materials for English Language Learners" by ELSF licensed under Creative Commons Attribution International 4.0 (CC BY-NC-SA.)

[3] This table, "Overview of Design Principles and Recommended Mathematical Language Routines" is a revision of "Principles for the Design of Mathematics Curricula: Promoting Language and Content Development" by Zwier, et. al licensed under Creative Commons Attribution International 4.0 (CC BY-NC-SA.)

lesson includes multiple opportunities for students to engage in discussion with one another, often through turn and talks, as they make sense of the content, and this sense-making is also supported through the use of concrete and pictorial models and a lesson visual anchor that captures student thinking and mathematical concepts and key vocabulary. Key concepts are highlighted as what and how key points for each lesson plan for teachers and the script directs teachers to stamp these key points through questioning and reverbalizations. The key stamping of these concepts help all students, but especially MLLs/ ELLs to be clear on the key take-aways for each lesson. Additionally, teachers are provided with student-friendly vocabulary definitions for all new vocabulary terms in the unit plan that can support MLLs/ ELLs further.	● **MLR 6: Three Reads:** Teachers support students in making sense of a situation or problem by reading three times, each time with a particular focus. *Purpose: To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning.* ● **MLR 8: Discussion Supports:**Teachers use a number of moves to help facilitate student discussion including revoicing, encouraging students to agree, disagree, build on, or ask questions of their peers, incorporating choral response to build vocabulary, showing concepts multi-modally, and modeling clear explanations/ think alouds. *Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.*
Design Principle 2: Optimize Output	
Principle in Action	**Recommended Language Routines** (see table below for guidance for implementation)
Lessons and the math stories protocols are strategically built to focus on student thinking. Students engage in each new task individually or with partners, have opportunities to discuss with one another, and then analyze student work samples as a whole class. These multiple opportunities to engage with the content focus on deepening and building on student understanding. All students benefit from the focus on the mathematical discourse and revising their own thinking, but this is especially true of MLLs/ ELLs who will benefit from hearing other students thinking and reasoning on the concepts and/or different methods of solving.	● **MLR 1: Stronger and Clearer Each Time:** Teachers provide students with multiple opportunities to articulate their mathematical thinking, with the opportunity to refine their language with each successive share. *Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output.* ● **MLR 3: Critique, Correct, and Clarify:** Teachers present students with a statement, an argument, an explanation, or a solution, and prompt them to analyze and discuss. *Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.* ● **MLR 4: Info Gap:** Students are put into pairs; each student in the pair is given partial information that when combined with their partner's information provides the full context needed to solve the problem. Students must communicate effectively in order to solve the problem. *Purpose: To create a need for students to communicate.* ● **MLR 5: Co-Craft Questions and Problems:** Teachers guide students to work with one another to create questions or situations for math problems or to create entire problems and then solve them. *Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students*

	to analyze how different mathematical forms can represent different situations. ● **MLR 7: Compare and Connect:** Teachers prompt students to understand one another's strategies by comparing and connecting other students' approaches to their own. *Purpose: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.*
Design Principle 3: Cultivate Conversation	
Principle in Action	**Recommended Language Routines** (see table below for guidance for implementation)
A key element of all lesson types is student discussion. Daily lesson plans and the math stories protocol rely heavily on the use of individual or partner think time, turn-and-talks with partners, and whole class discussion to answer key questions throughout the lesson script. The rationale for this is that all learners, but especially MLLs/ ELLs benefit from multiple opportunities to engage with the content. Students that are building their mastery of the language may struggle more with following a whole-class discussion; however, having an opportunity to ask questions and discuss with a strategic partner beforehand can help deepen their understanding and empower them to engage further in the class discussion. These smaller conversations also provide opportunities for teachers to strategically scan student work and listen to discussions to engage more learners.	● **MLR 1: Stronger and Clearer Each Time:** Teachers provide students with multiple opportunities to articulate their mathematical thinking, with the opportunity to refine their language with each successive share. *Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output.* ● **MLR 3: Critique, Correct, and Clarify:** Teachers present students with a statement, an argument, an explanation, or a solution, and prompt them to analyze and discuss. *Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.* ● **MLR 4: Info Gap:** Students are put into pairs; each student in the pair is given partial information that when combined with their partner's information provides the full context needed to solve the problem. Students must communicate effectively in order to solve the problem. *Purpose: To create a need for students to communicate.* ● **MLR 5: Co-Craft Questions and Problems:** Teachers guide students to work with one another to create questions or situations for math problems or to create entire problems and then solve them. *Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.* ● **MLR 7: Compare and Connect:** Teachers prompt students to understand one another's strategies by comparing and connecting other students' approaches to their own. *Purpose: To foster students' meta-awareness as they identify, compare, and*

	contrast different mathematical approaches, representations, concepts, examples, and language. ● **MLR 8: Discussion Supports:**Teachers use a number of moves to help facilitate student discussion including revoicing, encouraging students to agree, disagree, build on, or ask questions of their peers, incorporating choral response to build vocabulary, showing concepts multi-modally, and modeling clear explanations/ think alouds. *Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.*
Design Principle 4: Maximize Meta-Awareness	
Principle in Action	**Recommended Language Routines** (see table below for guidance for implementation)
Every daily lesson and math stories lesson is structured so that students have many opportunities to get "meta" about the mathematical processes they engage in. Students explain how they model and solve problems to the teacher and one another throughout the lesson, often through turn and talks in which they also evaluate their peers' strategies and thinking. Lesson scripts also encourage students to draw connections between new content and previous learning as well as between different strategies. As students share with one another, they develop meta-awareness of how they are communicating with one another, and as they make connections between concepts and new and previous learning, they are reflecting on their own and others' learning of the content.	● **MLR 2: Collect and Display:** The teacher captures student thinking and/or strategies visually and leads the class in a discussion. *Purpose: To capture students' oral words and phrases into a stable, collective reference.* ● **MLR 3: Critique, Correct, and Clarify:** Teachers present students with a statement, an argument, an explanation, or a solution, and prompt them to analyze and discuss. *Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.* ● **MLR 5: Co-Craft Questions and Problems:** Teachers guide students to work with one another to create questions or situations for math problems or to create entire problems and then solve them. *Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.* ● **MLR 6: Three Reads:** Teachers support students in making sense of a situation or problem by reading three times, each time with a particular focus. *Purpose: To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning.* ● **MLR 7: Compare and Connect:** Teachers prompt students to understand one another's strategies by comparing and connecting other students' approaches to their own.

	Purpose: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language. ● **MLR 8: Discussion Supports:** Teachers use a number of moves to help facilitate student discussion including revoicing, encouraging students to agree, disagree, build on, or ask questions of their peers, incorporating choral response to build vocabulary, showing concepts multi-modally, and modeling clear explanations/ think alouds. *Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.*

Implementing Language Routines[4]	
Mathematical Language Routine (MLR)	**Guidance for Implementation**
MLR 1: Stronger and Clearer Each Time *Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output.* Teachers provide students with multiple opportunities to articulate their mathematical thinking, with the opportunity to refine their language with each successive share.	Multiple opportunities to turn and talk are built into the daily lessons and math stories protocol; often students have many opportunities to refine their thinking and clarify as they reiterate key points through the turn and talk of the initial problem, the consolidate the learning turn and talk, and turn and talks in the mid-workshop interruption and discussion portions of the lessons. Teachers may further support MLLs/ ELLs by having students turn to multiple partners for each turn and talk, so that instead of just one partner for each question, they turn to at least 2, refining their language each time. Another option is to have students create 3 separate iterations of their mathematical arguments, each with a different focus. For example, they may have the initial turn and talk include only the "what," or the actual solution, the second may include the "what" and the "how," or the solution and how they came to it, and the third can include the "what," "how," and "why," or the solution, how they came to it, and why that works or makes sense. To further strengthen these routines for MLLs/ ELLs, teachers may also consider incorporating a pre-write and/or additional think time before each turn and talk.
MLR 2: Collect and Display *Purpose: To capture students' oral words and phrases into a stable, collective reference.* The teacher captures student thinking and/or strategies visually and leads the class in a discussion.	This routine can and should be incorporated into every game intro, exercise-based, and math stories lesson as teachers co-create the visual anchor with students when teachers capture student strategies and thinking as they share. Any time that a teacher captures student thinking and/or work visually on an anchor chart for the class and leads a discussion using the anchor chart, they are engaging in a version of this routine. To take this routine to the next level, teachers may also opt to chart the specific language and vocabulary students use to explain their strategies and lead the class in a discussion of how and which language and visuals

[4] This table, "Implementing Mathematical Language Routines" is a revision of "Principles for the Design of Mathematics Curricula: Promoting Language and Content Development" by Zwier, et. al licensed under Creative Commons Attribution International 4.0 (CC BY-NC-SA.)

	help to clearly communicate ideas.
MLR 3: Critique, Correct, and Clarify *Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.* Teachers present students with a statement, an argument, an explanation, or a solution, and prompt them to analyze and discuss.	This routine is worked into nearly every lesson as an option for the mid-workshop interruption and discussion when teachers lead students in the error analysis of a common misconception. To maximize the support for MLLs/ ELLs, teachers should allow students to work both individually and in pairs to identify and fix the error and then lead the class in sharing out whole group and refining their work. This routine can also be easily incorporated into many units, most naturally Geometry units but in others as well, in the form of a "sometimes/ always/ never" task. In this task, teachers provide statements and students determine if statements are sometimes, always, or never true. For example, teachers may present statements like, "A rectangle has sides of equal lengths," or "When we add two odd numbers, the sum is even," and students recognize that the former is sometimes true, while the latter is always true.
MLR 4: Info Gap *Purpose: To create a need for students to communicate.* Students are put into pairs; each student in the pair is given partial information that when combined with their partner's information provides the full context needed to solve the problem. Students must communicate effectively in order to solve the problem.	While this is not regularly incorporated into lesson materials, teachers can easily modify materials to incorporate this routine into the workshop of most lessons or into math stories if their students need more additional language development support. For any story problem or word problem, teachers can provide one student with the general problem or the context and the other with the data. The data can be presented in the form of tables, graphs, diagrams, etc. Students will need to read their information, which will give them the opportunity to practice reading English language, and then they will need to share with one another, asking clarifying questions, which will allow them to practice oral input and output. Outside of contextual problems, teachers can also set students up to play partner games in which one partner has information the other needs. For example, when solving for a missing addend or factor, one partner may know the known addend or factor and the other may know the whole or product; students need to communicate with one another to combine the known information and solve for the unknown.
MLR 5: Co-Craft Questions and Problems *Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.* Teachers guide students to work with one another to create questions or situations for math problems or to create entire problems	Most grade levels include at least some lessons in story problem units that task students with creating their own problems to solve. A low-lift way to incorporate more opportunities for students to engage in this language routine is to have it be a go-to routine for what to do when students finish early, particularly in the math stories block. When students finish any step of the protocol, they may use the additional time to craft their own story problems and then solve them. To scaffold for students, several variations of this routine may be used. Teachers may provide the situation and students may generate questions only. Alternatively, teachers may provide a mathematical model (equation, tape diagram, number bond, etc.) and then students may craft the situation.

and then solve them.	
MLR 6: Three Reads *Purpose: To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning.* Teachers support students in making sense of a situation or problem by reading three times, each time with a particular focus.	This routine can easily be incorporated into the Math Stories Protocol (or any lesson focused on math stories) by assigning students particular parts of the protocol for each reading. For example, the teacher may ask students to close their eyes and visualize for the first reading, to represent during the second reading, and to check their representation and/or to begin to think about solution strategies during the third reading. For students who are reading independently, teachers can consider asking students to strategically annotate for what they know during the first reading, what they need to figure out during the second reading, and to represent during the third reading.
MLR 7: Compare and Connect *Purpose: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.* Teachers prompt students to understand one another's strategies by comparing and connecting other students' approaches to their own.	Most daily lesson plans and the math stories protocol lend themselves easily to this routine because they include strategy and modeling shares and many already include questions aligned to the routine that ask students to compare and connect their own strategies to those shared whole group. (Often these connections are made along the CPA continuum as students are asked to relate pictorial strategies to concrete ones and abstract strategies to pictorial and concrete ones.) Any time that a lesson plan asks students to compare strategies after a strategy share or asks students how/why different strategies work, teachers are leading students in a version of this routine. To find more opportunities to engage students in this routine, teachers can add questions after strategy shares such as: - What is the same about these strategies? What is different? - Why do both strategies work to solve this particular problem? - What is particularly useful about this strategy? Why might someone choose to use this strategy over the other?
MLR 8: Discussion Supports *Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.* Teachers use a number of moves to help facilitate student discussion including revoicing, encouraging students to agree, disagree, build on, or ask questions of their peers, incorporating choral response to build	This routine is critical to strong problem-based instruction and successful teachers implementing the AF Math powered by Leap Educational Consulting Program incorporate it into their instruction daily. Many lesson scripts include discussion supports already, especially choral response and questions asking students to agree/ disagree with one another. Teachers can further incorporate this routine by modeling clear explanations as needed, particularly at the start of the year and providing sentence frames and starters to support students in clearly articulating their ideas. Explicit vocabulary instruction is also helpful in supporting discussion. Sentence frames and vocabulary are provided in unit overviews to support teachers in supporting student discussion.

vocabulary, showing concepts multi-modally, and modeling clear explanations/ think alouds.	