LEAP
Educational Consulting

Connecticut Mathematics Model Curricula Alignment

Resource Name:

AF Math powered by Leap Educational Consulting

Alignment Grade 3			
Model Unit Name	**Model Unit Standards**	**Resource Unit(s) Number and Resources Lessons**	**Pacing**
This is the title of the unit in the model curricula	*These are the standards addressed in the unit*	*This is the unit(s) that aligns with the model unit from the resource* *These are the lessons from the identified units that align to the standards within the model unit*	*This is the expected number of days for instruction*
Understanding Multiplication and Division	3.OA.A.1, 3.OA.A.2, 3.MD.B.3	**Unit 1: Multiplication, Division, and Area 1:** ● Lesson 1: Equal Groups as Multiplication- 3.OA.A.1 ● Lesson 2: Relate Multiplication to Arrays- 3.OA.A.1 ● Lesson 3: Relate Multiplication to Arrays Day 2- 3.OA.A.1 ● Lesson 4: Interpret the Meaning of Factors- 3.OA.A.1 ● Lesson 5: Write multiplication stories- 3.OA.A.1 ● Lesson 13: Unknown as Group Size Division- 3.OA.A.2* ● Lesson 14: Unknown as Number of Groups Division- 3.OA.A.2* ● Lesson 15: Mixed Practice: Unknown as Group Size or Number of Groups Division- 3.OA.A.2* ● Lesson 16: Write division stories- 3.OA.A.2* **Unit 2: Time and Graphing:** ● Lesson 1: Generate & Organize Data: Pictographs- 3.MD.B.3 ● Lesson 2: Collect & Organize Data – Tape Diagrams- 3.MD.B.3 ● Lesson 3: Create Scaled Bar Graphs- 3.MD.B.3 ● Lesson 4: Collect & Display Data Project- 3.MD.B.3 ● Lesson 5: Solve 2-step Story Problems About Graphs – Day 1- 3.MD.B.3 ● Lesson 6: Solve 2-step Story Problems About Graphs – Day 2- 3.MD.B.3 ● Lesson 7: Create Graphs and Questions Based on Graphs- 3.MD.B.3	16 days

		*These lessons also address standards 3.OA.A.4, 3.OA.B.5, 3.OA.B.6.	
Connecting and Using Multiplication and Division	3.OA.A.3, 3.OA.A.4, 3.OA.B.5, 3.OA.B.6, 3.OA.C.7	**Unit 1: Multiplication, Division, and Area 1:** ● Lesson 13: Unknown as Group Size Division- 3.OA.A.4, 3.OA.B.5, 3.OA.B.6** ● Lesson 14: Unknown as Number of Groups Division- 3.OA.A.4, 3.OA.B.5, 3.OA.B.6** ● Lesson 15: Mixed Practice: Unknown as Group Size or Number of Groups Division- 3.OA.A.4, 3.OA.B.5, 3.OA.B.6** ● Lesson 16: Write division stories- 3.OA.A.4, 3.OA.B.5, 3.OA.B.6** ● Lesson 17: Interpret unknowns using arrays (use variable/symbols to represent the unknown)- 3.OA.A.4, 3.OA.B.5, 3.OA.B.6 ● Lesson 18: Division as unknown factor arrays/tape diagrams- 3.OA.A.4, 3.OA.B.5, 3.OA.B.6 ● Lesson 19: Division as unknown factor arrays/tape diagrams Day 2- 3.OA.A.4, 3.OA.B.5, 3.OA.B.6 ● Lesson 20: Commutative & Associate Property with arrays- 3.OA.B.5 ● Lesson 21: Zero and identity property- 3.OA.B.5 **Unit 7: Multiplication, Division, and Area 2:** ● Lesson 1: Patterns with 2, 5, and 10- 3.OA.A.3, 3.OA.C.7* ● Lesson 2: Patterns with 2, 4, and 8- 3.OA.A.3, 3.OA.C.7* ● Lesson 3: Patterns with 3 and 6- 3.OA.A.3, 3.OA.C.7* ● Lesson 4: Patterns with 9- 3.OA.A.3, 3.OA.C.7* ● Lesson 5: Mixed Practice: All Fact Patterns- 3.OA.A.3, 3.OA.C.7* ● Lesson 6: Arrays and the Distributive Property- 3.OA.A.3, 3.OA.B.5, 3.OA.C.7* ● Lesson 7: Distributive Property Problems with all Digits as Factors- 3.OA.A.3, 3.OA.B.5, 3.OA.C.7* ● Lesson 8: Write Multiplication Stories- 3.OA.A.3, 3.OA.B.5, 3.OA.C.7* ● Lesson 9: Write Division Stories- 3.OA.A.3, 3.OA.B.5, 3.OA.C.7* ● Lesson 10: Understand Parentheses and Use to Solve- 3.OA.A.3, 3.OA.B.5, 3.OA.C.7* *These lessons also address standard 3.OA.D.9. **These lessons also address standard 3.OA.A.2.	19 days
Computing with Whole Numbers	3.NBT.A.1, 3.NBT.A.2, 3.NBT.A.3, 3.OA.C.7, 3.OA.D.8, 3.OA.D.9	**Unit 1: Multiplication, Division, and Area 1:** ● Lesson 22: Multiply with multiples of 10 (Concrete)- 3.NBT.A.3 ● Lesson 23: Multiply with multiples of 10 (Pictorial)- 3.NBT.A.3 ● Lesson 24: Multiply by multiples of 10 (Abstract)- 3.NBT.A.3 **Unit 3: Addition, Subtraction, and Time:** ● Lesson 1: Round to Nearest Ten Vertical Number Line- 3.NBT.A.1	47 lessons

- Lesson 2: Round to Nearest Hundred Vertical Number Line- 3.NBT.A.1
- Lesson 3: Round to Nearest Ten or Hundred- 3.NBT.A.1
- Lesson 4: Rounding Puzzles- 3.NBT.A.1
- Lesson 5: Add with Expanded Notation to Hundreds Place- 3.NBT.A.2
- Lesson 6: Add by Place to Hundreds Place- 3.NBT.A.2
- Lesson 7: Add by Place Multiple Addends- 3.NBT.A.2
- Lesson 8: Subtract with Expanded Notation to Hundreds Place- 3.NBT.A.2
- Lesson 9: Subtract with Number Line- 3.NBT.A.2
- Lesson 10: Add Subtract Round in Word Problems - 3.NBT.A.2

Unit 7: Multiplication, Division, and Area 2:

- Lesson 1: Patterns with 2, 5, and 10- 3.OA.C.7, 3.OA.D.9*
- Lesson 2: Patterns with 2, 4, and 8- 3.OA.C.7, 3.OA.D.9*
- Lesson 3: Patterns with 3 and 6- 3.OA.C.7, 3.OA.D.9*
- Lesson 4: Patterns with 9- 3.OA.C.7, 3.OA.D.9*
- Lesson 5: Mixed Practice: All Fact Patterns- 3.OA.C.7, 3.OA.D.9*
- Lesson 6: Arrays and the Distributive Property- 3.OA.C.7, 3.OA.D.9**
- Lesson 7: Distributive Property Problems with all Digits as Factors- 3.OA.C.7, 3.OA.D.9**
- Lesson 8: Write Multiplication Stories- 3.OA.C.7, 3.OA.D.9**
- Lesson 9: Write Division Stories- 3.OA.C.7, 3.OA.D.9**
- Lesson 10: Understand Parentheses and Use to Solve- 3.OA.C.7, 3.OA.D.9**

Unit 8: Story Problems:

- Lesson 1: Patterns in the Addition Table- 3.OA.D.9
- Lesson 2: Patterns in the Addition Table Day 2- 3.OA.D.9
- Lesson 3: Patterns in the Multiplication Table- 3.OA.D.9
- Lesson 4: Patterns in the Multiplication Table Day 2- 3.OA.D.9
- Lesson 5: Mixed Patterns- 3.OA.D.9
- Lesson 6: Two-Step Story Problems Day 1 Represent All Four Operations- 3.OA.D.8
- Lesson 7: Two-Step Story Problems All Four Operations Day 2s- 3.OA.D.8
- Lesson 8: Two-Step Story Problems Day 3s- 3.OA.D.8
- Lesson 9: Two-Step Story Problems Day 4 Multiples of Tens- 3.OA.D.8
- Lesson 10: Two-Step Story Problems Day 5 Multiples of Ten Additional Practices- 3.OA.D.8
- Lesson 11: Two-Step Story Problems Day – Area and Perimeter Day 1s- 3.OA.D.8
- Lesson 12: Two-Step Story Problems – Area and Perimeter Day 2s- 3.OA.D.8
- Lesson 13: Perimeter Robot Projects- 3.OA.D.8
- Lesson 14: Multi-Step Story Problems Extension <extension level>s- 3.OA.D.8

*These lessons also address standard 3.OA.A.3.

		**These lessons also address standard 3.OA.A.3. and 3.OA.B.5.	
Exploring Measurement and Data	3.MD.A.1, 3.MD.A.2, 3.MD.B.3, 3.MD.B.4	**Unit 2: Time and Graphing:** ● Lesson 1: Generate & Organize Data: Pictographs- 3.MD.B.3 ● Lesson 2: Collect & Organize Data – Tape Diagrams- 3.MD.B.3 ● Lesson 3: Create Scaled Bar Graphs- 3.MD.B.3 ● Lesson 4: Collect & Display Data Project- 3.MD.B.3 ● Lesson 5: Solve 2-step Story Problems About Graphs – Day 1- 3.MD.B.3 ● Lesson 6: Solve 2-step Story Problems About Graphs – Day 2- 3.MD.B.3 ● Lesson 7: Create Graphs and Questions Based on Graphs- 3.MD.B.3 **Unit 3: Addition, Subtraction, and Time:** ● Lesson 12: Time as continuous measurement- 3.MD.A.1 ● Lesson 13: Tell Time to the Nearest Minute- 3.MD.A.1 **Unit 4: Measurement:** ● Lesson 1: Measure and Estimate Mass- 3.MD.A.2 ● Lesson 2: Word Problems with Mass- 3.MD.A.2 ● Lesson 3: Measure and Estimate Metric Capacity- 3.MD.A.2 ● Lesson 4: Mixed Word Problems with Mass and Capacity- 3.MD.A.2 ● Lesson 5: Mixed Word Problems with Mass and Capacity Day 2- 3.MD.A.2 ● Lesson 6: Mixed Word Problems with Money- 3.MD.A.2 ● Lesson 7: Mixed Word Problems with Money, Mass and Capacity- 3.MD.A.2 **Unit 6: Length and Perimeter:** ● Lesson 1: Create Ruler and Measure to Nearest Quarter-Inch- 3.MD.B.4 ● Lesson 2: Measure with Broken Ruler- 3.MD.B.4 ● Lesson 3: Measure with a Ruler to Quarter inch- 3.MD.B.4 ● Lesson 4: Interpret Measurement Data from a Line Plot- 3.MD.B.4 ● Lesson 5: Create Line Plots- 3.MD.B.4 ● Lesson 6: Measure Create a Line Plot and Interpret Data- 3.MD.B.4	22 lessons
Understand Area and Perimeter	3.MD.C.5, 3.MD.C.6, 3.MD.C.7, 3.MD.D.8	**Unit 1: Multiplication, Division, and Area 1:** ● Lesson 6: Area as an Attribute- 3.MD.C.5, 3.MD.C.6 ● Lesson 7: Compare Area by Decomposing and Recomposing Shapes- 3.MD.C.5, 3.MD.C.6 ● Lesson 8: Model Tiling to Measure Area- 3.MD.C.5, 3.MD.C.6 ● Lesson 9: Relate Side Length to tiles on a side- 3.MD.C.5, 3.MD.C.6, 3.MD.C.7 ● Lesson 10: Relate Side Length to tiles on a side Day 2 and Error Analysis- 3.MD.C.5, 3.MD.C.6, 3.MD.C.7 ● Lesson 11: Draw rows and columns to tile determine area; use multiplication equations- 3.MD.C.5, 3.MD.C.6, 3.MD.C.7	24 lessons

		• Lesson 12: Determine area using multiplication and tiling, Day 2- 3.MD.C.5, 3.MD.C.6, 3.MD.C.7 **Unit 6: Length and Perimeter:** • Lesson 7: Find Perimeter Using String- 3.MD.D.8 • Lesson 8: Determine Perimeter All Sides Known- 3.MD.D.8 • Lesson 9: Determine Perimeter with Unknown Sides Simple Shapes- 3.MD.D.8 • Lesson 10: Determine Perimeter with Unknown Sides Complex Shapes- 3.MD.D.8 • Lesson 11: Same Perimeter Different Area- 3.MD.D.8 • Lesson 12: Same Area Different Perimeter- 3.MD.D.8 **Unit 7: Multiplication, Division, and Area 2:** • Lesson 11: Determine Area by Composing Rectangles- 3.MD.C.7* • Lesson 12: Distributive Property to Find Area of Large Rectangles- 3.MD.C.7* • Lesson 13: Distributive Property to Find Area of Large Rectangles Day 2- 3.MD.C.7* • Lesson 14: Area and Distributive Property Story Problems- 3.MD.C.7* • Lesson 15: Identifying Unknown Side Lengths of Rectilinear Shape- 3.MD.C.7* • Lesson 16: Area of Rectilinear Shapes- 3.MD.C.7* • Lesson 17: Area of Rectilinear Shapes Day 2- 3.MD.C.7* • Lesson 18: Area of Rectilinear Shapes- Subtracting the Cut Out- 3.MD.C.7* • Lesson 19: Area of Rectilinear Shapes- Subtracting the Cut Out Day 2- 3.MD.C.7* • Lesson 20: Area of Rectilinear Shapes- Mixed Practice- 3.MD.C.7* • Lesson 21: Area of Rectilinear Shapes- Mixed Practice and Error Analysis- 3.MD.C.7* *These lessons also address standard 3.OA.B.5.	
Reasoning About Two-dimensional Shapes	3.MD.D.8, 3.G.A.1, 3.G.A.2	**Unit 5: Fractions:** • Lesson 1: Seeing parts and a whole / unit fractions – Concrete- 3.G.A.2* • Lesson 2: Seeing parts and a whole / unit fractions - Fraction Strips- 3.G.A.2* • Lesson 3: Partition a Whole into Equal Parts / unit fractions - Pictorial- 3.G.A.2* • Lesson 4: Identify and Represent Unit Fractions- 3.G.A.2* • Lesson 5: Identify and Represent Unit Fractions Day 2- 3.G.A.2* • Lesson 6: Identify and Represent Non-Unit Fractions – Concrete- 3.G.A.2** • Lesson 7: Identify and Represent Non-Unit Fractions- 3.G.A.2* • Lesson 8: Identify and Represent Fractions Greater than One Whole- 3.G.A.2* **Unit 6: Length and Perimeter:** • Lesson 7: Find Perimeter Using String- 3.MD.D.8 • Lesson 8: Determine Perimeter All Sides Known- 3.MD.D.8	18 lessons

		- Lesson 9: Determine Perimeter with Unknown Sides Simple Shapes- 3.MD.D.8 - Lesson 10: Determine Perimeter with Unknown Sides Complex Shapes- 3.MD.D.8 - Lesson 11: Same Perimeter Different Area- 3.MD.D.8 - Lesson 12: Same Area Different Perimeter- 3.MD.D.8 **Unit 9: Geometry:** - Lesson 1: Define and Classify Polygons Based on Attributes- 3.G.A.1 - Lesson 2: Classifying quadrilaterals- 3.G.A.1 - Lesson 3: Comparing quadrilaterals- 3.G.A.1 - Lesson 4: Analyzing quadrilaterals- 3.G.A.1 *These lessons also address standard 3.NF.A.1.	
Understanding Fractions	3.NF.A.1, 3.NF.A.2	**Unit 5: Fractions:** - Lesson 1: Seeing parts and a whole / unit fractions – Concrete- 3.NF.A.1* - Lesson 2: Seeing parts and a whole / unit fractions - Fraction Strips- 3.NF.A.1* - Lesson 3: Partition a Whole into Equal Parts / unit fractions - Pictorial- 3.NF.A.1* - Lesson 4: Identify and Represent Unit Fractions- 3.NF.A.1* - Lesson 5: Identify and Represent Unit Fractions Day 2- 3.NF.A.1* - Lesson 6: Identify and Represent Non-Unit Fractions – Concrete- 3.NF.A.1* - Lesson 7: Identify and Represent Non-Unit Fractions- 3.NF.A.1* - Lesson 8: Identify and Represent Fractions Greater than One Whole- 3.NF.A.1* - Lesson 13: Place Unit Fractions on the Number Line- 3.NF.A.2 - Lesson 14: Place Any Fraction on the Number Line- 3.NF.A.2 - Lesson 15: Place Whole Number (and more than a whole) Fractions on the Number Line- 3.NF.A.2 - Lesson 16: Mixed Practice Placing Fractions on a Number Line- 3.NF.A.2 - Lesson 17: Mixed Practice Placing Fractions on a Number Line Day 2- 3.NF.A.2 - Lesson 18: Compare Fractions on a Number Line- 3.NF.A.2** - Lesson 19: Compare Fractions on a Number Line Day 2- 3.NF.A.2** *These lessons also address standard 3.G.A.2. **These lessons also address standard 3.NF.A.3.	15 lessons
Reasoning about Fraction Comparisons and Equivalence	3.NF.A.3, 3.G.A.2	**Unit 5: Fractions:** - Lesson 1: Seeing parts and a whole / unit fractions – Concrete- 3.G.A.2* - Lesson 2: Seeing parts and a whole / unit fractions - Fraction Strips- 3.G.A.2* - Lesson 3: Partition a Whole into Equal Parts / unit fractions - Pictorial- 3.G.A.2* - Lesson 4: Identify and Represent Unit Fractions- 3.G.A.2* - Lesson 5: Identify and Represent Unit Fractions Day 2- 3.G.A.2* - Lesson 6: Identify and Represent Non-Unit Fractions – Concrete- 3.G.A.2** - Lesson 7: Identify and Represent Non-Unit Fractions- 3.G.A.2*	25 lessons

		• Lesson 8: Identify and Represent Fractions Greater than One Whole- 3.G.A.2* • Lesson 9: Compare Fractions with the Same Numerator- 3.NF.A.3 • Lesson 10: Compare Fractions with the Same Denominator- 3.NF.A.3 • Lesson 11: Mixed Practice Comparing Fractions- 3.NF.A.3 • Lesson 12: Compare Fractions with Different Size Wholes- 3.NF.A.3 • Lesson 18: Compare Fractions on a Number Line- 3.NF.A.3** • Lesson 19: Compare Fractions on a Number Line Day 2- 3.NF.A.3** • Lesson 20: Equivalent Fractions with Different Shapes- 3.NF.A.3 • Lesson 21: Equivalent Fractions on a Number Line- 3.NF.A.3 • Lesson 22: Create Equivalent Fractions using Number Lines- 3.NF.A.3 • Lesson 23: Create Equivalent Fractions using Number Lines Day 2- 3.NF.A.3 • Lesson 24: Whole Numbers as Fractions on a Number Line- 3.NF.A.3 • Lesson 25: Mixed Practice Fractions and Number Lines- 3.NF.A.3 *These lessons also address standard 3.NF.A.1. **These lessons also address standard 3.NF.A.2.	

Scope and Sequence

If a district uses this resource to implement the state model curriculum for grade 3, the following scope and sequence should be followed to ensure alignment and attention to the progressions of mathematics.

Unit One: Multiplication, Division and Area 1

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Equal Groups as Multiplication** MWBAT represent equal groups with a multiplication number sentences that show the number of groups, the amount in each group, and the total.	3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5 × 7.
2	**Relate Multiplication to Arrays** MWBAT represent arrays with multiplication number sentences that show the number of rows, the amount in each row, and the total.	
3	**Relate Multiplication to Arrays Day 2** MWBAT represent arrays with multiplication number sentences that show the number of rows, the amount in each row, and the total.	
4	**Interpret the Meaning of Factors**	

	MWBAT explain the meaning of factors and products in multiplication equations by identifying the groups, amount in each group, and total.	
5	**Write multiplication stories** MWBAT write and solve multiplication stories using their understanding of multiplication as solving for a total of equal groups including groups and the amount in each group.	
6	**Area as an Attribute** MWBAT measure and compare the areas of shapes using standard and non-standard units and by accounting for gaps and overlaps.	3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. 3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7	**Compare Area by Decomposing and Recomposing Shapes** MWBAT measure and compare the areas of shapes by counting the number of square units that cover the shape and accounting for unit size.	
8	**Model Tiling to Measure Area** MWBAT measure the area of shapes using grid paper by counting the number of square units within the boundaries of a shape and accounting for partial units.	
9	**Relate Side Length to tiles on a side** MWBAT measure the area of a rectangle using multiplication by relating the side lengths of a rectangle to the array of square units that cover a rectangle.	3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. 3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). 3.MD.C.7.a Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
10	**Relate Side Length to tiles on a side Day 2 and Error Analysis** MWBAT measure the area of a rectangle using multiplication by relating the side lengths of a rectangle to the array of square units that cover a rectangle.	
11	**Draw rows and columns to tile determine area; use multiplication equations** MWBAT tile a shape given information about its side lengths and write a multiplication sentence that determines the shape’s area.	
12	**Determine area using multiplication and tiling, Day 2** MWBAT tile a shape given information about its side lengths and write a multiplication sentence that determines the shape’s area.	3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

		a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. 3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). 3.MD.C.7 Relate area to the operations of multiplication and addition.
13	**Unknown as Group Size Division** MWBAT represent division situations where the amount in each group is unknown by drawing models and writing division number sentences.	3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* 56 ÷ 8*.* 3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ÷ 3, 6 × 6 = ?.* 3.OA.B.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* 3.OA.B.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
14	**Unknown as Number of Groups Division** MWBAT represent division situations where the number of groups is unknown by drawing models and writing division number sentences.	
15	**Mixed Practice: Unknown as Group Size or Number of Groups Division** MWBAT represent division situations where the amount in each group is unknown or the number of groups are unknown by drawing models and writing division sentences.	
16	**Write division stories** MWBAT write and solve division stories using their understanding of division as splitting a total equally to solve for the number of groups OR amount in each group.	
17	**Interpret unknowns using arrays (use variable/symbols to represent the unknown)**	3.OA.A.4

	MWBAT describe an unknown quotient or factor in context as representing the same amount in related multiplication and division sentences.	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ÷ 3, 6 × 6 = ?.* 3.OA.B.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* 3.OA.B.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
18	**Division as unknown factor arrays/tape diagrams** MWBAT represent multiplication and division situations with tape diagrams that describe the number of groups, amount in each group, and the total.	
19	**Division as unknown factor arrays/tape diagrams Day 2** MWBAT represent multiplication and division situations with tape diagrams that describe the number of groups, amount in each group, and the total.	
20	**Commutative & Associate Property with arrays** MWBAT demonstrate the Commutative and Associative Properties of Multiplication by modeling arrays, writing multiplication sentences for arrays, and using arrays to find products.	3.OA.B.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
21	**Zero and identity property** MBWAT demonstrate the Zero and Identity Properties of Multiplication using models and multiplication and division sentences.	
22	**Multiply with multiples of 10 (Concrete)** MWBAT multiply by multiples of ten by using their understanding of place value and models.	3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
23	**Multiply with multiples of 10 (Pictorial)** MWBAT multiply by multiples of ten by using their understanding of place value, models, and a place value chart.	
24	**Multiply by multiples of 10 (Abstract)** MWBAT multiply by multiples of ten by using their understanding of place value and basic multiplication facts.	
25	**Unit Assessment**	

Unit Two: Time & Graphing

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Generate & Organize Data: Pictographs** MWBAT generate and organize data by conducting a survey and displaying the data in a pictograph. MWBAT choose a key for their pictograph that best displays the data, using half symbols as needed by thinking about the relationships between the numbers in their data.	3.MD.B.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
2	**Collect & Organize Data – Tape Diagrams** MWBAT display a data set by creating and labeling tape diagrams to represent values.	
3	**Create Scaled Bar Graphs** MWBAT create scaled bar graphs by drawing bars to represent values based on a given scale and labeling all components.	
4	**Collect & Display Data Project:** MWBAT collect and display data by conducting a survey and creating a scaled picture graph and bar graph to show the results.	
5	**Solve 2-step Story Problems About Graphs – Day 1** MWBAT solve 1- and 2-step story problems about graphs by determining the information from the graph needed to solve and representing the problem with equations.	
6	**Solve 2-step Story Problems About Graphs – Day 2** MWBAT solve 1- and 2-step story problems about graphs by determining the information from the graph needed to solve and representing the problem with equations.	
7	**Create Graphs and Questions Based on Graphs** MWBAT display data using a picture graph and bar graph and generate 1-and 2-step questions about the data by considering different problem types and using the data in their graph.	
8	**Unit Assessment**	

Unit Three: Addition, Subtraction, & Time

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Round to Nearest Ten Vertical Number Line** MWBAT round two- and three-digit numbers to the nearest ten using vertical number lines.	3.NBT.A.1 Use place value understanding to round whole numbers to the nearest 10 or 100.
2	**Round to Nearest Hundred Vertical Number Line** MWBAT round three-digit numbers to the nearest hundred using vertical number lines.	

3	**Round to Nearest Ten or Hundred** MWBAT round whole numbers to the nearest ten or hundred by asking what two tens or hundreds the number is in between, and which ten or hundred it is closer to.	
4	**Rounding Puzzles** MWBAT solve problems involving estimation using their understanding of rounding and place value.	
5	**Add with Expanded Notation to Hundreds Place** MWBAT add two- and three-digit numbers using add with expanded notation or a strategy that works for us.	3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
6	**Add by Place to Hundreds Place** MWBAT add two- and three-digit numbers using add by place or a strategy that works for us.	
7	**Add by Place Multiple Addends** MWBAT add three or more numbers using a strategy that works best for them and the problem.	
8	**Subtract with Expanded Notation to Hundreds Place** MWBAT subtract two- and three-digit numbers using subtract with expanded notation or a strategy that works for us.	
9	**Subtract with Number Line** MWBAT subtract two- and three-digit numbers using a number line to count up or count back.	
10	**Add Subtract Round in Word Problems** MWBAT compare estimates to exact sums or differences by thinking about how rounding affects values and calculations.	
11	**Add Subtract Round in Word Problems Exemplars** MWBAT compare estimates to exact sums or differences by thinking about how rounding affects values and calculations.	3.OA.D.8 Solve 2-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
12	**Time as continuous measurement** MWBAT measure amounts of time in seconds and minutes using stopwatches MWBAT solve problems about time using number lines to represent time as a continuous measurement.	3.MD.A.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
13	**Tell Time to the Nearest Minute**	

	MWBAT display and read times shown on clocks and number lines by relating the minutes on a clock to a number line that counts by 5s and 1s.	
14	**Find Elapsed Time with Clocks** MWBAT solve problems about time within one hour using a clock to count up or count back and by displaying their solution with the appropriate units.	
15	**Find Elapsed Time with a Number Line** MWBAT solve problems about elapsed time by counting up on a number line in hour and minute intervals.	
16	**Find End Time with a Number Line** MWBAT solve problems about end time by counting up on a number line in hour and minute intervals.	
17	**Find Start Time with a Number Line** MWBAT solve problems about start time by counting back on a number line in hour and minute intervals	
18	**Elapsed Time on a Time Line Exemplars** MWBAT solve problems with an unknown start, end or elapsed time by representing and then solving with a timeline.	
19	**Unit Assessment**	

Unit Four: Measurement

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Measure and Estimate Mass** MWBAT measure and estimate the weight of objects using standard units of mass (grams and kilograms) and by accurately reading a scale or using benchmark measures.	3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
2	**Word Problems with Mass** MWBAT solve problems about the actual or approximate mass of objects by visualizing, representing, and solving using all four operations and estimation.	
3	**Measure and Estimate Metric Capacity** MWBAT measure and estimate liquid volumes using standard units of volume (liters and milliliters) and a vertical number line.	
4	**Mixed Word Problems with Mass and Capacity**	

	MWBAT solve problems about measurement by visualizing, representing, and solving using all four operations and estimation.	
5	**Mixed Word Problems with Mass and Capacity Day 2** MWBAT solve problems about measurement by visualizing, representing, and solving using all four operations and estimation.	
6	**Mixed Word Problems with Money** MWBAT solve problems about money by visualizing, representing, and solving using all four operations and estimation.	
7	**Mixed Word Problems with Money, Mass and Capacity** MWBAT solve problems about measurement and money by visualizing, representing, and solving using all four operations and estimation.	
8	**Unit Assessment**	

Unit Five: Fractions

Lesson Number and Daily Aim		**Standard(s) in Lesson**
1	**Seeing parts and a whole / unit fractions – Concrete** MWBAT identify and represent unit fractions using models, pictures and fraction notation.	3.NF.A.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
2	**Seeing parts and a whole / unit fractions - Fraction Strips** MWBAT identify and represent unit fractions using models, pictures and fraction notation.	
3	**Partition a Whole into Equal Parts / unit fractions - Pictorial** MWBAT create and name unit fractions by partitioning wholes into equal parts.	
4	**Identify and Represent Unit Fractions** MWBAT identify and represent unit fractions using models, pictures and fraction notation.	
5	**Identify and Represent Unit Fractions Day 2** MWBAT identify and represent unit fractions using models, pictures and fraction notation.	
6	**Identify and Represent Non-Unit Fractions – Concrete** MWBAT identify and represent non-unit fractions using models, pictures and fraction notation.	
7	**Identify and Represent Non-Unit Fractions** MWBAT identify and represent non-unit fractions using pictures, words, and numbers.	
8	**Identify and Represent Fractions Greater than One Whole**	

	MWBAT identify and represent fractions equal to or greater than 1 whole using concrete and pictorial models.	
9	**Compare Fractions with the Same Numerator** MWBAT compare fractions by reasoning about their size using fraction strips and fraction models.	3.NF.A.3.d Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
10	**Compare Fractions with the Same Denominator** MWBAT compare fractions with the same denominator by reasoning about the number of parts using fraction strips and fraction models.	
11	**Mixed Practice Comparing Fractions** MWBAT compare fractions with the same denominator by reasoning about the number of parts or same numerator by reasoning about the size of the parts using fraction strips and fraction models.	
12	**Compare Fractions with Different Size Wholes** MWBAT identify that a shape with shaded parts can represent different fractions depending on the whole that is designated.	
13	**Place Unit Fractions on the Number Line** MWBAT place fractions on a number line between 0 and 1 by partitioning the line into equal intervals and labeling endpoints.	3.NF.A.2 Understand a fraction as a number on a number line diagram. a. Represent a fraction 1/ b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
14	**Place Any Fraction on the Number Line** MWBAT identify and represent fractions on a number line between 0 and 1 by partitioning the line into equal intervals and labeling endpoints.	
15	**Place Whole Number (and more than a whole) Fractions on the Number Line** MWBAT identify and represent fractions equal to and between whole numbers on a number line by labeling whole number fractions and partitioning the wholes into equal intervals.	
16	**Mixed Practice Placing Fractions on a Number Line** MWBAT identify and represent fractions equal to and between whole numbers on a number line by labeling whole number fractions and partitioning the wholes into equal intervals.	
17	**Mixed Practice Placing Fractions on a Number Line Day 2** MWBAT identify and represent fractions equal to and between whole numbers on a number line by labeling whole number fractions and partitioning the wholes into equal intervals.	
18	**Compare Fractions on a Number Line** MWBAT compare fractions on a number line by reasoning about the fractions' distance from 0.	3.NF.A.2 Understand a fraction as a number on a number line diagram. a. Represent a fraction 1/ b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
19	**Compare Fractions on a Number Line Day 2** MWBAT compare fractions on a number line by reasoning about the fractions' distance from 0.	

		b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. 3.NF.A.3.a,c,d Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
20	**Equivalent Fractions with Different Shapes** MWBAT recognize parts of a whole as equivalent if they are the same size and not just the same shape.	3.NF.A.3.b Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
21	**Equivalent Fractions on a Number Line** MWBAT identify equivalent fractions on the number line by partitioning the line into equal parts and finding fractions that are located on the same point.	3.NF.A.3.a-c Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
22	**Create Equivalent Fractions using Number Lines** MWBAT generate equivalent fractions by creating more or fewer equal parts/intervals in a model/number line.	
23	**Create Equivalent Fractions using Number Lines Day 2** MWBAT generate equivalent fractions by creating more or fewer equal parts/intervals in a model/number line.	
24	**Whole Numbers as Fractions on a Number Line**	3.NF.A.3.c

	MWBAT express whole numbers as fractions by recognizing the number of parts the whole is split into as the denominator and the number of parts described as the numerator.	Explain equivalence of fractions with denominators 2, 3, 4, 6, and 8 in special cases, and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
25	**Mixed Practice Fractions and Number Lines** MWBAT place fractions, find equivalent fractions, and compare fractions by using a number line and reasoning about fraction size.	
26	**Unit Assessment**	

Unit Six: Length & Perimeter

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Create Ruler and Measure to Nearest Quarter-Inch** MWBAT create a ruler that measures to the quarter-inch by partitioning equal inch intervals and use that ruler to measure lengths.	3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
2	**Measure with Broken Ruler** MWBAT identify the length of an object by counting whole inches and fractions of an inch on a broken ruler.	
3	**Measure with a Ruler to Quarter inch** MWBAT measure the length of an object to the nearest quarter-inch using a ruler and applying understandings of whole numbers and fractions.	
4	**Interpret Measurement Data from a Line Plot** MWBAT interpret the data displayed in a line plot by reading the title and labels to understand what information the graph is displaying.	
5	**Create Line Plots** MWBAT create line plots to represent data by writing a title and labels, completing a number line with the values represented, and using X's to plot values on the line.	
6	**Measure Create a Line Plot and Interpret Data** MWBAT generate measurement data by measuring objects to the nearest quarter-inch. MWBAT create line plots to display the data and ask and answer questions to interpret plots.	
7	**Find Perimeter Using String** MWBAT recognize and measure the perimeter of shapes using string to find the length around the outside/boundary of the shape.	3.MD.D.8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting

8	**Determine Perimeter All Sides Known** MWBAT determine the perimeter of a shape by adding the lengths of all sides of the shape.	rectangles with the same perimeter and different areas or with the same area and different perimeters.
9	**Determine Perimeter with Unknown Sides Simple Shapes** MWBAT determine the perimeter of a common shape, using standard units, by thinking about the attributes of given shapes and how sides relate to one another.	
10	**Determine Perimeter with Unknown Sides Complex Shapes** MWBAT calculate the perimeter of irregular shapes by determining the lengths of missing sides using the attributes of the shape and the given side lengths.	
11	**Same Perimeter Different Area** MWBAT draw conclusions about the difference between area and perimeter by exploring the area of shapes that have the same perimeter but different dimensions.	
12	**Same Area Different Perimeter** MWBAT draw conclusions about the difference between area and perimeter by exploring the perimeter of shapes that have the same area but different dimensions.	
13	**Unit Assessment**	

Unit Seven: Multiplication, Division, & Area 2

Lesson Number and Daily Aim		**Standard(s) in Lesson**
1	**Patterns with 2, 5, and 10** MWBAT multiply with 2, 5, and 10 as factors by identifying patterns in the ones place of products and applying these patterns to check the reasonableness of a solution.	3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of
2	**Patterns with 2, 4, and 8** MWBAT multiply with 2, 4 and 8 as factors by using doubling.	
3	**Patterns with 3 and 6** MWBAT multiply with 3 and 6 as factors by using doubles.	
4	**Patterns with 9** MWBAT multiply with 9 as a factor by identifying patterns and applying these patterns to skip-count and check the reasonableness of a solution.	
5	**Mixed Practice: All Fact Patterns** MWBAT multiply with all factors by identifying patterns and applying these patterns to skip-count and check the reasonableness of a solution.	

		operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
6	**Arrays and the Distributive Property** MWBAT solve multiplication problems using the distributive property to break facts into parts and combine the products of these parts to find the original product.	3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. 3.OA.B.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
7	**Distributive Property Problems with all Digits as Factors** MWBAT solve multiplication problems using the distributive property to break facts into parts and combine the products of these parts to find the original product.	
8	**Write Multiplication Stories** MWBAT write multiplication stories to describe abstract situations with equal groups.	
9	**Write Division Stories** MWBAT write division stories to describe a situation where either the number of groups or the amount in each group is unknown.	
10	**Understand Parentheses and Use to Solve** MWBAT demonstrate and explain how the place of the parentheses in an equation can change the solution.	
11	**Determine Area by Composing Rectangles** MWBAT determine the area of larger rectangles composed of separate, smaller rectangles using multiplication and addition.	3.OA.B.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* 3.MD.7.b-d Relate area to the operations of multiplication and addition.
12	**Distributive Property to Find Area of Large Rectangles** MWBAT determine the area of rectangles using the distributive property to break apart the larger rectangles into smaller arrays of square units.	
13	**Distributive Property to Find Area of Large Rectangles Day 2** MWBAT determine the area of rectangles using the distributive property to break apart the larger rectangles into smaller arrays of square units.	

14	**Area and Distributive Property Story Problems** MWBAT solve story problems by using the distributive property.	b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.
15	**Identifying Unknown Side Lengths of Rectilinear Shapes** MWBAT identify the unknown side length of a rectilinear shape by reasoning with their knowledge of geometry.	
16	**Area of Rectilinear Shapes** MWBAT determine the area of rectilinear shapes by decomposing the shapes into squares and rectangles whose area can be found using multiplication and then combining these areas.	
17	**Area of Rectilinear Shapes Day 2** MWBAT determine the area of rectilinear shapes by decomposing the shapes into squares and rectangles whose area can be found using multiplication and then combining these areas.	
18	**Area of Rectilinear Shapes- Subtracting the Cut Out** MWBAT find the area of a rectilinear shape by thinking about the shape as a large rectangle with a rectangular cut out.	
19	**Area of Rectilinear Shapes- Subtracting the Cut Out Day 2** MWBAT find the area of a rectilinear shape by thinking about the shape as a large rectangle with a rectangular cut out.	
20	**Area of Rectilinear Shapes- Mixed Practice** MWBAT find the area of rectilinear shapes by choosing an appropriate strategy.	
21	**Area of Rectilinear Shapes- Mixed Practice and Error Analysis** MWBAT find the area of rectilinear shapes by choosing an appropriate strategy.	
22	**Unit Assessment**	

Unit Eight: Story Problems

Lesson Number and Daily Aim		**Standard(s) in Lesson**
1	**Patterns in the Addition Table** MWBAT use addend-addend sum tables to model addition patterns and explain why patterns make sense.	3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*
2	**Patterns in the Addition Table Day 2** MWBAT use addend-addend sum tables to model addition patterns and explain why patterns make sense.	

3	**Patterns in the Multiplication Table** MWBAT identify and describe patterns in the multiplication and division table and why they work.	
4	**Patterns in the Multiplication Table Day 2** MWBAT identify and describe patterns in the multiplication and division table and why they work.	
5	**Mixed Patterns** MWBAT find and describe patterns and explain why they work.	
6	**Two-Step Story Problems Day 1 Represent All Four Operations** SWBAT annotate and represent multi-step word problems by thinking about the big problem and the little questions they need to answer first.	3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
7	**Two-Step Story Problems All Four Operations Day 2** MWBAT solve two-step story problems by representing and solving all parts of the problem to answer the big question using letters to represent the unknown.	
8	**Two-Step Story Problems Day 3** MWBAT solve two-step story problems by representing and solving all parts of the problem to answer the big question using letters to represent the unknown.	
9	**Two-Step Story Problems Day 4 Multiples of Ten** MWBAT solve two-step story problems by representing and solving all parts of the problem to answer the big question using letters to represent the unknown.	
10	**Two-Step Story Problems Day 5 Multiples of Ten Additional Practice** MWBAT solve two-step story problems by representing and solving all parts of the problem to answer the big question using letters to represent the unknown.	
11	**Two-Step Story Problems Day – Area and Perimeter Day 1** MWBAT solve multi-step problems involving area and perimeter by applying their understanding of these measurement concepts to real-world situations.	
12	**Two-Step Story Problems – Area and Perimeter Day 2** MWBAT solve multi-step problems involving area and perimeter by applying their understanding of these measurement concepts to real-world situations.	
13	**Perimeter Robot Project** MWBAT create a rectilinear robot by solving perimeter problems using what they know about addition patterns and the perimeter formula.	
14	**Multi-Step Story Problems Extension** MWBAT solve multi-step story problems by representing and solving all parts of the problem to answer the big question using letters to represent the unknown.	

15	**Unit Assessment**	

Unit Nine: Geometry

Lesson Number and Daily Aim		Standard(s) in Lesson
1	**Define and Classify Polygons Based on Attributes** MWBAT identify and describe different attributes of polygons by observing their sides and angles. MWBAT classify polygons based on the number of sides and angles.	3.G.A.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2	**Classifying quadrilaterals** MWBAT identify and classify quadrilaterals based on their attributes (sides and angles).	
3	**Comparing quadrilaterals** MWBAT compare and draw quadrilaterals based on their attributes (sides and angles).	
5	**Analyzing quadrilaterals** MWBAT analyze quadrilaterals by deciding whether or not their attributes make them part of another category.	
6	**Unit Assessment**	

Supports of Diversity, Equity and Inclusion
Please provide any information relative to supporting culturally responsive instruction, multi-language learners, and students with disabilities
A unique aspect of the AF Math powered by Leap Educational Consulting Program is that we do not simply offer a curriculum. We know that rigorous materials are only one component of meeting the needs of all students. In most cases, the work is not around adopting a curriculum, but rather executing the content with excellence. In order to support this excellence and to support schools and districts in the change management work of a new curriculum, Leap Educational Consulting offers a range of comprehensive coaching services. **Our Vision**: We envision a world in which all students experience an education that empowers them to lead lives of choice and opportunity. **Our Mission**: We work in deep partnership with educators committed to dismantling systemic racism to accelerate their impact. We do this by providing comprehensive supports that build their capacity to drive equitable and sustainable academic outcomes for all students. **Our Approach:** At Leap, we believe that all students can thrive in math. The formula for success is simple – access to grade level math curriculum coupled with excellent instruction will equal equitable outcomes for kids. Our curriculum was rated amongst the highest on EdReports.

Our Theory of Change: If Leap provides effective, best-in-class support to school leaders…Then school leaders will lead with equity, have a high bar for what students can do, be equipped with the technical and adaptive skills to effectively develop teachers quickly, and create systems that will ensure strong student math outcomes...Then teachers will provide more rigorous, standards-aligned, inquiry-based, engaging math instruction to all students...Then students will have access to grade-level math content and an educational experience where they are deeply engaged, powerful doers and critical thinkers of mathematics and are well-positioned for college and beyond.

Our Results:
From our most recent test results in elementary school (ES) Math:

- 44% achieved at least 10 points growth in state results in one year
- 78% achieved at least 5 points growth in state results in one year

From our most recent test results in middle school (MS) Math:

- 53% achieved at least 10 points growth in state results in one year
- 93% achieved at least 5 points growth in state results in one year

What this looks like in action: Schools and districts partner with Leap Educational Consulting for support in their adoption and implementation of the AF Math powered by Leap Educational Consulting Program. This support is highly customizable. We recommend new schools and districts elect the “full partnership,” which includes:

- **Weekly personalized coaching of the school leader** that is leading the implementation of the AF Math powered by Leap Educational Consulting Program: We believe that by building the skillset of the instructional leader, this will lead to rapid progress with teachers and students. This personalized coaching includes skills such as effective observations, teacher coaching meetings, leading intellectual preparation with teachers, leading looking at student work protocols with teachers, facilitating professional development, and leading change. The leader receives support in leveraging these skills to ensure strong implementation of the AF Math powered by Leap Educational Consulting Program.
- **In person and virtual site visits throughout the school year** to do a “deep dive” on progress towards strong implementation of the AF Math powered by Leap Educational Consulting Program, review of student data, and action planning based on progress to lead to rapid growth.
- **Access to resources,** including all curriculum and assessments, a bank of pre-designed professional development sessions for teachers, videos of lessons in execution by teachers across the country, a network of other leaders implementing the AF Math powered by Leap Educational Consulting Program, and more.
- **Professional development** for the school leader in a virtual setting of peers across the country. This development includes an initial summer launch training as well as ongoing sessions throughout the school year. Some of the professional developments include materials that can be then used in teacher professional development.
- **Differentiated support,** including support preparing for the state exam.

More information is available at https://www.leapeducationalconsulting.org/.

Program Overview: Differentiation and Working with Special Populations

AF Math powered by Leap Educational Consulting is built on the belief that all learners, especially students with disabilities or special needs and Multilingual and English language Learners (MLLs/ ELLs), deserve great curriculum. Therefore, our curriculum is strategically built to align to some of the best practices for supporting ALL learners.

One tenet of the AF Math powered by Leap Educational Consulting program is that students are grouped heterogeneously for instruction. This follows the recommendations of the California Mathematics Framework (2021) and is based on research that has shown that students of all levels and backgrounds, including minority students, students of low socioeconomic status, and students at all initial achievement levels (including high achievers), perform better in mixed-level classrooms (Burris et al., 2006). While we strongly recommend that students are heterogeneously grouped for core instruction, we recognize the need for interventions and extensions for students in order to meet their learning needs, and our framework for supporting special populations are detailed below.

Supporting Students with Disabilities

Without strong support, students with disabilities can easily struggle with learning mathematics and feel unsuccessful. Therefore, it is critical that strong curricular materials are designed to provide support for all student learners, especially those with diagnosed disabilities (Hott et al., 2014). The AF Math powered by Leap Educational Consulting Program was designed with this in mind and is based on several bodies of research about best practices for the instruction of students with math disabilities, including the work of the What Works Clearinghouse (an investment of the Institute of Education Sciences within the U.S. Department of Education) and the Council for Learning Disabilities (an international organization composed of professionals who represent diverse disciplines).

Unit Overviews and lesson level materials include guidance around working with students with disabilities, including daily suggested interventions in the Workshop Section of the daily lesson plan. Teachers should reference these materials in conjunction with the information that follows in this Implementation Guide when planning instruction in order to best support all students.

The What Works Clearinghouse makes several recommendations for students struggling with mathematics in elementary and middle school, several of which are incorporated into the AF Math powered by Leap Educational Consulting Program and all of which are recommended for use by teachers and schools. The Council for Learning Disabilities (2014) also recommends 5 elements of "explicit and systematic instruction" (several of which overlap with the recommendations of the WWC) to be utilized in core mathematics instruction. These two sets of recommendations and guidance about how to incorporate them into instruction using the AF Math powered by Leap Educational Consulting Program are outlined below:

Research-Based Recommendations from the What Works Clearinghouse (2009)	
Recommendations	**Guidance for Incorporating this Recommendation into Instruction**
Recommendation 1: Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	Each Unit Overview includes a pre-assessment. We recommend that teachers use this as a screening tool for all students prior to each unit. Teachers should then use data from the pre-assessment to inform tier 1 instruction and tier 2 and 3 interventions. We also recommend that in combination with data from pre-assessments, teachers use state assessment data to inform instruction and interventions for grades at which data is available.

Recommendation 2: Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	In alignment with this recommendation, the core focus of instruction in the elementary grades is on building understanding of whole numbers, with an average of over 80% of AF Math powered by Leap Educational Consulting Program K-5 instructional materials, including intervention materials, focused on the major content of the grade level. When students are engaged in a unit focused on additional or supporting standards, teachers may choose to prioritize major content for interventions, even if the interventions are then not fully aligned to the unit being taught. Developing students' foundational proficiencies, paying special attention to counting, place value, and addition, subtraction, multiplication, and division, is essential, and thus, alignment with the core curriculum is not necessary. (In cases where intervention is not aligned to the core curriculum, that intervention needs to be supplemental, however, not in place of core instruction.)
Recommendation 3: Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	According to the National Mathematics Advisory Panel (2008), explicit instruction includes "teachers providing clear models for solving a particular problem type using an array of examples," students receiving extensive practice, including many opportunities to think aloud or verbalize their strategies as they work, and students being provided with extensive feedback." General instruction using the AF Math powered by Leap Educational Consulting Program rarely involves the teacher explicitly modeling how to solve a problem type, but does encourage teachers to guide students to explicitly model problem-solving approaches for one another and to explicitly model techniques themselves when students do not naturally discover/ apply them. Many of the suggested interventions included in lesson plans for small group interventions direct teachers to explicitly model strategies more frequently; this is in alignment with the recommendations of the National Mathematics Advisory Panel, which specifies that not all mathematics instruction should be explicit, but rather that struggling students receive some explicit instruction regularly. Teachers should feel comfortable explicitly modeling strategies for students as needed, so long as students are able to articulate how and why the strategies work. (This understanding should be built by introducing and relating strategies along the CPA continuum as discussed above.) General instruction for all students involves many rounds of practice, opportunities to discuss thinking through turn and talks, and feedback; students in small group interventions naturally receive more opportunities for feedback given the lowered teacher to student ratio. We also recommend that teachers incorporate additional opportunities for students to get "meta" about their strategies through extra turn and talks during intervention and/or through conferencing one-on-one with teachers. Checks for understanding that ask students to articulate their thinking and explain their strategies are included in lesson plans to help teachers think of questions they may ask.

Recommendation 4: Interventions should include instruction on solving word problems that is based on common underlying structures.	The Math Stories Guide should be used as a resource for interventions for students struggling with math stories. Experts recommend teaching students who are struggling with math stories to recognize problem types/ structures and to use their understanding of those to determine appropriate solutions (Yan Ping Xin et al., 2005). The math stories protocol outlined in the guide is designed based on the recognition that the breakdown for students is often in understanding the problem. By strategically separating the operating (reading and identifying the operation used to solve) from the calculating doing the math with the numbers to get an answer) into separate steps of the protocol, teachers are able to better support struggling students. The steps of visualizing and representing the story problem support reading and listening comprehension as a bridge to then behind able to identify structures and choose a calculation strategy. While there are opportunities to discuss computation and number sense within the protocol, the overarching intention of math stories is to understand why decontextualized computation *makes sense* given a context through representation. Each story problem type in the Math Stories Guide is coded by problem type to support teachers in understanding the various structures and helping students to do so as well. A guide to each problem type is also included for teachers. The representation step of the math stories protocol encourages visual models; using visual representations to model story problems helps students recognize the structure of various problem types and to relate various problem types and structures so that they can transfer known solution methods from familiar to unfamiliar problems. When students are struggling with story problems, we encourage teachers to focus more heavily on concrete and pictorial representations to build understanding. Additionally, to support student understanding of different modeling and solution strategies in different contexts, several sample discussion questions and key points are provided in the Math Stories Guide.
Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	As noted above, the AF Math powered by Leap Educational Consulting Program is designed to move students along the concrete-pictorial-abstract (CPA) continuum as they develop conceptual understanding of grade level mathematics. This is 100% in alignment with this recommendation from the WWC. All unit overviews and the Math Stories Guide include an outline of concrete-pictorial-abstract strategies that students may use to solve problems. Teachers first move when intervening should be to move students back along the continuum. If a student is struggling with abstract application of a concept, the intervention should move back to pictorial models and solution strategies and then use questioning to relate the pictorial to abstract. If a student is struggling with the pictorial, teachers should move back to concrete.

	Beyond supporting students with learning disabilities, use of the CPA method of instruction has been shown to reduce mathematical anxiety of students in primary school (Putri et al., 2020b) and to increase students' sense of self-efficacy (Yuliyanto et al., 2019) and thus is recommended for use in general instruction as well.
Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	10-15 minutes of Calendar/ Practice is a recommended component of the Program. Teachers may opt to use some or all of this time to pull small groups for targeted practice to build fluency. Additionally, every grade level includes a practice workbook that is largely focused on fluency. Additional recommendations for building fluency from WWC include explicitly introducing efficient strategies for counting to add and subtract for grades K-2. Strategies such as counting on, up, and back, along with efficient strategies to add and subtract based on derived facts are central in many lessons in lower elementary. As teachers provide interventions in these lessons, they should be sure to encourage students to work toward the most efficient strategies in order to help students develop fluency.
Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Exit tickets and unit assessments are provided to help teachers monitor the progress of students receiving intervention. Teachers should feel free to re-use exit tickets and/or to re-administer unit pre- and post-assessments as well.
Recommendation 8. Include motivational strategies in tier 2 and tier 3 interventions.	Motivational strategies are not explicitly built into the curriculum, but the AF Math powered by Leap Educational Consulting Program recognizes the importance and impact of such strategies on student achievement and encourages teachers to incorporate these into their instruction for all students, especially those receiving tier 2 and 3 interventions. Some strategies we have seen teachers use successfully and recommend are (in order of ease of implementation/ level of intervention): 1. Basic Verbal and Nonverbal Praise: Things as simple as high fives, smiles, and positive narration can have a positive impact on student motivation. We recommend that these moments of praise are linked to engagement, focus, and effort. 2. A Point System: Students may earn points either individually or as a small group for focus and effort. These points can be prizes unto themselves or may be linked to a reward. Rewards might be items, phone calls home, or a group activity like extra time with pattern blocks or a dance party. 3. Student Goal Setting and Monitoring: Students can set their own academic and behavioral goals at the start of a year, unit, or term. Each day or every several days, they reflect on their progress toward that goal. This is most effective for older students, and we recommend pairing it with one of the above recommendations.
5 Elements of Explicit and Systematic Instruction Recommended by The Council for Learning Disabilities (Hott et al., 2014)	

Recommended Element	Guidance for Incorporating this Recommendation into Instruction
Element 1: Specific and Clear Teacher Models	As outlined above (WWC Recommendation 3), specific and clear teacher models are rarely built into materials for general instruction, as the curriculum is intended to be problem-based and student-driven; nonetheless, the AF Math powered by Leap Educational Consulting Program does encourage the use of specific and clear student models that are shared whole group for the purpose of clearly modeling mathematical strategies and thinking for students who may be struggling. (In some instances, teachers also are directed to explicitly model if a particular focal strategy is not used by students.) Lesson plans include guidance for teachers, including questions to ask, to help students clearly articulate their thinking and give examples so that their peers may make sense of their models and apply them. Similarly, the lesson plans include questions teachers may ask to ensure student understanding of the models after they are shared; these are often in the form of a turn and talk after the strategy/ modeling share and/or the consolidate the learning question at the end of the introduction. Additionally, as noted above, many of the suggested interventions included in lesson plans for small group interventions direct teachers to explicitly model strategies more frequently; this is in alignment with the recommendations of the National Mathematics Advisory Panel, which specifies that not all mathematics instruction should be explicit, but rather that struggling students receive some explicit instruction regularly. Teachers should feel comfortable explicitly modeling strategies for students as needed, so long as students are able to articulate how and why the strategies work.
Element 2: Examples that are Sequenced in Level of Difficulty	Examples and problems introduced in whole and small groups are strategically sequenced along the concrete-pictorial-abstract continuum to build conceptual understanding. Sometimes the level of difficulty builds over the course of the year, other times over the course of a unit or several days, and sometimes it builds within a single lesson. As described above (WWC Recommendation 5), teachers should always feel comfortable moving flexibly along the continuum based on student data. Most materials can be accessed anywhere along the continuum. (For example, a problem asking students to combine 5 and 6 can be tackled concretely using cubes, pictorially drawing pictures, or abstractly by counting on or making a ten.) Within student work, examples are also categorized by level of difficulty. In Game Intro Lessons (K-2 only), this takes the form of adjusting the game; suggestions for how to adjust the game to increase or decrease difficulty level within the standard and without increasing/ decreasing workload are provided in the workshop section of the plan under suggested interventions and extensions. In Exercise-based lessons (2-4 only), problems within the independent work are strategically sequenced to help students build on their mastery of the concept from basic to more advanced problems; the problems are also labeled by difficulty. By clearly marking these problems in terms of difficulty, teachers can easily differentiate for student needs and allow all students to complete the

	same number of problems aligned to the same standards and objectives, but at the appropriate difficulty level.
Element 3: Scaffolding	All lesson plans include a teacher script included scaffolded questions to be used as needed based on student understanding. These are often in the forms of sub-bullets or are sometimes labeled "back-pocket questions" in the plan. These additional questions are designed to provide scaffolding to the original question in the script to be used only if needed if students are struggling. These questions may be useful for differentiation for students with disabilities and could be used for co-teaching models or additional one-on-one support for students. Additionally, nearly every question in the lesson plan includes an exemplar student response. These exemplar responses are included so that teachers can recognize when students are not articulating the understanding required by the lesson and can plan to scaffold up. As a part of the lesson internalization process, teachers should anticipate student responses to the key questions in the lesson plan and backwards plan any additional scaffolds they might use to move students from the anticipated response to the exemplar one. The common misconceptions sections of unit overviews are a useful resource for this internalization process.
Element 4: Consistent Feedback	A key tenet of all lesson types is student discussion and feedback. Throughout each lesson, students have multiple opportunities to work independently, with partners, and engage in class discussion and analysis of student work. One purpose of this is to ensure all students, but especially those with disabilities or who struggle in mathematics, have multiple opportunities to revise and check their own mastery. Additionally, the lesson plans include guidance to teachers about circulating to provide feedback. In the Workshop section of the plan, there are suggestions to teachers about what to focus each "lap" of feedback on as they circulate the room. The first lap often focuses on procedural skills, while additional laps focus on conceptual skills. (The lesson plans suggest a focus for 2 laps, but teachers may circulate as many times as they can within a block.) The suggestions for the focus of feedback can be applied to both workshop and independent practice, and as teachers circulate, they should have pen-in-hand in order to give immediate and efficient feedback. Guidance for what teachers should look for and questions they can ask to ensure understanding as they circulate are provided within this section of the plan.
Element 5: Frequent Opportunity for Cumulative Review	Cumulative Review is a component of the AF Math powered by Leap Educational Consulting Program. It takes the form of calendar math/ practice in lower elementary and practice and cumulative review in upper elementary. Students are provided time for skills fluency and mixed practice to ensure mastery and fluency is developed with previously mastered content and

	opportunities are strategically built in for teachers to include re-teaches and/or practice with previously unmastered content.

Supporting Advanced Students

Part of supporting all learners is ensuring that advanced students also have opportunities to learn and grow by engaging with the grade level content at higher levels of complexity.

A problem-based approach is naturally differentiated as students choose the strategies they use to model and solve the problem. Teachers highlight particular strategies for the class, but they always affirm any strategy that works, regardless of its level of complexity. In a classroom implementing the AF Math powered by Leap Educational Consulting Program, students are expected to work with a variety of tools and strategies even as they work through the same set of problems; this allows advanced students to engage with the content at higher levels of complexity.

Daily lessons also provide suggested extension activities for students in the Workshop Section of the lesson plan so that teachers can encourage students to engage with the content at a higher level of complexity if they are not doing so naturally but are ready to. These extension suggestions include variations of the game that encourage more sophisticated strategies in Game Intro Lessons (K-2) and variations of the tasks or suggested strategies or tools students may use in Exercise Based Lesson (2-4). The independent practice for grades Exercise Based Lessons also include problems labeled by difficulty. Teachers should differentiate for student needs by assigning the most challenging problems to advanced students while allowing them to skip some of the simpler ones, so that they can engage with the same number of problems, but at the appropriate difficulty level.

Additionally, the Discussion section of the daily lesson plans always include a potential whole class extension/ application problem. These are often additional problems or tasks that ask students to apply the mathematical concepts taught that day, and like the focal problem of the day, students should be encouraged to use the strategy that makes sense to them in order to solve, once again allowing students to engage with the grade level content at a level that is appropriate to them.

Unit Overviews also provide aligned tasks. If teachers opt to use these tasks to supplement instruction, all students should engage in the task, but similar to the focal problems in the introduction, students can use their own strategies to solve, which allows for natural differentiation.

Supporting Multilingual and English Language Learners[1]

The AF Math powered by Leap Educational Consulting Program appreciates the importance of creating a classroom environment in which Multilingual and English language learners (MLLs/ ELLs) can thrive socially, emotionally, and academically. MLLs/ ELLs have the double-task of learning mathematics while continuing to build their language mastery. Therefore, additional support and thoughtful curriculum is often needed to ensure their mastery and support in learning. Our materials are designed to help teachers recognize and serve the unique educational needs of MLLs/ ELLs while also celebrating the assets they bring to the learning environment, both culturally and linguistically.

[1] We use both the terms Multilingual Learner (MLL) and English Language Learner (ELL) to refer to students who are learning English in addition to any other language(s) they speak or hear at home. The AF Math powered by Leap Educational Consulting Program appreciates the importance of taking an asset-based approach with students who are developing their at-home language or languages in addition to English. Rather than choose a single preferred term on behalf of the diverse group of learners who are developing additional languages, we encourage schools to choose which term they wish to use based on the preferences of their student populations.

In designing the materials to ensure they adequately support teachers of MLLs/ ELLs, we relied heavily on the math guidelines and resources provided by the English Learners Success Forum.[2] Both the Game Introduction Lessons in lower elementary and the Exercise Based Lessons in upper elementary along with the Math Stories Protocols used in Math Stories at all grade levels build on the four design principles for promoting mathematical language use and development in curriculum and instructions outlined by Stanford's Graduate School of Education (Zwiers et al., 2017), Understanding Language/SCALE and recommended by the English Language Success Forum, and we have strategically included several mathematical language routines (MLRs) to support the language and content development of MLLs/ ELLs in all lesson plans and called them out explicitly for teachers in a third of lesson plans . We encourage teachers to incorporate more of these language routines into daily lessons (using the lessons that explicitly identify them already as guides) and the Math Stories block as student data indicates the need for more. These design principles and the MLRs to support MLLs/ ELLs are outlined below, along with guidance about how to implement each routine more regularly into the curriculum if needed:

Overview of Design Principles and Recommended Mathematical Language Routines (MLRs) [3]	
Design Principle 1: Support sense-making	
Principle in Action	**Overview of Recommended MLRs** (see table below for guidance for implementation)
Daily lesson plan scripts and the math stories protocols intentionally amplify rather than simplify student language by anticipating where students may have difficulty accessing the concepts and language and providing multiple ways for them to come to understanding. Every lesson includes multiple opportunities for students to engage in discussion with one another, often through turn and talks, as they make sense of the content, and this sense-making is also supported through the use of concrete and pictorial models and a lesson visual anchor that captures student thinking and mathematical concepts and key vocabulary. Key concepts are highlighted as what and how key points for each lesson plan for teachers and the script directs teachers to stamp these key points through questioning and reverbalizations. The key stamping of these concepts help all students, but especially MLLs/ ELLs to be clear on the key take-aways for each lesson. Additionally, teachers are provided with student-friendly vocabulary definitions for all new vocabulary terms in the unit plan that can support MLLs/ ELLs further.	● **MLR 2: Collect and Display:** The teacher captures student thinking and/or strategies visually and leads the class in a discussion. *Purpose: To capture students' oral words and phrases into a stable, collective reference.* ● **MLR 6: Three Reads:** Teachers support students in making sense of a situation or problem by reading three times, each time with a particular focus. *Purpose: To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning.* ● **MLR 8: Discussion Supports:**Teachers use a number of moves to help facilitate student discussion including revoicing, encouraging students to agree, disagree, build on, or ask questions of their peers, incorporating choral response to build vocabulary, showing concepts multi-modally, and modeling clear explanations/ think alouds. *Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.*

[2] "Guidelines for Improving Math Materials for English Language Learners" by ELSF licensed under Creative Commons Attribution International 4.0 (CC BY-NC-SA.)

[3] This table, "Overview of Design Principles and Recommended Mathematical Language Routines" is a revision of "Principles for the Design of Mathematics Curricula: Promoting Language and Content Development" by Zwier, et. al licensed under Creative Commons Attribution International 4.0 (CC BY-NC-SA.)

Design Principle 2: Optimize Output	
Principle in Action	**Recommended Language Routines** (see table below for guidance for implementation)
Lessons and the math stories protocols are strategically built to focus on student thinking. Students engage in each new task individually or with partners, have opportunities to discuss with one another, and then analyze student work samples as a whole class. These multiple opportunities to engage with the content focus on deepening and building on student understanding. All students benefit from the focus on the mathematical discourse and revising their own thinking, but this is especially true of MLLs/ ELLs who will benefit from hearing other students thinking and reasoning on the concepts and/or different methods of solving.	● **MLR 1: Stronger and Clearer Each Time:** Teachers provide students with multiple opportunities to articulate their mathematical thinking, with the opportunity to refine their language with each successive share. *Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output.* ● **MLR 3: Critique, Correct, and Clarify:** Teachers present students with a statement, an argument, an explanation, or a solution, and prompt them to analyze and discuss. *Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.* ● **MLR 4: Info Gap:** Students are put into pairs; each student in the pair is given partial information that when combined with their partner's information provides the full context needed to solve the problem. Students must communicate effectively in order to solve the problem. *Purpose: To create a need for students to communicate.* ● **MLR 5: Co-Craft Questions and Problems:** Teachers guide students to work with one another to create questions or situations for math problems or to create entire problems and then solve them. *Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.* ● **MLR 7: Compare and Connect:** Teachers prompt students to understand one another's strategies by comparing and connecting other students' approaches to their own. *Purpose: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.*
Design Principle 3: Cultivate Conversation	
Principle in Action	**Recommended Language Routines** (see table below for guidance for implementation)

A key element of all lesson types is student discussion. Daily lesson plans and the math stories protocol rely heavily on the use of individual or partner think time, turn-and-talks with partners, and whole class discussion to answer key questions throughout the lesson script. The rationale for this is that all learners, but especially MLLs/ ELLs benefit from multiple opportunities to engage with the content. Students that are building their mastery of the language may struggle more with following a whole-class discussion; however, having an opportunity to ask questions and discuss with a strategic partner beforehand can help deepen their understanding and empower them to engage further in the class discussion. These smaller conversations also provide opportunities for teachers to strategically scan student work and listen to discussions to engage more learners.	● **MLR 1: Stronger and Clearer Each Time:** Teachers provide students with multiple opportunities to articulate their mathematical thinking, with the opportunity to refine their language with each successive share. *Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output.* ● **MLR 3: Critique, Correct, and Clarify:** Teachers present students with a statement, an argument, an explanation, or a solution, and prompt them to analyze and discuss. *Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.* ● **MLR 4: Info Gap:** Students are put into pairs; each student in the pair is given partial information that when combined with their partner's information provides the full context needed to solve the problem. Students must communicate effectively in order to solve the problem. *Purpose: To create a need for students to communicate.* ● **MLR 5: Co-Craft Questions and Problems:** Teachers guide students to work with one another to create questions or situations for math problems or to create entire problems and then solve them. *Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.* ● **MLR 7: Compare and Connect:** Teachers prompt students to understand one another's strategies by comparing and connecting other students' approaches to their own. *Purpose: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.* ● **MLR 8: Discussion Supports:**Teachers use a number of moves to help facilitate student discussion including revoicing, encouraging students to agree, disagree, build on, or ask questions of their peers, incorporating choral response to build vocabulary, showing concepts multi-modally, and modeling clear explanations/ think alouds. *Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.*
Design Principle 4: Maximize Meta-Awareness	
Principle in Action	**Recommended Language Routines**

	(see table below for guidance for implementation)
Every daily lesson and math stories lesson is structured so that students have many opportunities to get “meta” about the mathematical processes they engage in. Students explain how they model and solve problems to the teacher and one another throughout the lesson, often through turn and talks in which they also evaluate their peers’ strategies and thinking. Lesson scripts also encourage students to draw connections between new content and previous learning as well as between different strategies. As students share with one another, they develop meta-awareness of how they are communicating with one another, and as they make connections between concepts and new and previous learning, they are reflecting on their own and others' learning of the content.	● **MLR 2: Collect and Display:** The teacher captures student thinking and/or strategies visually and leads the class in a discussion. *Purpose: To capture students’ oral words and phrases into a stable, collective reference.* ● **MLR 3: Critique, Correct, and Clarify:** Teachers present students with a statement, an argument, an explanation, or a solution, and prompt them to analyze and discuss. *Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.* ● **MLR 5: Co-Craft Questions and Problems:** Teachers guide students to work with one another to create questions or situations for math problems or to create entire problems and then solve them. *Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.* ● **MLR 6: Three Reads:** Teachers support students in making sense of a situation or problem by reading three times, each time with a particular focus. *Purpose: To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning.* ● **MLR 7: Compare and Connect:** Teachers prompt students to understand one another’s strategies by comparing and connecting other students’ approaches to their own. *Purpose: To foster students’ meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.* ● **MLR 8: Discussion Supports:**Teachers use a number of moves to help facilitate student discussion including revoicing, encouraging students to agree, disagree, build on, or ask questions of their peers, incorporating choral response to build vocabulary, showing concepts multi-modally, and modeling clear explanations/ think alouds. *Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.*

Implementing Language Routines[4]	
Mathematical Language Routine (MLR)	**Guidance for Implementation**
MLR 1: Stronger and Clearer Each Time *Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output.* Teachers provide students with multiple opportunities to articulate their mathematical thinking, with the opportunity to refine their language with each successive share.	Multiple opportunities to turn and talk are built into the daily lessons and math stories protocol; often students have many opportunities to refine their thinking and clarify as they reiterate key points through the turn and talk of the initial problem, the consolidate the learning turn and talk, and turn and talks in the mid-workshop interruption and discussion portions of the lessons. Teachers may further support MLLs/ ELLs by having students turn to multiple partners for each turn and talk, so that instead of just one partner for each question, they turn to at least 2, refining their language each time. Another option is to have students create 3 separate iterations of their mathematical arguments, each with a different focus. For example, they may have the initial turn and talk include only the "what," or the actual solution, the second may include the "what" and the "how," or the solution and how they came to it, and the third can include the "what," "how," and "why," or the solution, how they came to it, and why that works or makes sense. To further strengthen these routines for MLLs/ ELLs, teachers may also consider incorporating a pre-write and/or additional think time before each turn and talk.
MLR 2: Collect and Display *Purpose: To capture students' oral words and phrases into a stable, collective reference.* The teacher captures student thinking and/or strategies visually and leads the class in a discussion.	This routine can and should be incorporated into every game intro, exercise-based, and math stories lesson as teachers co-create the visual anchor with students when teachers capture student strategies and thinking as they share. Any time that a teacher captures student thinking and/or work visually on an anchor chart for the class and leads a discussion using the anchor chart, they are engaging in a version of this routine. To take this routine to the next level, teachers may also opt to chart the specific language and vocabulary students use to explain their strategies and lead the class in a discussion of how and which language and visuals help to clearly communicate ideas.
MLR 3: Critique, Correct, and Clarify *Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.* Teachers present students with a statement, an argument, an explanation, or a solution, and prompt them to analyze and discuss.	This routine is worked into nearly every lesson as an option for the mid-workshop interruption and discussion when teachers lead students in the error analysis of a common misconception. To maximize the support for MLLs/ ELLs, teachers should allow students to work both individually and in pairs to identify and fix the error and then lead the class in sharing out whole group and refining their work. This routine can also be easily incorporated into many units, most naturally Geometry units but in others as well, in the form of a "sometimes/ always/ never" task. In this task, teachers provide statements and students determine if statements are sometimes, always, or never true. For example, teachers may present statements like, "A rectangle has sides of equal lengths," or "When we add two odd numbers, the sum is even," and students recognize that the former is sometimes true, while the latter is always true.

[4] This table, "Implementing Mathematical Language Routines" is a revision of "Principles for the Design of Mathematics Curricula: Promoting Language and Content Development" by Zwier, et. al licensed under Creative Commons Attribution International 4.0 (CC BY-NC-SA.)

MLR 4: Info Gap *Purpose: To create a need for students to communicate.* Students are put into pairs; each student in the pair is given partial information that when combined with their partner's information provides the full context needed to solve the problem. Students must communicate effectively in order to solve the problem.	While this is not regularly incorporated into lesson materials, teachers can easily modify materials to incorporate this routine into the workshop of most lessons or into math stories if their students need more additional language development support. For any story problem or word problem, teachers can provide one student with the general problem or the context and the other with the data. The data can be presented in the form of tables, graphs, diagrams, etc. Students will need to read their information, which will give them the opportunity to practice reading English language, and then they will need to share with one another, asking clarifying questions, which will allow them to practice oral input and output. Outside of contextual problems, teachers can also set students up to play partner games in which one partner has information the other needs. For example, when solving for a missing addend or factor, one partner may know the known addend or factor and the other may know the whole or product; students need to communicate with one another to combine the known information and solve for the unknown.
MLR 5: Co-Craft Questions and Problems *Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.* Teachers guide students to work with one another to create questions or situations for math problems or to create entire problems and then solve them.	Most grade levels include at least some lessons in story problem units that task students with creating their own problems to solve. A low-lift way to incorporate more opportunities for students to engage in this language routine is to have it be a go-to routine for what to do when students finish early, particularly in the math stories block. When students finish any step of the protocol, they may use the additional time to craft their own story problems and then solve them. To scaffold for students, several variations of this routine may be used. Teachers may provide the situation and students may generate questions only. Alternatively, teachers may provide a mathematical model (equation, tape diagram, number bond, etc.) and then students may craft the situation.
MLR 6: Three Reads *Purpose: To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning.* Teachers support students in making sense of a situation or problem by reading three times, each time with a particular focus.	This routine can easily be incorporated into the Math Stories Protocol (or any lesson focused on math stories) by assigning students particular parts of the protocol for each reading. For example, the teacher may ask students to close their eyes and visualize for the first reading, to represent during the second reading, and to check their representation and/or to begin to think about solution strategies during the third reading. For students who are reading independently, teachers can consider asking students to strategically annotate for what they know during the first reading, what they need to figure out during the second reading, and to represent during the third reading.

MLR 7: Compare and Connect *Purpose: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.* Teachers prompt students to understand one another's strategies by comparing and connecting other students' approaches to their own.	Most daily lesson plans and the math stories protocol lend themselves easily to this routine because they include strategy and modeling shares and many already include questions aligned to the routine that ask students to compare and connect their own strategies to those shared whole group. (Often these connections are made along the CPA continuum as students are asked to relate pictorial strategies to concrete ones and abstract strategies to pictorial and concrete ones.) Any time that a lesson plan asks students to compare strategies after a strategy share or asks students how/why different strategies work, teachers are leading students in a version of this routine. To find more opportunities to engage students in this routine, teachers can add questions after strategy shares such as: - What is the same about these strategies? What is different? - Why do both strategies work to solve this particular problem? - What is particularly useful about this strategy? Why might someone choose to use this strategy over the other?
MLR 8: Discussion Supports *Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.* Teachers use a number of moves to help facilitate student discussion including revoicing, encouraging students to agree, disagree, build on, or ask questions of their peers, incorporating choral response to build vocabulary, showing concepts multi-modally, and modeling clear explanations/ think alouds.	This routine is critical to strong problem-based instruction and successful teachers implementing the AF Math powered by Leap Educational Consulting Program incorporate it into their instruction daily. Many lesson scripts include discussion supports already, especially choral response and questions asking students to agree/ disagree with one another. Teachers can further incorporate this routine by modeling clear explanations as needed, particularly at the start of the year and providing sentence frames and starters to support students in clearly articulating their ideas. Explicit vocabulary instruction is also helpful in supporting discussion. Sentence frames and vocabulary are provided in unit overviews to support teachers in supporting student discussion.