

## Connecticut Mathematics Model Curriculum Alignment

**Resource Name: Imagine Learning Illustrative Mathematics Grade 7**

Model Unit Name	Model Unit Standards	Resource Unit(s) Number and Lessons	Standard Frequency
<i>This is the title of the unit in the model curricula</i>	<i>These are the standards addressed in the unit</i>	<i>This is the unit(s) that aligns with the model unit from the resource</i>	<i>This is the total number of lessons the standard is addressed</i>
<b>Pacing</b> - Illustrative Mathematics 6-8 lessons are designed to fit within a 45-50 minute block. Pacing guidance for each activity is provided in the lesson plans.			
Operating with Rational Numbers (Addition and Subtraction)			
	7.NS.A.1	<a href="#">Unit 5, Lesson 4: Money and Debts</a>	9 Days - 9 Spotlight Lessons
		<a href="#">Unit 5, Lesson 1: Interpreting Negative Numbers</a>	

	7.NS.A.3	<a href="#">Unit 5, Lesson 14: Solving Problems with Rational Numbers</a>	9 Days - 7 Spotlight Lessons
		<a href="#">Unit 5, Lesson 13: Expressions with Rational Numbers</a>	
Operating with Rational Numbers (Multiplication and Division)			
	7.NS.A.2	<a href="#">Unit 5, Lesson 9: Multiplying with Rational Numbers</a>	
		<a href="#">Unit 5, Lesson 11: Dividing Rational Numbers</a>	8 Days - 7 Spotlight Lessons
	7.NS.A.3	<a href="#">Unit 5, Lesson 15: Solving Equations with Rational Numbers</a>	9 Days - 7 Spotlight Lessons
		<a href="#">Unit 5, Lesson 16: Representing Contexts with Equations</a>	

	7.EE.A.2	<a href="#">Unit 6, Lesson 12: Solving Problems about Percent Increase or Decrease</a>	1 Day - 1 Spotlight Lesson
		<a href="#">7.6.12 Spotlight Lesson: Solving Problems about Percent Increase or Decrease</a>	
	7.EE.B.3	<a href="#">Unit 3, Lesson 11: Stained-Glass Windows</a>	10 Days - 10 Spotlight Lessons
		<a href="#">Unit 5, Lesson 12: Negative Rates</a>	
Two- and Three-Dimensional Geometry			
	7.G.A.2	<a href="#">Unit 7, Lesson 6: Building Polygons (Part 1)</a>	7 Days - 7 Spotlight Lessons
		<a href="#">Unit 7, Lesson 17: Building Prisms</a>	

	7.G.A.3	<a href="#">Unit 7, Lesson 11: Slicing Solids</a>	2 Days - 2 Spotlight Lessons
		<a href="#">Unit 7, Lesson 13: Decomposing Bases for Area</a>	
	7.G.B.4	<a href="#">Unit 3, Lesson 5: Circumference and Wheels</a>	11 Days - 8 Spotlight Lessons
		<a href="#">Unit 3, Lesson 10: Distinguishing Circumference and Area</a>	
	7.G.B.5	<a href="#">Unit 7, Lesson 4: Solving for Unknown Angles</a>	4 Days - 4 Spotlight Lessons
		<a href="#">Unit 7, Lesson 5: Using Equations to Solve for Unknown Angles</a>	

	7.G.B.6	<a href="#">Unit 7, Lesson 12: Volume of Right Prisms</a>	12 Days - 9 Spotlight Lessons
		<a href="#">Unit 7, Lesson 15: Distinguishing Volume and Surface Area</a>	
Proportional Reasoning			
	7.RP.A.1	<a href="#">Unit 2, Lesson 8: Comparing Relationships with Equations</a>	4 Days - 3 Spotlight Lessons
		<a href="#">Unit 4, Lesson 2: Ratios and Rates with Fractions</a>	
	7.RP.A.2	<a href="#">Unit 2, Lesson 2: Introducing Proportional Relationships with Tables</a>	24 Days - 22 Spotlight Lessons
		<a href="#">Unit 2, Lesson 3: More about Constant of Proportionality</a>	
	7.RP.A.3	<a href="#">Unit 4, Lesson 8: Percent Increase and Decrease with Equations</a>	19 Days - 12 Spotlight Lessons

		<a href="#">Unit 4, Lesson 6: Increasing or Decreasing</a>	
	7.G.A.1	<a href="#">Unit 1, Lesson 1: What are Scaled Copies?</a>	18 Days - 16 Spotlight Lessons
		<a href="#">Unit 1, Lesson 4: Scaled Relationships</a>	
Algebraic Reasoning II			
	7.EE.A.1	<a href="#">Unit 6, Lesson 18: Subtraction in Equivalent Expressions</a>	6 Days - 5 Spotlight Lessons
		<a href="#">Unit 6, Lesson 20: Combining Like Terms (Part 1)</a>	
	7.EE.A.2	<a href="#">Unit 6, Lesson 12: Solving Problems about Percent Increase or Decrease</a>	1 Day - 1 Spotlight Lesson

		<a href="#">7.6.12 Spotlight Lesson: Solving Problems about Percent Increase or Decrease</a>	
	7.EE.B.4	<a href="#">Unit 6, Lesson 13: Reintroducing Inequalities</a>	18 Days - 17 Spotlight Lessons
		<a href="#">Unit 6, Lesson 14: Finding Solutions to Inequalities in Context</a>	
Probability			
	7.SP.C.5	<a href="#">Unit 8, Lesson 2: Chance Experiments</a>	5 Days - 5 Spotlight Lessons
		<a href="#">Unit 8, Lesson 4: Estimating Probabilities Through Repeated Experiments</a>	
	7.SP.C.6	<a href="#">Unit 8, Lesson 3: What Are Probabilities?</a>	5 Days - 5 Spotlight Lessons

		<a href="#">Unit 8, Lesson 6: Estimating Probabilities Using Simulations</a>	
	7.SP.C.7	<a href="#">Unit 8, Lesson 5: More Estimating Probabilities</a>	6 Days - 6 Spotlight Lessons
		<a href="#">Unit 8, Lesson 14: Sampling in a Fair Way</a>	
	7.SP.C.8	<a href="#">Unit 8, Lesson 9: Multi-Step Experiments</a>	

		<a href="#">Unit 8, Lesson 8: Keeping Track of All Possible Outcomes</a>	5 Days 5 Spotlight Lessons
<b>Inferences and Populations</b>			
	7.SP.A.1	<a href="#">Unit 8, Lesson 12: Larger Populations</a>	5 Days - 5 Spotlight Lessons



	<a href="#">Unit 8, Lesson 15: Estimating Population Measures of Center</a>	
7.SP.A.2	<a href="#">Unit 8, Lesson 13: What Makes a Good Sample?</a>	6 Days - 6 Spotlight Lessons
	<a href="#">Unit 8, Lesson 17: More about Sampling Variability</a>	
7.SP.B.3	<a href="#">Unit 8, Lesson 11: Comparing Groups</a>	2 Days - 2 Spotlight Lessons
	<a href="#">Unit 8, Lesson 18: Comparing Populations Using Samples</a>	
7.SP.B.4	<a href="#">Unit 8, Lesson 19: Comparing Populations With Friends</a>	6 Days - 5 Spotlight Lessons

	<a href="#">Unit 8, Lesson 20: Memory Test</a>	

Scope and Sequence			
If a district uses this resource to implement the state model curriculum for grade 6, the following scope and sequence should be followed to ensure alignment and attention to the progressions of mathematics.			
Unit Number/Title	Lesson Title	Lesson Objectives	# of Days/Weeks (assume 1 hour of instruction)
Unit 1: Scale Drawings			13 - 15 Days of Instruction -- 4 Weeks
	Scaled Copies		
	Lesson 1	Describe (orally) characteristics of scaled and non-scaled copies.	
		Identify scaled copies of a figure and justify (orally and in writing) that the copy is a scaled copy.	

	Lesson 2	Comprehend the phrase “scale factor” and explain (orally) how it relates corresponding lengths of a figure and its scaled copy.	
		Explain (orally) what it means to say one part in a figure “corresponds” to a part in another figure.	
		Identify and describe (orally and in writing) corresponding points, corresponding segments, or corresponding angles in a pair of figures.	
	Lesson 3	Critique (orally and in writing) different strategies (expressed in words and through other representations) for creating scaled copies of a figure.	
		Draw a scaled copy of a given figure using a given scale factor.	
		Generalize (orally and in writing) that the relationship between the side lengths of a figure and its scaled copy is multiplicative, not additive.	
	Lesson 4	Explain (orally and in writing) that corresponding angles in a figure and its scaled copies have the same measure.	

		Identify (orally and in writing) corresponding distances or angles that can show that a figure is not a scaled copy of another.	
		Recognize that corresponding distances in a figure and its scaled copy are related by the same scale factor as corresponding sides.	
	Lesson 5	Describe (orally and in writing) how scale factors of 1, less than 1, and greater than 1 affect the size of scaled copies.	
		Explain and show (orally and in writing) how to recreate the original figure given a scaled copy and its scale factor.	
		Recognize (orally and in writing) the relationship between a scale factor of a scaled copy to its original figure is the “reciprocal” of the scale factor of the original figure to its scaled copy.	
	Lesson 6 - Optional	Calculate and compare (orally and in writing) the areas of multiple scaled copies of the same shape.	

		Generalize (orally) that the area of a scaled copy is the product of the area of the original figure and the “square” of the scale factor.	
		Recognize that a two-dimensional attribute, like area, scales at a different rate than one-dimensional attributes, like length and distance.	
	Scale Drawings		
	Lesson 7	Describe (orally) what a “scale drawing” is.	
		Explain (orally and in writing) how to use scales and scale drawings to calculate actual and scaled distances.	
		Interpret the “scale” of a scale drawing.	
Lesson 8			
	Justify (orally and in writing) which of two objects was moving faster.		

		Use a scale drawing to estimate the distance an object traveled, as well as its speed or elapsed time, and explain (orally and in writing) the solution method.	
	Lesson 9	Compare and contrast (orally) different scale drawings of the same object, and describe (orally) how the scale affects the size of the drawing.	

		Create a scale drawing, given the actual dimensions of the object and the scale.	
		Determine the scale used to create a scale drawing and generate multiple ways to express it (in writing).	
	Lesson 10	Determine how much actual area is represented by one square unit in a scale drawing.	
		Generalize (orally) that as the actual distance represented by one unit on the drawing increases, the size of the scale drawing decreases.	
		Reproduce a scale drawing at a different scale and explain (orally) the solution method.	
	Lesson 11	Explain (orally and in writing) how to use scales without units to determine scaled or actual distances.	

		Interpret scales expressed without units, e.g., “1 to 50,” (in spoken and written language).	
	Lesson 12	Comprehend that the phrase “equivalent scales” refers to different scales that relate scaled and actual measurements by the same scale factor.	
		Generate a scale without units that is equivalent to a given scale with units, or vice versa.	
		Justify (orally and in writing) that scales are equivalent, including scales with and without units.	

	Let's Put It to Work		
	Lesson 13	Compare, contrast, and critique (orally) scale drawings of the classroom.	
		Generate an appropriate scale to represent an actual distance on a limited drawing size, and explain (orally) the reasoning.	
		Make simplifying assumptions and determine what information is needed to create a scale drawing of the classroom.	

Unit 2: Introducing Proportional Relationships			17 Days of Instruction -- 4 Weeks
	Representing Proportional Relationships with Tables		
	Lesson 1	Choose and create representations to compare ratios in the context of recipes or scaled copies.	
		Coordinate (orally) different representations of a situation involving equivalent ratios, e.g., discrete diagrams, tables, or double number line diagrams.	
		Determine which recipes or geometric figures involve equivalent ratios, and justify (orally, in writing, and through other representations) that they are equivalent.	
	Lesson 2	Comprehend that the phrase “proportional relationship” (in spoken and written language) refers to when two quantities are related by multiplying by a “constant of proportionality.”	
		Describe (orally and in writing) relationships between rows or between columns in a table that represents a proportional relationship.	



	Explain (orally) how to calculate missing values in a table that represents a proportional relationship.
Lesson 3	Compare, contrast, and critique (orally and in writing) different ways to express the constant of proportionality for a relationship.
	Explain (orally) how to determine the constant of proportionality for a proportional relationship represented in a table.
	Interpret the constant of proportionality for a relationship in the context of constant speed.
Representing Proportional Relationships with Equations	
Lesson 4	Generalize a process for finding missing values in a proportional relationship, and justify (orally) why this can be abstracted as $y=kx$ , where $k$ is the constant of proportionality.
	Generate an equation of the form $y=kx$ to represent a proportional relationship in a familiar context.
	Write the constant of proportionality to complete a row in the table of a proportional relationship where the value for the first quantity is 1.
Lesson 5	Use the word “reciprocal” to explain (orally and in writing) that there are two related constants of proportionality for a proportional relationship.

		Write two equations that represent the same proportional relationship, i.e., $y=kx$ and $x=(1/k)y$ , and explain (orally) what each equation means.	
	Lesson 6	Generate an equation for a proportional relationship, given a description of the situation but no table.	
		Interpret (orally) each part of an equation that represents a proportional relationship in an unfamiliar context.	
		Use an equation to solve problems involving a proportional relationship, and explain (orally) the reasoning.	
	Comparing Proportional and Nonproportional Relationships		
	Lesson 7	Calculate and compare the quotients of the values in each row of a given table.	
		Generate a different recipe for lemonade and describe (orally) how it would taste in comparison to a given recipe.	
		Justify (orally) whether the values in a given table could or could not represent a proportional relationship.	
	Lesson 8	Compare and contrast (orally) equations that do and do not represent proportional relationships.	

		Generalize that an equation equivalent to the form $y=kx$ can represent a proportional relationship.	
		Use a table to determine whether a given equation represents a proportional relationship, and justify (in writing) the decision.	
	Lesson 9	Decide whether it makes sense to represent a situation with a proportional relationship, and explain (orally) the reasoning.	
		Determine what information is needed to solve a problem involving proportional relationships. Ask questions to elicit that information.	
		Write an equation to represent a proportional relationship, and use the equation to solve problems about the situation.	
	Representing Proportional Relationships with Graphs		
	Lesson 10	Compare and contrast (orally) graphs of relationships.	
		Generalize (orally and in writing) that a proportional relationship can be represented in the coordinate plane by a line that includes the “origin” or by a collection of points that lie on such a line.	
		Justify (orally) that a table and a graph represent the same relationship.	

	Lesson 11	Create the graph of a proportional relationship given only one pair of values, by drawing the line that connects the given point and $(0, 0)$ .	
		Identify the constant of proportionality from the graph of a proportional relationship.	
		Interpret (orally and in writing) points on the graph of a proportional relationship.	
	Lesson 12	Create and interpret graphs that show two different proportional relationships on the same axes.	
		Generalize (orally and in writing) that when two different proportional relationships are graphed on the same axes, the steeper line has the greater constant of proportionality.	
	Lesson 13	Coordinate (orally and in writing) tables, graphs, and equations that represent the same proportional relationship.	
		Interpret two different graphs that represent the same proportional relationship, but have reversed which quantity is represented on each axis.	
		Write an equation to represent a proportional relationship given only one pair of values or one point on the graph.	

Let's Put it to Work			
Lesson 14	Calculate the constant of proportionality for a proportional relationship in an unfamiliar context, and express it (in writing) using the correct units.		
	Critique (orally and in writing) presentations of proportional and nonproportional relationships.		
	Invent and describe (in writing and using other representations) a proportional relationship and a nonproportional relationship.		
Lesson 15	Apply reasoning developed in this unit to determine whether a proportional relationship models a situation about water usage.		
		Make simplifying assumptions and determine what information is needed to solve a problem about water usage.	
		Use proportional relationships to analyze (orally and in writing) a problem about water usage.	
Unit 3: Measuring Circles			11 - 13 Days of Instruction -- 4 Weeks
	Circumference of a Circle		

	Lesson 1	Create and describe (in writing) graphs that show measurements of squares.	
		Justify (orally and in writing) whether the relationship shown on a graph is close enough to a straight line through the origin that it might be a proportional relationship with some measurement error.	
		Recognize that when we measure the quantities in a proportional relationship, measurement error can cause the graph to be not perfectly straight and the quotients to be not exactly constant.	
	Lesson 2	Compare (orally) different ways to measure a circle, and generalize the relationship between radius and diameter.	
		Comprehend the terms “diameter,” “center,” “radius,” and “circumference” in reference to parts of a circle.	
		Describe (orally and in writing) the defining characteristics of a circle.	
	Lesson 3	Comprehend the word “pi” and the symbol $\pi$ to refer to the constant of proportionality between the diameter and circumference of a circle.	

		Create and describe (in writing) graphs that show measurements of circles.	
		Generalize that the relationship between diameter and circumference is proportional and that the constant of proportionality is a little more than 3.	
	Lesson 4	Apply understanding of circumference to calculate the perimeter of a shape that includes circular parts, and explain (orally and in writing) the solution method.	
		Compare and contrast (orally) values for the same measurements that were calculated using different approximations for $\pi$ .	
		Explain (orally) how to calculate the radius, diameter, or circumference of a circle, given one of these three measurements.	
	Lesson 5 - Optional	Compare wheels of different sizes and explain (orally) why a larger wheel needs fewer rotations to travel the same distance.	
		Generalize that the distance a wheel rolls in one rotation is equal to the circumference of the wheel.	
		Write an equation to represent the proportional relationship between the number of rotations and the distance a wheel travels.	

	Area of a Circle		
	Lesson 6	Estimate the area of a complex, real-world region, e.g., a state or province, by approximating it with an irregular polygon, and indicate that	

		it is an approximation when expressing the answer (orally and in writing).	
		Explain (orally and in writing) how to calculate the area of an irregular polygon by decomposing it.	
		Interpret floor plans and maps in order to identify the information needed to calculate area.	
	Lesson 7	Create a table and a graph that represent the relationship between the diameter and area of circles of various sizes, and justify (using words and other representations) that this relationship is not proportional.	
		Estimate the area of a circle on a grid by decomposing and approximating it with polygons.	
	Lesson 8	Generalize a process for finding the area of a circle, and justify (orally) why this can be abstracted as $\pi r^2$ .	



		Show how a circle can be decomposed and rearranged to approximate a polygon, and justify (orally and in writing) that the area of this polygon is equal to half of the circle's circumference multiplied by its radius.
Lesson 9		Calculate the area of a shape that includes circular or semi-circular parts, and explain (orally and in writing) the solution method.
		Comprehend and generate expressions in terms of $\pi$ to express exact measurements related to a circle.
Let's Put it to Work		

	Lesson 10	Critique (orally and in writing) claims about the radius, diameter, circumference, or area of a circle in a real-world situation.	
		Decide whether to calculate the circumference or area of a circle to solve a problem in a real-world situation, and justify (orally) the decision.	
		Estimate measurements of a circle in a real-world situation, and explain (orally and in writing) the estimation strategy.	
	Lesson 11 - Optional	Apply circumference and area of circles to calculate the cost of a stained-glass window, and explain (orally and in writing) the solution method.	

		Design a stained-glass window that could be built for a given dollar amount, and present (orally, in writing, and through other representations) a justification that it costs less than the limit.	
		Make simplifying assumptions to solve problems about a stained-glass window.	
Unit 4: Proportional Relationships and Percentages			17 - 18 Days of Instruction -- 4 Weeks
	Proportional Relationships with Fractions		
	Lesson 1	Compare (orally and in writing) the dimensions and scale factors of multiple scaled copies of the same figure.	
		Explain (orally) how to estimate or calculate the percentage of a rectangular area that is covered by another region.	
		Generate the dimensions for a scaled copy of an original figure that has fractional side lengths.	

Lesson 2	Compare and contrast (orally and in writing) different strategies for solving a problem involving equivalent ratios with fractional quantities.	
	Explain (orally and in writing) how to find and use a unit rate to solve a problem involving fractional quantities.	
Lesson 3	Calculate and interpret (orally) the constant of proportionality for a proportional relationship involving fractional quantities.	
	Explain (orally and in writing) how to use a table with only two rows to solve a problem involving a proportional relationship.	
	Write an equation to represent a given proportional relationship with a fractional constant of proportionality.	
Lesson 4	Apply the distributive property to generate algebraic expressions that represent a situation involving adding or subtracting a fraction of the initial value, and explain (orally) the reasoning.	

		Coordinate tables, equations, tape diagrams, and verbal descriptions that represent a relationship involving adding or subtracting a fraction of the initial value.	
		Generalize a process for finding the value that is “half as much again,” and justify (orally and in writing) why this can be abstracted as $(3/2)x$ or equivalent.	

	Lesson 5	Comprehend and use the term “repeating” (in spoken language) and the notation ____ (in written language) to refer to a decimal expansion that keeps having the same number over and over forever.	
		Coordinate fraction and decimal representations of situations involving adding or subtracting a fraction of the initial value.	
		Use long division to generate a decimal representation of a fraction, and describe (in writing) the decimal that results.	
	Percent Increase and Decrease		
	Lesson 6	Coordinate statements about “percent increase” or “percent decrease” with comparisons to the original amount, e.g. a 20% increase means the new value is 120% of the original value.	

		Draw and label a tape diagram to represent a situation that involves adding or subtracting a percentage of the initial value.	
		Explain (orally and in writing) how to calculate the new amount given the original amount and a percentage of increase or decrease.	
	Lesson 7	Critique (orally and in writing) double number line diagrams that represent situations involving percent increase or decrease.	
		Generalize (orally) that the original amount corresponds to 100% and the new amount corresponds to more or less than 100%, depending on whether the situation involves an increase or decrease.	

		Interpret a description of a situation to identify the original amount, the new amount, the change, and corresponding percentages. Label these on a double number line diagram.	
	Lesson 8	Explain (orally and in writing) how to calculate the original amount given the new amount and a percentage of increase or decrease.	
		Generate algebraic expressions that represent a situation involving percent increase or decrease, and justify (orally) the reasoning.	
	Lesson 9	Comprehend that percentages do not have to be a whole number.	
		Recognize that 0.1% of a number is $\frac{1}{10}$ of 1% of the number.	

		Use reasoning about place value to calculate percentages that are not whole numbers, and explain (orally) the strategy.
Applying Percentages		
Lesson 10	Comprehend “sales tax” and “tip” as two contexts that involve adding a percentage of the initial amount.	
	Explain (orally) how to calculate the total cost including a tax or tip, given the “subtotal” and the percentage.	
	Explain (orally) how to determine the percentage of the subtotal that a tax or tip is.	
Lesson 11	Comprehend “interest,” “markup,” “markdown,” and “commission” as other contexts that involve adding or subtracting a percentage of the initial amount.	

		Determine the original dollar amount before a markup, markdown, or commission.	
		Explain (orally) how to calculate the new dollar amount after a markup, markdown, or commission.	
	Lesson 12	Determine what information is needed to solve a problem involving sales tax and discounts. Ask questions to elicit that information.	

		Explain (orally) how to calculate the percentage, given the dollar amounts before and after a sales tax, tip, or discount.	
		Interpret (orally and in writing) tape diagrams that represent situations involving a sales tax, tip, or discount.	
	Lesson 13	Compare and contrast (orally) multiple measurements of the same length that result from using rulers with different levels of precision.	
		Describe (orally) possible sources of “measurement error” when measuring lengths.	
		Generalize a process for calculating measurement error and expressing it as a percentage of the actual length.	
	Lesson 14	Calculate the percent error, correct amount, or erroneous amount, given the other two of these three quantities, and explain (orally and using other representations) the solution method.	
		Compare and contrast (orally) strategies used for solving problems about percent error with strategies used for solving problems about percent increase or decrease.	
	Lesson 15 - Optional	Comprehend that manufacturers often define a maximum acceptable percent error for characteristics of their products.	

		Determine what information is needed to solve a problem involving percent error. Ask questions to elicit that information.	
		Generate values that fall within the acceptable range for a measurement, given a maximum percent error.	
Let's Put it to Work			
Lesson 16		Generate questions (orally and in writing) about a real-world situation involving percent increase or decrease.	
		Interpret news headlines or advertisements that include statements about percent increase and decrease.	
		Solve a problem about a real-world situation involving percent increase or decrease and present the solution method (in writing and through other representations).	
Unit 5: Rational Number Arithmetic			19 Days of Instruction -- 4 Weeks
	Interpreting Negative Numbers		



Lesson 1	Interpret signed numbers in the contexts of temperature and elevation.	
	Order rational numbers, and justify (orally) the comparisons.	
	Plot points on a vertical or horizontal number line to represent rational numbers.	

Adding and Subtracting Rational Numbers		
Lesson 2	Determine the final temperature given the starting temperature and the change in temperature, and explain (orally and using other representations) the solution method.	
	Explain (orally) how to create a number line diagram that represents adding signed numbers.	
	Write an addition equation to represent a situation involving a temperature increase or decrease.	
Lesson 3	Comprehend the term “opposite” (in spoken and written language) refers to numbers with the same magnitude but different signs.	
	Create and interpret equations and diagrams that represent adding signed numbers in the context of elevation.	

		Generalize (orally) a method for determining the sum of two signed numbers.	
	Lesson 4	Apply addition of signed numbers to calculate an account balance after a deposit or withdrawal, and explain (orally and using other representations) the solution method.	
		Explain (orally and in writing) how signed numbers can be used to represent situations involving money, including deposits or withdrawals, and assets or debts.	
		Write an equation with an unknown addend to represent a situation where the amount of change is unknown.	
	Lesson 5	Generalize (orally and in writing) that subtracting a number results in the same value as adding the additive inverse.	
		Interpret a number line diagram that represents subtracting signed numbers as adding with an unknown addend.	
		Use a number line diagram to find the difference of signed numbers, and explain (orally) the reasoning.	
	Lesson 6	Compare and contrast (orally) subtraction expressions that have the same numbers in the opposite order.	

	Recognize that the “difference” of two numbers can be positive or negative, depending on the order they are listed, while the “distance” between two numbers is always positive.
	Subtract signed numbers, and explain (orally) the reasoning.
Lesson 7	Apply addition and subtraction of signed numbers to solve problems in an unfamiliar context, and explain (orally and in writing) the solution method.
	Interpret signed numbers used to represent gains or losses in an unfamiliar context.
Multiplying and Dividing Rational Numbers	
Lesson 8	Explain (orally and in writing) how signed numbers can be used to represent positions and speeds in opposite directions.

		Generalize (orally) that the product of a negative number and a positive number is negative.	
		Write a multiplication equation to represent a situation involving constant speed with direction.	
	Lesson 9	Generalize (orally) that the product of two negative numbers is positive.	

		Interpret signed numbers used to represent elapsed time before or after a chosen reference point.	
		Use patterns to find the product of signed numbers, and explain (orally and using other representations) the reasoning.	
	Lesson 10	Identify multiplication expressions that are equal, and justify (orally) that they are equal.	
		Multiply rational numbers, including multiplication expressions with three factors, and explain (orally and in writing) the reasoning.	
	Lesson 11	Apply multiplication and division of signed numbers to solve problems involving constant speed with direction, and explain (orally) the reasoning.	
		Generalize (orally) a method for determining the quotient of two signed numbers.	
		Generate a division equation that represents the same relationship as a given multiplication equation with signed numbers.	
	Lesson 12	Apply operations with signed numbers to solve problems involving constant rates, and explain (orally) the solution method.	

	<p>Explain (orally and in writing) how signed numbers can be used to represent situations involving constant rates.</p> <p>Write an equation of the form <math>y=-kx</math> to represent a situation that involves descending at a constant rate.</p>
Four Operations with Rational Numbers	
Lesson 13	<p>Evaluate an expression for given values of the variable, including negative values, and compare (orally) the resulting values of the expression.</p> <p>Generalize (orally) about the relationship between additive inverses and about the relationship between multiplicative inverses.</p> <p>Identify numerical expressions that are equal, and justify (orally) that they are equal.</p>
	<p>Lesson 14</p> <p>Apply operations with rational numbers to solve problems involving repeated gains or losses, and present (orally, in writing, and using other representations) the solution method.</p>
	Solving Equations Where There Are Negative Numbers
Lesson 15 (Optional)	<p>Explain (orally and in writing) how to solve an equation of the form <math>x+p=q</math> or <math>px=q</math>, where <math>p</math>, <math>q</math>, and <math>x</math> are rational numbers.</p>

		Generalize (orally) the usefulness of additive inverses and multiplicative inverses for solving equations of the form $x+p=q$ or $px=q$ .	
		Generate an equation of the form $x+p=q$ or $px=q$ to represent a situation involving rational numbers.	
	Lesson 16	Coordinate (orally and in writing) verbal descriptions, equations, and diagrams that represent the same situation involving an unknown amount in the context of temperature or elevation.	
		Write equations of the form $x+p=q$ or $px=q$ to represent and solve a problem in an unfamiliar context, and present the solution method (using words and other representations).	
	Let's Put it to Work		
	Lesson 17	Apply operations with rational numbers to calculate a stock's new value, change in value, or change expressed as a signed percentage of the previous value.	
		Compare (orally and in writing) changes in the value of different stocks, including the dollar amount and the percentage of the previous value.	
		Interpret (orally) tables that represent the values of different stocks in the stock market.	
Unit 6: Expression, Equations, and Inequalities			26 Days of Instruction -- 5 Weeks
	Representing Situations of the Form $px + q = r$ and $p(x + q) = r$		

	Lesson 1	Determine unknown values in a relationship that is not proportional, and explain (orally and in writing) the solution method.	
		Interpret and describe (orally and in writing) relationships that are predictable, but not proportional.	
		Justify (orally) that a given relationship is not proportional.	
	Lesson 2	Draw and label a tape diagram to represent relationships between quantities in a situation.	
		Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in a situation.	
		Interpret a tape diagram that represents a relationship of the form $px+q=r$ or $p(x+q)=r$ .	
	Lesson 3	Coordinate tape diagrams and equations of the form $px+q=r$ or $p(x+q)=r$ .	
		Create a tape diagram to represent an equation of the form $px+q=r$ or $p(x+q)=r$ , and use it to solve the equation.	
		Identify equivalent equations, and justify (using words and other representations) that they are equivalent.	

	Lesson 4	Coordinate tape diagrams, equations of the form $px+q=r$ , and verbal descriptions of the situations.	
		Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in an equation of the form $px+q=r$ .	
		Interpret (in writing) the solution to an equation in the context of the situation it represents.	
	Lesson 5	Coordinate tape diagrams, equations of the form $p(x+q)=r$ , and verbal descriptions of the situations.	
		Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in an equation of the form $p(x+q)=r$ .	
		Interpret (in writing) the solution to an equation in the context of the situation it represents.	
	Lesson 6	Categorize equations of the forms $px+q=r$ and $p(x+q)=r$ , and describe (orally) the categories.	
		Interpret a verbal description of a situation (in written language), and write an equation of the form $px+q=r$ or $p(x+q)=r$ to represent it.	
Solving Equations of the Form $px + q = r$ and $p(x + q) = r$ and Problems That Lead to Those Equations			



	Lesson 7	Compare and contrast (orally) different strategies for solving an equation of the form $px+q=r$ .	
		Explain (orally and in writing) how to use a balanced hanger diagram to solve an equation of the form $px+q=r$ .	
		Interpret a balanced hanger diagram, and write an equation of the form $px+q=r$ to represent the relationship shown.	

	Lesson 8	Compare and contrast (orally) different strategies for solving an equation of the form $p(x+q)=r$ .	
		Explain (orally and in writing) how to use a balanced hanger diagram to solve an equation of the form $p(x+q)=r$ .	
		Interpret a balanced hanger diagram with multiple groups, and justify (in writing) that there is more than one way to write an equation that represents the relationship shown.	
	Lesson 9	Generalize (orally) that doing the same thing to each side of an equation generates an equivalent equation.	
		Solve equations of the form $px+q=r$ or $p(x+q)=r$ that involve negative numbers, and explain (orally and in writing) the solution method.	

	Lesson 10	Critique (orally and in writing) a given solution method for an equation of the form $p(x+q)=r$ .	
		Evaluate (orally) the usefulness of different approaches for solving a given equation of the form $p(x+q)=r$ .	
		Recognize that there are two common approaches for solving an equation of the form $p(x+q)=r$ , i.e., expanding using the distributive property or dividing each side by $p$ .	
	Lesson 11	Interpret and coordinate tape diagrams, equations, and verbal descriptions for situations involving signed numbers.	

		Solve an equation of the form $px+q=r$ or $p(x+q)=r$ to determine an unknown quantity in a situation, and present the solution method (orally, in writing, and through other representations).	
		Write an equation of the form $px+q=r$ or $p(x+q)=r$ to represent a situation involving signed numbers.	
	Lesson 12	Solve word problems leading to equations of the form $px+q=r$ or $p(x+q)=r$	
	Inequalities		

	Lesson 13	Comprehend the terms “less than or equal to” and “greater than or equal to” (in spoken and written language) and the symbols $\leq$ and $\geq$ (in written language).	
		Recognize that more than one value for a variable makes the same inequality true.	
		Use substitution to determine whether a given value for a variable makes an inequality true, and justify (orally) the answer.	
	Lesson 14	Interpret inequalities that represent situations with a constraint.	
		Solve an equation of the form $px+q=r$ to determine the boundary point for an inequality of the form $px+q>r$ or $px+q<r$ .	
		Use substitution or reasoning about the context to justify (orally and in writing) whether the values that make an inequality true are greater than or less than the boundary point.	
	Lesson 15	Compare and contrast (orally) solutions to equations and solutions to inequalities.	
		Draw and label a graph on the number line that represents all the solutions to an inequality.	

		Generalize (orally) that you can solve an inequality of the form $px+q>r$ or $px+q<r$ by solving the equation $px+q=r$ and then testing a value to determine the direction of the inequality in the solution.	
	Lesson 16	Critique (orally) a solution method for a problem involving an inequality.	
		Identify the inequality that represents a situation, and justify (in writing) the choice.	
		Present (orally, in writing, and using other representations) the solution method for a problem involving an inequality, and interpret the solution.	
	Lesson 17	Critique (orally) the solution to an inequality, including whether fractional or negative values are reasonable.	
		Determine what information is needed to solve a problem involving a quantity constrained by a maximum or minimum acceptable value. Ask questions to elicit that information.	
		Write and solve an inequality of the form $px+q>r$ or $px+q<r$ to answer a question about a situation with a constraint.	
	Writing Equivalent Expressions		

Lesson 18	Explain (orally, in writing, and using other representations) how the distributive and commutative properties apply to expressions with negative coefficients.
	Justify (orally and in writing) whether expressions are equivalent, including rewriting subtraction as adding the opposite.
Lesson 19	Apply the distributive property to expand or factor an expression that includes negative coefficients, and explain (orally and using other representations) the reasoning.
	Comprehend the terms “expand” and “factor” (in spoken and written language) in relation to the distributive property.
Lesson 20	Apply properties of operations to justify (orally and in writing) that expressions are equivalent.
	Generate an expression that is equivalent to a given expression with fewer terms.
	Interpret different methods for determining whether expressions are equivalent, and evaluate (orally) their usefulness.
Lesson 21	Critique (in writing) methods for generating equivalent expressions with fewer terms.

		Generate expressions that are not equivalent, but differ only in the placement of parentheses, and justify (orally) that they are not equivalent.	
		Write expressions with fewer terms that are equivalent to a given expression that includes negative coefficients and parentheses.	
	Lesson 22	Explain (orally and in writing) how to write an equivalent expression with fewer terms.	
		Generalize (orally) about what strategies are useful and what mistakes are common when writing equivalent expressions with fewer terms.	
		Identify equivalent expressions, and justify (orally and in writing) that they are equivalent.	
	Let's Put it to Work		
	Lesson 23	Determine which order for applying multiple coupons gives the better discount and explain (orally and in writing) the reasoning.	

		Justify (orally, in writing, and using other representations) that two different sequences of calculations give the same result.	
Unit 7: Angles, Triangles and Prisms			19 Days of Instruction -- 4 Weeks
	Angle Relationships		
	Lesson 1	Comprehend and use the word “degrees” (in spoken and written language) and the symbol ° (in written language) to refer to the amount of turn between two different directions.	

		Recognize 180° and 360° angles, and identify when adjacent angles add up to these amounts.	
		Use reasoning about adjacent angles to determine the angle measures of pattern blocks, and justify (orally) the reasoning.	
	Lesson 2	Comprehend the terms “complementary” and “supplementary” (in spoken and written language) as they describe pairs of angles.	
		Explain (orally and in writing) how to find an unknown angle measure, given adjacent complementary or supplementary angles.	

		Generalize (orally) that when a straight angle or a right angle is decomposed, the measures of the resulting angles add up to $180^\circ$ or $90^\circ$ , respectively.	
	Lesson 3	Comprehend the term “vertical angles” (in spoken and written language) refers to a pair of angles created by two intersecting lines.	
		Generalize (orally and in writing) that the opposite angles created by two intersecting lines have equal angle measures.	
		Use reasoning about angle measures to identify complementary or supplementary angles that are not adjacent.	
	Lesson 4	Coordinate (orally and in writing) diagrams and equations that represent the same relationship between angle measures.	
		Solve multi-step problems involving complementary, supplementary, and vertical angles, and explain (orally) the reasoning.	
	Lesson 5	Critique whether a given equation represents the relationship between angles in a diagram.	
		Solve an equation that represents a relationship between angle measures, and explain (in writing and using other representations) the reasoning.	



		Write an equation of the form $px+q=r$ or $p(x+q)=r$ to represent the relationship between angles in a given diagram.	
	Drawing Polygons with Given Conditions		
	Lesson 6	Comprehend that two shapes are considered “identical copies” if they can be placed on top of each other and match up exactly.	
		Recognize that four side lengths do not determine a unique quadrilateral, but that three side lengths can determine a unique triangle.	
		Use manipulatives to create a polygon with given side lengths, and describe (orally) the resulting shape.	
	Lesson 7	Explain (in writing) how to use circles to locate the point where the sides of a triangle with known side lengths should meet.	
		Use manipulatives to justify when it is not possible to make a triangle with three given side lengths.	
		Use manipulatives to show that there is a minimum and maximum length the third side of a triangle could be, given the other two side lengths.	

	Lesson 8	Describe, compare, and contrast (orally and in writing) triangles that share three common measures of angles or sides.	
		Justify (orally and using other representations) whether triangles are identical copies or are “different” triangles.	
		Recognize that examining which side lengths and angle measures are adjacent can help determine whether triangles are identical copies.	
	Lesson 9	Draw triangles with two given angle measures and one side length, and describe (orally) how many different triangles could be drawn with the given conditions.	
		Use drawings to justify (in writing) whether two given angle measures and one side length determine one unique triangle.	
	Lesson 10	Draw triangles with two given side lengths and one angle measure or three given angle measures, and describe (orally) how many different triangles could be drawn with the given conditions.	
		Use drawings to justify (in writing) whether two given side lengths and one angle measure determine one unique triangle.	
Solid Geometry			
Lesson 11	Categorize images of planes intersecting pyramids and prisms, and describe (orally) the categories.		

		Comprehend that the term “cross section” (in spoken and written language) refers to the two-dimensional face that results from slicing a three-dimensional figure.	
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		Describe, compare, and contrast (orally and in writing) different cross sections that could result from slicing the same pyramid or prism.	
	Lesson 12	Determine the volume of a right prism by counting how many unit cubes it takes to build one layer and then multiplying by the number of layers.	
		Generalize (orally) the relationship between the volume of a prism, the area of its base, and its height.	
		Identify whether a given figure is a prism, and if so, identify its base and height.	
	Lesson 13	Critique (orally) different methods for decomposing and calculating the area of a prism’s base.	
		Explain (orally and in writing) how to decompose and calculate the area of a prism’s base, and then use it to calculate the prism’s volume.	
	Lesson 14	Calculate the surface area of a prism, and explain (in writing) the solution method.	

		Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units.	
		Interpret different methods for calculating the surface area of a prism, and evaluate (orally and in writing) their usefulness.	
	Lesson 15	Compare and contrast (orally and in writing) problems that involve surface area and volume of prisms.	

		Decide whether to calculate the surface area or volume of a prism to solve a problem in a real-world situation, and justify (orally) the decision.	
		Estimate measurements of a prism in a real-world situation, and explain (orally) the estimation strategy.	
	Lesson 16	Apply reasoning about surface area and volume of prisms as well as proportional relationships to calculate how much the material to build something will cost, and explain (orally and in writing) the solution method.	
	Let's Put it to Work		

	Lesson 17	Compare and contrast (orally) triangular prisms, including comparisons of their height, cross sections, surface area, and volume.	
		Compose two triangular prisms into a new prism, and describe (orally and in writing) the composite shape.	
		Draw and assemble a net of a triangular prism, given two side lengths of the prism's base and one angle measure.	
Unit 8: Probability and Sampling			21 - 23 Days of Instruction -- 5 Weeks
	Probabilities of Single Step Events		
	Lesson 1	Compare outcomes for different experiments, predict which experiment is more likely to produce a desired result, and justify (orally and in writing) the prediction.	
		Describe (orally) how we can use the outcomes from previous experiments to help determine the relative likelihood of future events.	

	Lesson 2	Comprehend and use the terms “impossible,” “unlikely,” “equally likely as not,” “likely,” and “certain” (in spoken and written language) to describe the likelihood of an event.	
		Interpret percentages, fractions, and decimals that represent the likelihood of events.	
		Order a given set of events from least likely to most likely, and justify (orally) the reasoning.	
	Lesson 3	Generalize (orally) the relationship between the probability of an event and the number of possible outcomes in the sample space, for an experiment in which each outcome in the sample space is equally likely.	
		List (in writing) the sample space of a simple chance experiment.	
		Use the sample space to determine the probability of an event, and express it as a fraction, decimal, or percentage.	
	Lesson 4	Describe (orally and in writing) patterns observed on a table or graph that shows the relative frequency for a repeated experiment.	
		Generalize (orally) that the cumulative relative frequency approaches the probability of the event as an experiment is repeated many times.	
		Generate possible results that would or would not be surprising for a repeated experiment, and justify (orally) the reasoning.	

	Lesson 5	Describe (orally and in writing) reasons why the relative frequency from an experiment may not exactly match the actual probability of the event.	
		Recognize that sometimes the outcomes in a sample space are not equally likely.	
		Use the results from a repeated experiment to estimate the probability of an event, and justify (orally and in writing) the estimate.	
	Lesson 6	Comprehend the term “simulation” (in written and spoken language) refers to a chance experiment used to represent a real-world situation.	
		Describe (orally and in writing) a simple chance experiment that could be used to simulate a real-world event.	
		Perform a simulation, and use the results to estimate the probability of a simple event in a real-world situation (using words and other representations).	
	Probabilities of Multi-step Events		
Lesson 7	Coordinate (orally) a real-world situation and a chance event that could be used to simulate that situation.		

		Perform a multi-step simulation, and use the results to estimate the probability of a compound event in a real-world situation (using words and other representations).	
	Lesson 8	Compare and contrast (in writing) different methods for representing the sample space of a compound event, and evaluate (orally) their usefulness.	

		Determine the total number of possible outcomes for a compound event, and justify the reasoning (orally, in writing, and using other representations).	
		Interpret or create a list, table, or tree diagram that represents the sample space of a compound event.	
	Lesson 9	Choose a method for representing the sample space of a compound event, and justify (orally) the choice.	
		Use the sample space to determine the probability of a compound event, and explain (orally, in writing, and using other representations) the reasoning.	
	Lesson 10	Describe a multi-step experiment that could be used to simulate a compound event in a real-world situation, and justify (orally and in writing) that it represents the situation.	



		Perform a simulation to estimate the probability of a compound event, and explain (orally and in writing) how the simulation could be improved.	
	Sampling		
	Lesson 11	Calculate the mean and mean absolute deviation for a data set, and interpret (orally) these measures.	
		Compare and contrast (orally and in writing) populations represented on dot plots in terms of their shape, center, spread, and visual overlap.	

		Justify (in writing) whether two populations are “very different” based on the difference in their means expressed as a multiple of the mean absolute deviation.	
	Lesson 12	Comprehend the terms “population” and “sample” (in spoken and written language) to refer to the whole group and a part of the group under consideration.	
		Describe (orally and in writing) a sample for a given population.	
		Explain (orally) that a sample may be used when it is unreasonable to gather data about an entire population.	

	Lesson 13	Calculate the mean or median of various samples, and compare them with the mean or median of the population.	
		Comprehend that the term “representative” (in spoken and written language) refers to a sample with a distribution that closely resembles the population’s shape, center, and spread.	
		Given dot plots, determine whether a sample is representative of the population, and explain (orally and in writing) the reasoning.	
	Lesson 14	Describe (orally and in writing) methods to obtain a random sample from a population.	
		Justify (orally) whether a given sampling method is fair.	
		Recognize that random sampling tends to produce representative samples and support valid inferences.	
	Using Samples		
	Lesson 15	Calculate and interpret (orally and in writing) the mean absolute deviation of a sample.	

		Generalize that an estimate for the center of a population distribution is more likely to be accurate when it is based on a random sample with less variability.	
		Use the mean of a random sample to make inferences about the population, and explain (orally and in writing) the reasoning.	
	Lesson 16	Compare (orally) proportions for the same category from different samples of a population.	
		Comprehend that the term “proportion” refers to a number between 0 and 1 that represents the fraction of the data within a certain category.	
		Use the proportion of a random sample that is within a certain category to make inferences about the population, and explain (orally and in writing) the reasoning.	
	Lesson 17 - Optional	Compare and contrast (orally) a distribution of sample means and the distribution of the population.	
		Generalize that an estimate for the center of a population distribution is more likely to be accurate when it is based on a larger random sample.	
		Interpret (orally and in writing) a dot plot that displays the means of multiple samples from the same population.	

	Lesson 18	Calculate the difference between the mean or median of two samples from different populations, and express it as a multiple of the MAD or IQR.
		Interpret a pair of box plots, including the amount of visual overlap between the two distributions.
		Justify (orally and in writing) whether there is likely to be a meaningful difference between two populations, based on a sample from each population.
	Lesson 19	Apply reasoning about center and spread to determine whether two populations are likely to be meaningfully different, and explain (orally and in writing) the reasoning.
		Coordinate (orally) visual displays of data with descriptions of shape, measures of center, and measures of spread.
		Determine what information is needed to solve problems about using samples to compare populations. Ask questions to elicit that information.
	Let's Put it to Work	
Lesson 20 - Optional	Describe (orally) connections between sampling and probability.	

		Generate a random sample, and use it to make inferences (in writing) about the population.	
		Justify (orally and in writing) whether a given method produces a random sample.	

Unit 9: Putting It All Together (Optional Unit)			0 - 13 Days of Instruction -- 3 Weeks
	Running a Restaurant		
	Lesson 1 - Optional	Create a recipe that meets the requirements to be considered low calorie, low fat, or low sodium, and justify (orally) the reasoning.	
		Determine whether one serving of a recipe meets the requirements to be considered low calorie, low fat, or low sodium, and explain (orally) the reasoning.	
		Use proportional reasoning to calculate nutritional values of one serving of a recipe.	
	Lesson 2 - Optional	Comprehend the term “spreadsheet” (in written and spoken language) is a computer program in which data is arranged in the rows and columns of a grid and can be manipulated and used in calculations.	

		Create formulas in a spreadsheet to perform repeated calculations.	
		Use a spreadsheet to calculate the cost per serving of all the ingredients in a recipe.	
	Lesson 3 - Optional	Create an equation to represent certain expenses for a restaurant, and interpret (orally and in writing) the solution.	
		Determine whether a relationship is proportional and explain (orally) the reasoning.	

		Determine whether a restaurant is making a profit using estimates of ongoing expenses, number of meals sold, average price per meal, and average cost per meal.	
Lesson 4 - Optional		Choose an appropriate scale and create a scale drawing for a restaurant floor plan.	
		Use proportional reasoning to solve problems about the area or volume of different elements of a floor plan and explain (orally) the solution method.	
Making Connections			

	Lesson 5 - Optional	Compare and contrast the density of uniformly distributed dots in squares.	
		Create an equation and a graph that represent the proportional relationship between the area of a square and the number of dots enclosed by the square.	
		Interpret the constant of proportionality in models of housing per square kilometer or population of people per square kilometer.	
	Lesson 6 - Optional	Calculate a rough estimate for quantities that are difficult or impossible to measure directly and explain (orally) the reasoning.	
		Choose an appropriate level of accuracy when reporting estimates of quantities.	

		Make simplifying assumptions to solve problems about estimating quantities.	
	Lesson 7 - Optional	Create expressions and equations to represent a linear relationship with two or more related quantities in context.	
		Determine the solution to an equation of the form $px+q=r$ and explain (orally) the solution method.	

		Interpret expressions, equations, and solutions that represent a linear relationship with two or more related quantities.	
	Lesson 8 - Optional	Estimate a measurement and determine the largest possible percent error of the estimate.	
		Generalize (orally) a process for calculating the maximum percent error of measurements that are added together.	
	Lesson 9 - Optional	Comprehend that percent error is greatest when the actual values are smaller than the measured values for an object.	
		Generalize (orally) a process for calculating the maximum percent error for the area of a rectangle and volume of a rectangular prism.	
Designing a Course			
	Lesson 10 - Optional	Choose a method and appropriate measuring tools to measure the length of a path.	
		Compare and contrast (orally and in writing) methods for measuring distances.	
		Compare measurements of a path and represent (in writing) the difference between measurements as a percentage.	



	Lesson 11 - Optional	Create a trundle wheel and use it to calculate the length of the classroom.	
		Explain (orally and in writing) how a trundle wheel is used to measure long distances.	
	Lesson 12 - Optional	Calculate the distance of a path using the circumference and number of rotations of a trundle wheel.	
		Compare measurement calculations and express differences between measurements as a percentage.	
		Critique (orally) methods for measuring a long distance.	
	Lesson 13 - Optional	Calculate the distance of a path using the circumference and number of rotations of a trundle wheel.	
		Create a scale drawing of a 5K course and present (using words and other representations) the map and course details.	
		Use proportional reasoning to calculate the number of laps of a course that is equal to 5 kilometers.	

**Supports of Diversity, Equity and Inclusion**

*Please provide any information relative to supporting culturally responsive instruction, multi-language learners, and students with disabilities*

**Review Site Information:**

URL: review-ct.ilclassroom.com

Username: CT@example.com

Password: teacher

**Culturally Responsive Instruction:**

Illustrative Mathematics includes culturally relevant materials and culturally responsive teaching and instructional practices. Materials are inclusive of a variety of cultures and ethnicities and are free from bias in the portrayal of ethnic groups, gender, age, class, cultures, religions, and people with disabilities.

We address racial, cultural, and religious bias in the following ways:

- The materials contain racial/ethnic balance in the main characters and illustrations.
- Minorities are represented as central figures in text and illustrations.
- Minority figures reflect qualities such as leadership, intelligence, imagination, and courage.
- The materials provide an opportunity for a variety of racial, ethnic, and cultural perspectives.
- The vocabulary or depiction of racism is avoided (i.e., insulting overtones).
- Race/culture stereotyping language is avoided.
- Biographical or historical content includes minority figures and their discoveries and contributions to society.

**Multi-Language Learners:**

In a problem-based mathematics classroom, sense-making and language are interwoven. Mathematics classrooms are language-rich, and therefore language demanding learning environments for every student. The linguistic demands of doing mathematics include reading, writing, speaking, listening, conversing, and representing (Aguirre & Bunch, 2012). Students are expected to say or write mathematical explanations, state assumptions, make conjectures, construct mathematical arguments, and listen to and respond to the ideas of others. In an effort to advance the mathematics and language learning of all students, the materials purposefully engage

students in sense-making and using language to negotiate meaning with their peers. To support students who are learning English in their development of language, this curriculum includes instruction devoted to fostering language development alongside mathematics learning, fostering language-rich environments where there is space for all students to participate.

This interwoven approach is grounded in four design principles that promote mathematical language use and development:

**Principle 1. Support sense-making: Scaffold tasks and amplify language so students can make their own meaning.** Students need multiple opportunities to talk about their mathematical thinking, negotiate meaning with others, and collaboratively solve problems with targeted guidance from the teacher. Teachers can make language more accessible by amplifying rather than simplifying speech or text. Simplifying includes avoiding the use of challenging words or phrases. Amplifying means anticipating where students might need support in understanding concepts or mathematical terms and providing multiple ways to access them.

**Principle 2. Optimize output: Strengthen opportunities for students to describe their mathematical thinking to others, orally, visually, and in writing.** All students benefit from repeated, strategically optimized, and supported opportunities to articulate mathematical ideas into linguistic expression, to communicate their ideas to others. Opportunities for students to produce output should be strategically optimized for both (a) important concepts of the unit or course, and (b) important disciplinary language functions (for example, explaining reasoning, critiquing the reasoning of others, making generalizations, and comparing approaches and representations).

**Principle 3. Cultivate conversation: Strengthen opportunities for constructive mathematical conversations.** Conversations are back-and-forth interactions with multiple turns that build up ideas about math. Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning, communicate that meaning, and refine the way content understandings are communicated. During effective discussions, students pose and answer questions, clarify what is being asked and what is happening in a problem, build common understandings, and share experiences relevant to the topic. Meaningful conversations depend on the teacher using activities and routines as opportunities to build a classroom culture that motivates and values efforts to communicate.

**Principle 4. Maximize meta-awareness: Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language.** Meta-awareness, consciously thinking about one's own thought processes or language use, develops when students consider how to improve their communication and reasoning about mathematical concepts. When students are using language in ways that are purposeful and meaningful for themselves, in their efforts to understand—and be understood by—each other, they are motivated to attend to ways in which language can be both clarified and clarifying. Students learning English benefit from being aware of how language choices are related to the purpose of the task and the intended audience, especially if oral or written work is required. Both metacognitive and metalinguistic awareness are powerful tools to help students self-regulate their academic learning and language acquisition.



These design principles and related mathematical language routines, described below, ensure language development is an integral part of planning and delivering instruction. Moreover, they work together to guide teachers to amplify the most important language that students are expected to know and use in each unit.

### **Mathematical Language Routines**

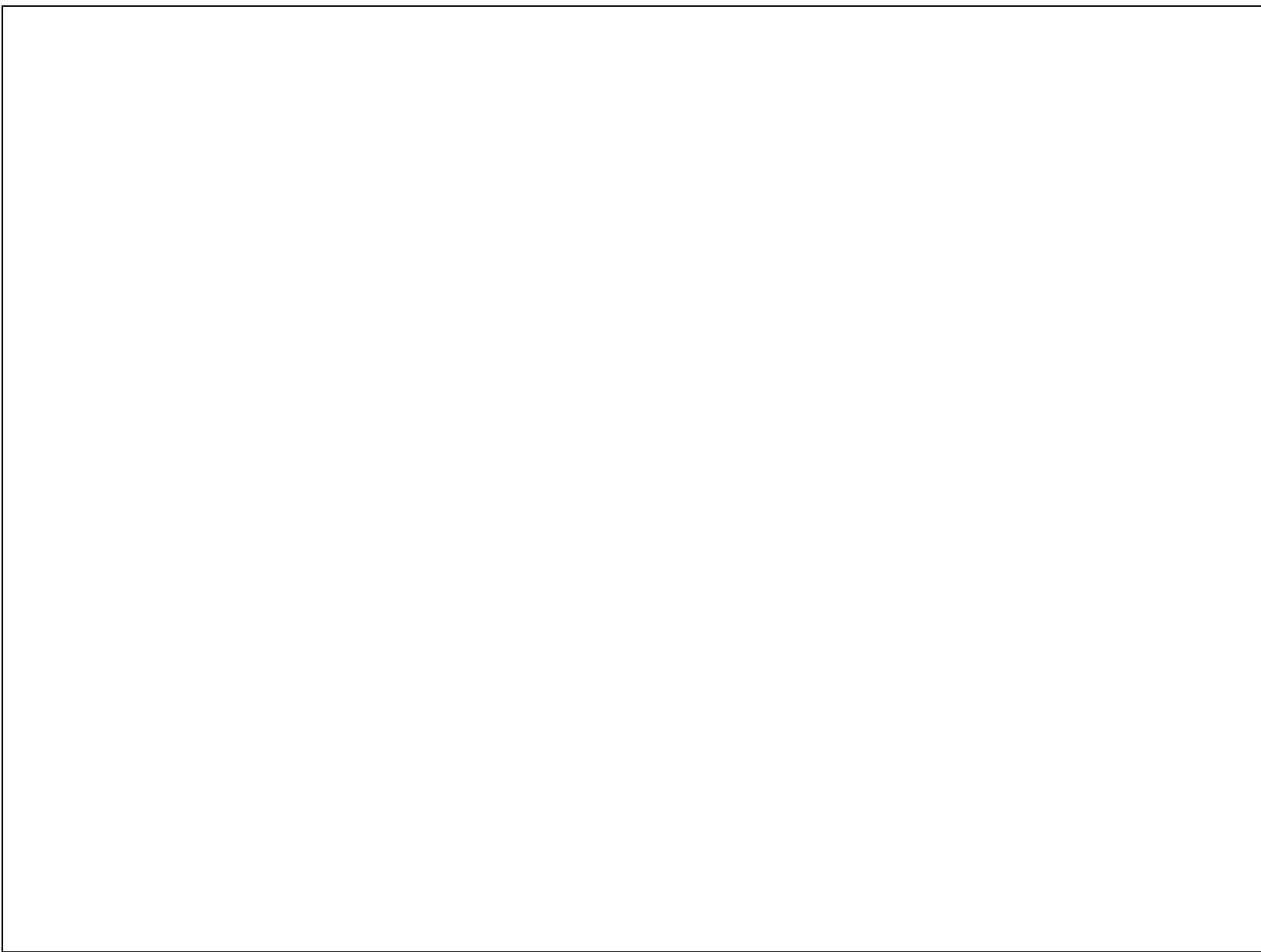
Mathematical Language Routines (MLRs) are instructional routines that provide structured but adaptable formats for amplifying, assessing, and developing students' language. The MLRs included in this curriculum were selected because they simultaneously support students' learning of mathematical practices, content, and language. They are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English. These routines are flexible and can be adapted to support students at all stages of language development in using and improving their English and disciplinary language use.

These routines are included in the Curriculum Guide and noted below:

- MLR 1: Stronger and Clearer Each Time
- MLR 2: Collect and Display
- MLR 3: Clarify, Critique, Correct
- MLR 4: Information Gap
- MLR 5: Co-Craft Questions
- MLR 6: Three Reads
- MLR 7: Compare and Connect ● MLR 8: Discussion Supports

MLRs are included in select activities in each unit to provide all students with explicit opportunities to develop mathematical and academic language proficiency. These “embedded” MLRs are described in the teacher notes for the lessons in which they appear.

Each lesson also includes optional, suggested MLRs that can be used to support access and language development for English learners, based on the language demands students will encounter. They are described in the activity narrative, under the heading “Access for English Learners.” Teachers can use the suggested MLRs and language strategies as appropriate to provide students with access to an activity without reducing the mathematical demand of the task. When using these supports, teachers should take into account the language demands of the specific activity and the language needed to engage the content more broadly, in relation to their students' current ways of using language to communicate ideas as well as their students' English language proficiency. Using these supports can help maintain student engagement in mathematical discourse and ensure that struggle remains productive. All of the supports are designed to be used as needed, and use should fade out as students develop understanding and fluency with the English language.



In addition to the comprehensive pedagogical design of the program, Spanish translations are available for the educator components, including teacher slides, and the student components, including the student workbook (print version).

Materials are also available in Spanish as follows:

### What's in Spanish for IM?

K-5	6-8	AGA
<ul style="list-style-type: none"> <li>Printed: Student Workbooks</li> <li>eBook/PDF: Student, Teacher, Teacher Resource Pack</li> <li>Spanish Lesson Cards</li> </ul> <p><b>Other Materials</b> <i>(no solutions translated)</i></p> <ul style="list-style-type: none"> <li>Task Statements (PDF)</li> <li>Cool-Down (PDF)</li> <li>Practice Problems (PDF)</li> <li>Unit Assessments (PDF)</li> <li>Section Checkpoint Quizzes (PDF)</li> <li>Family Supports (PDF)</li> <li>Center Materials (PDF)</li> <li>Glossary entries</li> </ul>	<p><u><b>6-8 Courses Only (Not Acc.)</b></u></p> <ul style="list-style-type: none"> <li>Printed: Student Workbooks</li> <li>eBook/PDF: Student</li> </ul> <p><b>Other Materials</b> <i>(no solutions translated)</i></p> <ul style="list-style-type: none"> <li>Task Statements (PDF)</li> <li>Cool-Down (PDF)</li> <li>Practice Problems (PDF)</li> <li>Unit Assessments Option B, (PDF)</li> <li>Glossary entries</li> </ul>	<p><u><b>Algebra 1 Only</b></u></p> <p>eBook/PDF: Student Workbook</p> <p>*Print coming for BTS 2023</p> <p><b>Other Materials</b> <i>(no solutions translated)</i></p> <ul style="list-style-type: none"> <li>Task Statements (PDF)</li> <li>Cool-Down (PDF)</li> <li>Practice Problems (PDF)</li> <li>Unit Assessments (PDF)</li> <li>Modeling prompts</li> <li>Glossary entries</li> </ul>

### Exceptional Learners:

Imagine Learning Illustrative Mathematics materials empower all students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students, positioning all learners as competent, valued contributors. When planning to support access, teachers should consider the strengths and needs of their particular students.

Each lesson is carefully designed to maximize engagement and accessibility for all students. Purposeful design elements that support access for all learners, but that are especially helpful for students with disabilities, include:

### **Lesson Structures are Consistent**





The structure of every lesson is the same: warm-up, activities, synthesis, cool-down. By keeping the components of each lesson similar from day to day, the flow of work in class becomes predictable for students. This reduces cognitive demand and enables students to focus on the mathematics at hand rather than the mechanics of the lesson.

### **Concepts Develop from Concrete to Abstract**

Mathematical concepts are introduced simply, concretely, and repeatedly, with complexity and abstraction developing over time. Students begin with concrete examples, and transition to diagrams and tables before relying exclusively on symbols to represent the mathematics they encounter.

### **Individual to Pair, or Small Group to Whole Class Progression**

Providing students with time to think through a situation or question independently before engaging with others allows students to carry the weight of learning, with support arriving just in time from the community of learners. This progression allows students to first activate what they already know, and continue to build from this base with others.

### **Opportunities to Apply Mathematics to Real-World Contexts**

Giving students opportunities to apply the mathematics they learn clarifies and deepens their understanding of core math concepts and skills and provides motivation and support. Mathematical modeling is a powerful activity for all students, but especially students with disabilities. Each unit has a culminating activity designed to explore, integrate, and apply all the big ideas of the unit. Centering instruction on these contextual situations can provide students with disabilities an anchor on which to base their mathematical understandings.

Supplemental instructional strategies that can be used to increase access, reduce barriers and maximize learning are included in each lesson, listed in the activity narratives under “*Access for Students with Disabilities.*” Each support is aligned to the Universal Design for Learning Guidelines and based on one of the three principles of UDL, to provide alternative means of *engagement, representation, or action and expression*. These supports provide teachers with additional ways to adjust the learning environment so that students can access activities, engage in content, and communicate their understanding. Supports are tagged with the areas of cognitive functioning they are designed to address to help teachers identify and select appropriate supports for their students. Designed to facilitate access to Tier 1 instruction by capitalizing on student strengths to address challenges related to cognitive functions or disabilities, these strategies and supports are appropriate for any students who need additional support to access rigorous, grade-level content.

Teachers are encouraged to use what they know about their students’ IEPs, strengths and challenges, and a UDL approach to ensure access.

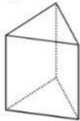
There are embedded supports for exceptional students in most lessons. Teachers will find these in the **Teaching Notes** section. As of

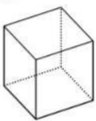
June 2020, Illustrative Mathematics 6-8 student facing materials meet Section 508 compliance standards, meaning that students can use

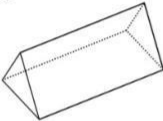
assistive technology to navigate the site. Illustrative Mathematics K-5 digital materials were added during the 21-22 School Year and are 508 compliant as well. Outlined in the Curriculum Guide, there are features, supports, and strategies available.

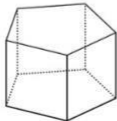
813.2 Activity: Prisms and Pyramids

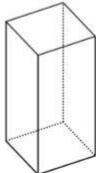
1. Here are some polyhedra called **prisms**.

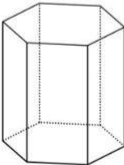
A

B

C

D

E

F

When talking about the polygons that make up their polyhedra, as well as the characteristics of their polyhedra (e.g., triangle, rectangle, square, hexagon, pentagon, vertex, edge, face). Collect this language, with corresponding drawings, and display it for all students to see. Remind students to borrow language from the display as they describe the features of prisms and pyramids. This will help students produce mathematical language to describe and define characteristics of polyhedra.

**Design Principle(s):** Support sense-making

**Support for students with disabilities**

- **Representation: Access for Perception.** Provide access to concrete manipulatives. Provide prisms and pyramids for students to view or manipulate. These hands-on models will help students identify characteristics or features, and support net building for each polyhedra.

Supports accessibility for: Visual-spatial processing; Conceptual processing

The curriculum authors drew heavily on the UDL framework in the design of these materials. A number one design principle of the curriculum is “Access for all.” This foundational principle draws from the UDL framework and shapes the instructional goals, recommended practices, lesson plans, and assessments to support a flexible approach to instruction, ensuring all students have an equitable opportunity to learn.

Imagine Learning software is browser-based so it will work with any browser-based text-to-speech tools. Fonts can be adjusted in type and size. Non-text navigation elements can be adjusted in size. Math equation editing is available on assessment items and practice problems.

Imagine Learning can provide a NIMAS-compatible version of Illustrative Mathematics content. These files may be used for the production of alternate formats as permitted under the law for students with disabilities.