Connecticut Mathematics Model Curriculum Alignment

Resource Name: Imagine Learning Illustrative Mathematics Grade 6

Model Unit Name	Model Unit Standards	Resource Unit(s) Number and Lessons	Standard Frequency
This is the title of the unit in the model curricula	*These are the standards addressed in the unit*	*This is the unit(s) that aligns with the model unit from the resource*	*This is the total number of lessons the standard is addressed*
Pacing - Illustrative Mathematics 6-8 lessons are designed to fit within a 45-50 minute block. Pacing guidance for each activity is provided in the lesson plans.			
Operating with Positive Rational Numbers			
	6.NS.A.1	Unit 4, Lesson 7: What Fraction of a Group?	16 days - 15 Spotlight Lessons
		Unit 4, Lesson 17: Fitting Boxes into Boxes.	
	6.NS.B.2	Unit 5, Lesson 9: Using the Partial Quotients Method	3 days - 3 Spotlight Lessons

		Unit 5, Lesson 10: Using Long Division	
	6.NS.B.3	Unit 5, Lesson 2: Using Diagrams to Represent Addition and Subtraction	13 days - 12 Spotlight Lessons

		Unit 5, Lesson 12: Dividing Decimals by Whole Numbers	
	6.NS.B.4	Unit 7, Lesson 16: Common Factors	3 days - 3 Spotlight Lessons
		Unit 7, Lesson 18: Using Common Multiples and Common Factors	
	6.G.A.2	Unit 4, Lesson 14: Fractional Lengths in Triangles and Prisms	4 days - 3 Spotlight Lessons
		Unit 1, Lesson 15: More Nets, More Surface Area	
Understanding Positive and Negative Numbers			

	6.NS.C.5	Unit 7, Lesson 1: Positive and Negative Numbers	2 days - 2 Spotlight Lessons
		Unit 7, Lesson 5: Using Negative Numbers to Make Sense of Contexts	
	6.NS.C.6	Unit 7, Lesson 2: Points on the Number Line	5 days - Spotlight Lessons
		Unit 7, Lesson 4: Ordering Rational Numbers	
	6.NS.C.7	Unit 7, Lesson 6: Absolute Value of Numbers	3 Days - 3 Spotlight Lessons
		Unit 7, Lesson 7: Comparing Numbers and Distance from Zero	

	6.NS.C.8	Unit 7, Lesson 11: Points on the Coordinate Plane	5 days - 5 Spotlight Lessons
		Unit 7, Lesson 13: Interpreting Points on a Coordinate Plane	

Using Expressions and Equations			
	6.EE.A.1	Unit 6, Lesson 13: Expressions with Exponents	6 days - 6 Spotlight Lessons
		Unit 1, Lesson 17: Squares and Cubes	
	6.EE.A.2	Unit 6, Lesson 10: The Distributive Property, Part 2	3 days - 3 Spotlight Lessons
		Unit 6, Lesson 19: Tables, Equations, and Graphs, Oh My!	
	6.EE.A.3	Unit 6, Lesson 9: The Distributive Property, Part 1	3 days - 3 Spotlight Lessons
		Unit 6, Lesson 11: The Distributive Property, Part 3	
	6.EE.A.4	Unit 6, Lesson 8: Equal and Equivalent	4 days - 4 Spotlight Lessons
		Unit 5, Lesson 13: Dividing Decimals by Decimals	

	6.EE.B.5	Unit 6, Lesson 2: Truth and Equations	8 days - 8 Spotlight Lessons
		Unit 6, Lesson 3: Staying in Balance	
	6.EE.B.6	Unit 6, Lesson 1: Tape Diagrams and Equations	8 days - 8 Spotlight Lessons
		Unit 6, Lesson 5: A New Way to Interpret a over b	
	6.EE.B.7	Unit 6, Lesson 4: Practice Solving Equations and Representing Situations with Equations	5 days - 5 Spotlight Lessons
		Unit 6, Lesson 7: Revisit Percentages	
	6.EE.B.8	Unit 7, Lesson 8: Writing and Graphing Inequalities	3 days - 3 Spotlight Lessons
		Unit 7, Lesson 9: Solutions of Inequalities	

Applications of Geometry			
	6.G.A.1	Unit 1, Lesson 2: Finding Area by Decomposing and Rearranging	12 days - 12 Spotlight Lessons
		Unit 1, Lesson 3: Reasoning to Find Area	
	6.G.A.3	Unit 7, Lesson 15: Shapes on the Coordinate Plane	2 days - 2 Spotlight Lessons
		Unit 7, Lesson 19: Drawing on the Coordinate Plane	
	6.G.A.4	Unit 1, Lesson 14: Nets and Surface Area	7 days - 7 Spotlight Lessons
		Unit 1, Lesson 15: More Nets, More Surface Area	
Ratios and Rates			

	6.RP.A.1	Unit 2, Lesson 1: Introducing Ratios and Ratio Language	7 days - 6 Spotlight Lessons
		Unit 2, Lesson 3: Recipes	
	6.RP.A.2	Unit 2, Lesson 10: Comparing Situations by Examining Ratios	6 days - 5 Spotlight Lessons
		Unit 3, Lesson 5: Comparing Speeds and Prices	
	6.RP.A.3	Unit 3, Lesson 9: Solving Rate Problems	17 days - 14 Spotlight Lessons
		Unit 3, Lesson 7: Equivalent Ratios Have the Same Unit Rates	
Algebraic Reasoning			

	6.EE.B.6	Unit 6, Lesson 4: Practice Solving Equations and Representing Situations with Equations	8 days - 8 Spotlight Lessons
		Unit 6, Lesson 6: Write Expressions Where Letters Stand for Numbers	
	6.EE.B.7	Unit 6, Lesson 5: A New Way to Interpret a over b	5 days - 5 Spotlight Lessons
		Unit 6, Lesson 3: Staying in Balance	
	6.EE.C.9	Unit 6, Lesson 16: Two Related Quantities, Part 1	4 days - 4 Spotlight Lessons
		Unit 6, Lesson 18: More Relationships	

Statistics and Distributions

	6.SP.A.1	Unit 8, Lesson 2: Statistical Questions	5 days - 5 Spotlight Lessons
		Unit 8, Lesson 7: Using Histograms to Answer Statistical Questions	
	6.SP.A.2	Unit 8, Lesson 5: Using Dot Plots to Answer Statistical Questions	6 days - 6 Spotlight Lessons
		Unit 8, Lesson 18: Using Data to Solve Problems	
	6.SP.A.3	Unit 8, Lesson 10: Finding and Interpreting the Mean as the Balance Point	4 days - 4 Spotlight Lessons
		Unit 8, Lesson 11: Variability and MAD	

	6.SP.B.4	Unit 8, Lesson 3: Representing Data Graphically	8 days - 8 Spotlight Lessons
		Unit 8, Lesson 17: Using Box Plots	
	6.SP.B.5	Unit 8, Lesson 4: Dot Plots	

		Unit 8, Lesson 6: Histograms	1 day - 1 Spotlight Lesson
Scope and Sequence			
If a district uses this resource to implement the state model curriculum for grade 6, the following scope and sequence should be followed to ensure alignment and attention to the progressions of mathematics.			

Unit Number/Title	Lesson Title	Lesson Objectives	# of Days/Weeks (assume 1 hour of instruction)
Unit 1: Area and Surface Area			**21 - 22 Days of Instruction -- 4 Weeks**
	Reasoning to Find Area		
	Lesson 1	Compare (orally) areas of the shapes that make up a geometric pattern.	
		Comprehend that the word “area” (orally and in writing) refers to how much of the plane a shape covers	
	Lesson 2	Calculate the area of a region by decomposing it and rearranging the pieces, and explain (orally and in writing) the solution method.	
		Recognize and explain (orally) that if two figures can be placed one on top one other so that they match up exactly, they must have the same area.	

		Show that area is additive by composing polygons with a given area.	
	Lesson 3	Calculate the area of a region by decomposing it and rearranging the pieces, and explain (orally and in writing) the solution method.	
		Recognize and explain (orally) that if two figures can be placed one on top of one other so that they match up exactly, they must have the same area.	
		Show that area is additive by composing polygons with a given area.	
	Parallelograms		
	Lesson 4	Compare and contrast (orally) different strategies for determining the area of a parallelogram.	
		Describe (orally and in writing) observations about the opposite sides and opposite angles of parallelograms.	
		Explain (orally and in writing) how to find the area of a parallelogram by rearranging or enclosing it in a rectangle.	

	Lesson 5	Comprehend the terms “base” and “height” to refer to one side of a parallelogram and the perpendicular distance between that side and the opposite side.	
		Generalize (orally) a process for finding the area of a parallelogram, using the length of a base and the corresponding height.	
		Identify a base and the corresponding height for a parallelogram, and understand that there are two different base-height pairs for any parallelogram.	
	Lesson 6	Apply the formula for area of a parallelogram to find the area, the length of the base, or the height, and explain (orally and in writing) the solution method.	
		Choose which measurements to use for calculating the area of a parallelogram when more than one base or height measurement is given, and explain (orally and in writing) the choice.	
	Triangles		
	Lesson 7	Describe (orally and in writing) ways in which two identical triangles can be composed, i.e., into a parallelogram or into a rectangle.	

		Show how any parallelogram can be decomposed into two identical triangles by drawing a diagonal, and generalize (in writing) that this property applies to all parallelograms, but not all quadrilaterals.	

	Lesson 8	Draw a diagram to show that the area of a triangle is half the area of an associated parallelogram.	
		Explain (orally and in writing) strategies for using the base and height of an associated parallelogram to determine the area of a triangle.	
	Lesson 9	Compare, contrast, and critique (orally) different strategies for determining the area of a triangle.	
		Generalize a process for finding the area of a triangle, and justify (orally and in writing) why this can be abstracted as $1/2.b.h$	
		Recognize that any side of a triangle can be considered its base, choose a side to use as the base when calculating the area of a triangle, and identify the corresponding height.	
	Lesson 10	Draw and label the height that corresponds to a given base of a triangle, making sure it is perpendicular to the base and the correct length.	

		Evaluate (orally) the usefulness of different base-height pairs for finding the area of a given triangle.	
	Polygons		
	Lesson 11	Compare and contrast (orally) different strategies for finding the area of a polygon.	

		Describe (orally and in writing) the defining characteristics of polygons.	
		Find the area of a polygon, by decomposing it into rectangles and triangles, and present the solution method (using words and other representations).	
	Surface Area		
	Lesson 12	Calculate the surface area of a rectangular prism and explain (orally and in writing) the solution method.	
		Comprehend that the term “surface area” (in written and spoken language) refers to how many square units it takes to cover all the faces of a three-dimensional object.	

	Lesson 13	Compare and contrast (orally and in writing) features of prisms and pyramids.	
		Comprehend and use the words “face”, “edge”, “vertex”, and “base” to describe polyhedra (in spoken and written language).	
		Understand that the word “net” refers to a two-dimensional figure that can be assembled into a polyhedron, and create a net for a given polyhedron	
	Lesson 14	Match polyhedra with their nets and justify (orally) that they match.	

		Use a net with gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.	
		Visualize and identify the polyhedron that can be assembled from a given net.	
	Lesson 15	Draw and assemble a net for the prism or pyramid shown in a given drawing.	
		Interpret (using words and other representations) two-dimensional representations of prisms and pyramids.	

		Use a net without gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.	
	Lesson 16 (Optional)	Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units.	
		Describe (orally and in writing) shapes built out of cubes, including observations about their surface area and volume.	
		Determine the surface area and volume of shapes made out of cubes.	
	Squares and Cubes		
	Lesson 17	Generalize a process for finding the volume of a cube, and justify (orally) why this can be abstracted as s3	
		Include appropriate units (orally and in writing) when reporting lengths, areas, and volumes, e.g. cm, cm2,cm3	
		Interpret and write expressions with exponents2 and 3 to represent the area of a square or the volume of a cube.	

	Lesson 18	Generalize a process for finding the surface area of a cube, and justify (orally) why this can be abstracted as $6 \cdot s^2$	
		Interpret (orally) expressions that include repeated addition, multiplication, repeated multiplication, or exponents.	
		Write expressions, with or without exponents, to represent the surface area of a given cube.	
	Lesson 19	Apply understanding of surface area to estimate the amount of fabric in a tent, and explain (orally and in writing) the estimation strategy.	
		Compare and contrast (orally) different tent designs.	
		Interpret information (presented in writing and through other representations) about tents and sleeping bags.	

Unit 2: Introducing Ratios			**19 Days of Instruction -- 4 Weeks**
	What Are Ratios?		

	Lesson 1	Comprehend the word “ratio” (in written and spoken language) and the notation a:b (in written language) to refer to an association between quantities.	
		Describe (orally and in writing) associations between quantities using the language “For every a of these, there are b of those” and “The ratio of these to those is a:b (or a to b).”	
	Lesson 2	Coordinate discrete diagrams and multiple written sentences describing the same ratios.	
		Draw and label discrete diagrams to represent situations involving ratios.	
		Practice reading and writing sentences describing ratios, e.g., “The ratio of these to those is a:b. The ratio of these to those is a to b. For every a of these, there are b of those.”	
	Equivalent Ratios		
	Lesson 3	Draw and label a discrete diagram with circled groups to represent multiple batches of a recipe.	
		Explain equivalent ratios (orally and in writing) in terms of different sized batches of the same recipe having the same taste.	

		Understand that doubling or tripling a recipe involves multiplying the amount of each ingredient by the same number, yielding something that tastes the same	
	Lesson 4	Comprehend and respond (orally and in writing) to questions asking whether two ratios are equivalent, in the context of color mixtures.	
		Draw and label a discrete diagram with circled groups to represent multiple batches of a color mixture.	
		Explain equivalent ratios (orally and in writing) in terms of the amounts of each color in a mixture being multiplied by the same number to create another mixture that is the same shade.	
	Lesson 5	Generate equivalent ratios and justify that they are equivalent.	
		Present (in words and through other representations) a definition of equivalent ratios, including examples and non-examples.	

	Representing Equivalent Ratios	

Lesson 6	Compare and contrast (orally and in writing) discrete diagrams and double number line diagrams representing the same situation.
	Explain (orally) how to use a double number line diagram to find equivalent ratios.
	Label and interpret a double number line diagram that represents a familiar context.
Solving Ratio and Rate Problems	
Lesson 7	Comprehend and use the word “per” (orally and in writing) to mean “for each.”
	Draw and label a double number line diagram from scratch, with parallel lines and equally-spaced tick marks.
	Use double number line diagrams to find a wider range of equivalent ratios.
Lesson 8	Calculate equivalent ratios between prices and quantities and present the solution method (using words and other representations).

		Calculate unit price and express it using the word “per” (orally and in writing).	

		Understand the phrase “at this rate” indicates that equivalent ratios are involved.	
	Lesson 9	Calculate the distance an object travels in 1 unit of time and express it using a phrase like “meters per second” (orally and in writing).	
		For an object moving at a constant speed, use a double number line diagram to represent equivalent ratios between the distance traveled and elapsed time.	
		Justify (orally and in writing) which of two objects is moving faster, by identifying that it travels more distance in the same amount of time or that it travels the same distance in less time.	
	Lesson 10	Choose and create diagrams to help compare two situations and explain whether they happen at the same rate.	

		Justify that two situations do not happen at the same rate by finding a ratio to describe each situation where the two ratios share one value but not the other, i.e., a:b and a:c, or x:z and y:z	
		Recognize that a question asking whether two situations happen “at the same rate” is asking whether the ratios are equivalent.	

	Lesson 11	Comprehend the words “row” and “column” (in written and spoken language) as they are used to describe a table of equivalent ratios.	
		Explain (orally and in writing) how to find a missing value in a table of equivalent ratios.	
		Interpret a table of equivalent ratios that represents different sized batches of a recipe.	
	Part-Part-Whole Ratios		
	Lesson 12	Choose multipliers strategically while solving multi-step problems involving equivalent ratios.	

		Describe (orally and in writing) how a table of equivalent ratios was used to solve a problem about prices and quantities.	
		Remember that dividing by a whole number is the same as multiplying by an associated unit fraction.	
	Lesson 13	Compare and contrast (orally) double number line diagrams and tables representing the same situation.	
		Draw and label a table of equivalent ratios from scratch to solve problems about constant speed.	

	Lesson 14	Determine what information is needed to solve a problem involving equivalent ratios. Ask questions to elicit that information.	
		Understand the structure of a what-why info gap activity.	
	Lesson 15	Comprehend the word “parts” as an unspecified unit in sentences (written and spoken) describing ratios.	
		Draw and label a tape diagram to solve problems involving ratios and the total amount. Explain (orally) the solution method.	

	Lesson 16	Choose and create diagrams to help solve problems involving ratios and the total amount.	
		Compare and contrast (orally) different representations of and solution methods for the same problem.	
	Let's Put it To Work		
	Lesson 17	Apply reasoning developed throughout this unit to an unfamiliar problem.	
		Decide what information is needed to solve a real-world problem.	
		Make simplifying assumptions about a real-world situation.	

Unit 3: Unit Rates and Percentages			**18 - 19 Days of Instruction -- 4 Weeks**
	Units of Measurement		
	Lesson 1	Evaluate (orally) the usefulness of calculating a rate per 1 when solving problems involving unfamiliar rates.	

		Explain (orally, in writing, and through other representations) how to solve a problem involving rates in a less familiar context, e.g., minutes per window.	
		Compare (orally) the relative size of different units of measure for one attribute, i.e., length, volume, weight or mass.	
		Comprehend the approximate size of 1 “inch,” “foot,” “yard,” “mile,” “millimeter,” “centimeter,” “meter,” “kilometer,” “ounce,” “pound,” “ton,” “gram,” “kilogram,” “cup,” “quart,” “gallon,” “milliliter,” and “liter.”	
		Identify which unit is closest to the length, volume, weight, or mass of a given object, and explain (orally) the reasoning.	
	Unit Conversion		
		Comprehend the approximate size of 1 “inch,” “foot,” “yard,” “mile,” “millimeter,” “centimeter,” “meter,” “kilometer,” “ounce,” “pound,”	

	Lesson 2	“ton,” “gram,” “kilogram,” “cup,” “quart,” “gallon,” “milliliter,” and “liter.”	

		Identify which unit is closest to the length, volume, weight, or mass of a given object, and explain (orally) the reasoning.	
		Compare (orally) the relative size of different units of measure for one attribute, i.e., length, volume, weight or mass.	
	Lesson 3	Generalize (orally and in writing) that it takes more of a smaller unit or fewer of a larger unit to measure the same quantity.	
		Given a measurement in one unit, estimate what would be the same amount expressed in a different unit, and explain (orally) the reasoning.	
	Lesson 4	Choose and create a double number line diagram or table to solve problems involving unit conversion.	
		Explain (orally) how to use a “rate per 1” to solve problems involving unit conversion.	
		Recognize that when we measure things in two different units, the pairs of measurements are equivalent ratios.	
	Rates		
	Lesson 5	Explain (orally and in writing) that if two ratios have the same rate per 1, they are equivalent ratios.	

		Justify (orally and in writing) comparisons of speeds or prices.	
		Recognize that calculating how much for 1 of the same unit is a useful strategy for comparing rates. Express these rates (in spoken and written language) using the word “per” and specifying the unit.	
	Lesson 6	Calculate and interpret the two unit rates associated with a ratio, i.e., ab and ba for the ratio a:b	
		Choose which unit rate to use to solve a given problem and explain the choice (orally and in writing).	
		Comprehend the term “unit rate” (in spoken and written language) refers to a rate per 1.	
	Lesson 7	Apply reasoning about unit rates to complete a table of equivalent ratios, and explain (orally and in writing) the solution method.	
		Explain (orally) that if two ratios are equivalent, they have the same rate per 1.	

		Generalize that the unit rate is the factor that takes you from one column to the other column in a table of equivalent ratios.	

	Lesson 8	Calculate unit rates that represent speed or pace, use them to determine unknown distances or elapsed times, and explain (orally) the solution method.	
		Interpret a verbal (written) description of a situation involving two objects moving at constant speeds, and create a diagram or table to represent the situation.	
	Lesson 9	Apply reasoning about ratios and rates to convert and compare (in writing) distances expressed in different units.	
		Apply reasoning about ratios and rates to justify (orally) whether a given price is a good deal.	
		Practice grade 5 arithmetic with fractions and decimals.	
	Percentages		

	Lesson 10	Comprehend the word “percentage” (in written and spoken language) and the symbol “%” (in written language) to mean a rate per 100.	
		Draw and label a double number line diagram to represent percentages of a dollar and to find corresponding monetary values or percentages.	

	Lesson 11	Comprehend a phrase like “A% of B” (in written and spoken language) to refer to the value that makes a ratio with B that is equivalent to A : 100.	
		Explain (orally) how to use a double number line diagram or table to solve problems such as A% of B is ? and A% of ? is C.	
		State explicitly what one is finding the percentage of.	
	Lesson 12	Choose and create diagrams to solve problems such as A% of B is ? and A% of ? is C.	
		Draw and label a tape diagram to represent a situation involving percentages.	

		Interpret tape diagrams that represent multiplicative comparisons and express such comparisons using fractions and percentages.	
	Lesson 13	Explain (orally and in writing) how to solve problems involving the percentages 10%, 25%, 50%, and 75% by reasoning about the fractions 1/10, ¼, 1/2, and ¾.	
		Generalize (orally) processes for calculating 10%, 25%, 50%, and 75% of a quantity.	

	Lesson 14	Choose and create a tape diagram, double number line diagram, or table to solve problems involving percentages and explain (orally) the solution method.	
		Determine what information is needed to solve a problem involving percentages. Ask questions to elicit that information.	
	Lesson 15	Choose and create diagrams to calculate A% of B, and explain (orally) the solution method.	
		Generalize a process for finding A% of B and justify (orally) why this can be abstracted as A/100 ⋅B	

		Identify equivalent expressions that could be used to find A% of B and justify (orally) that they are equivalent.	
	Lesson 16	Critique or justify (orally) statements about percentages and equivalent numerical expressions.	
		Generalize a process for finding the percentage that C is of B and justify (orally) why this can be abstracted as C/B .100	
	Let's Put it to Work		
	Lesson 17	Apply rates and percentages to calculate how long it will take and how much it will cost to complete a painting project, and explain (orally) the reasoning.	

		Make simplifying assumptions and determine what information is needed to solve a problem about painting a room.	
Unit 4: Dividing Fractions			**20 Days of Instruction -- 4 Weeks**
	Making Sense of Division		

	Lesson 1	Comprehend the terms “dividend” and “divisor” (in spoken language) to refer to the numbers in a division problem.	
		Explain (orally) how to estimate quotients, by comparing the size of the dividend and divisor.	
		Generalize about the size of a quotient, i.e., predicting whether it is a very large number, a very small number, or close to 1.	
	Lesson 2	Identify or generate a multiplication equation that represents the same relationship as a division expression, and explain (orally) the reasoning.	
		Interpret and create tape diagrams that represent situations involving equal-sized groups.	
		Recognize there are two different ways to interpret a division expression, i.e., asking “how many groups?” or “how many in each group?”	

	Lesson 3	Create an equation and a diagram to represent a multiplication or division situation involving fractions, and coordinate these representations (orally).	
		Explain (using words and other representations) how to find the unknown quantity in a multiplication or division situation involving fractions.	
		Interpret a verbal description of a multiplication situation (in spoken or written language), and identify which quantity is unknown, i.e., the number of groups, the amount in one group, or the total amount.	
	Meanings of Fraction Division		
	Lesson 4	Coordinate multiplication equations and pattern block diagrams in which the yellow hexagon represents one whole.	
		Create a diagram to represent and solve a problem asking “How many groups?” in which the divisor is a unit fraction, and explain (orally) the solution method.	
	Lesson 5	Coordinate multiplication and division equations and pattern block diagrams in which the red trapezoid represents one whole.	

		Create a diagram to represent and solve a problem asking “How many groups?” in which the divisor is a non-unit fraction, and explain (orally) the solution method.	

		Identify or generate a multiplication or division equation that represents a given situation involving a fractional divisor.	
	Lesson 6	Explain (orally) how to create a tape diagram to represent and solve a problem asking “How many groups?”	
		Justify (orally and using other representations) the answer to a problem asking “How many groups?” in which the divisor is a non-unit fraction and the quotient is a fraction greater than 1.	
	Lesson 7	Comprehend the phrase “What fraction of a group?” (in spoken and written language) as a variation of the question “How many groups?” that is used when the quotient is less than 1.	

		Create a tape diagram to represent and solve a problem asking “How many groups?” in which the quotient is a fraction less than 1.	
		Write multiplication and division equations to represent a problem asking “How many times as long?”	
	Lesson 8	Compare and contrast (orally) strategies for solving problems about “how many groups?” and “how much in 1 group?”	

		Create a tape diagram to represent and solve a problem asking “How much in 1 group?” where the dividend, divisor, and quotient may be fractions, and explain (orally) the solution method.	
		Write multiplication and division equations to represent a problem asking “How much in 1 group?”	

	Lesson 9	Interpret a situation (presented in written language or using other representations) involving equal-sized groups, and generate mathematical questions that could be asked about it.	
		Solve a problem involving division of fractions, and present the solution method (orally, in writing, and using other representations).	
	Algorithm for Fraction Division		
	Lesson 10	Interpret and critique explanations (in spoken and written language, as well as in other representations) of how to divide by a fraction.	
		Use a tape diagram to represent dividing by a non-unit fraction a/b and explain (orally) why this produces the same result as multiplying the number by b and dividing by a.	
		Use a tape diagram to represent dividing by a unit fraction 1/b and explain (orally and in writing) why this is the same as multiplying by b.	

	Lesson 11	Coordinate (orally) different strategies for dividing by a fraction.	

		Find the quotient of two fractions, and explain (orally, in writing, and using other representations) the solution method.	
		Generalize a process for dividing a number by a fraction, and justify (orally) why this can be abstracted as n.(b/a).	
	Fractions in Lengths, Areas, and Volumes		
	Lesson 12	Apply dividing by fractions to solve a problem about comparing lengths or measuring with non-standard units, and explain (orally and in writing) the solution method.	
		Interpret a question (in written language) about multiplicative comparison, e.g., “How many times as long?” and write a division equation to represent it.	
	Lesson 13	Apply dividing by fractions to calculate the side length of a rectangle, given its area and the other side length.	
		Coordinate (orally) diagrams and equations that represent the area of a rectangle with fractional side lengths.	
		Draw and label a diagram to justify the area of a rectangle with fractional side lengths.	

	Lesson 14	Apply dividing by fractions to calculate the base or height of a triangle, given its area and the other measurement.	
		Determine the volume of a rectangular prism by counting how many 12-inch or 13-inch cubes it takes to build, and explain (orally and in writing) the solution method.	
		Generalize that the volume of a rectangular prism with fractional edge lengths can be found by multiplying the edge lengths.	
	Lesson 15	Apply dividing by fractions to calculate one edge length of a rectangular prism, given its volume and the other two edge lengths.	
		Explain (orally, in writing, and using other representations) how to solve a problem involving the volume of a rectangular prism with fractional edge lengths.	
		Generalize that it takes more smaller cubes or fewer larger cubes to fill the same volume.	
	Let's Put it to Work		

	Lesson 16	Apply operations with fractions to solve problems in a variety of situations, and explain (orally and in writing) the reasoning.	

		Generate an equation to represent a situation involving fractions, and justify (orally) the operation chosen.	
	Lesson 17	Compare and contrast (orally and using other representations) different ways jewelry boxes could be packed inside larger shipping boxes.	
		Determine which size shipping box is least expensive, and present (orally and in writing) a justification.	
		Make simplifying assumptions and determine what information is needed to solve a problem about shipping costs.	
Unit 5: Arithmetic in Base Ten			**16 - 18 Days of Instruction -- 4 Weeks**
	Warming Up to Decimals		

	Lesson 1	Calculate sums and products of decimals in the context of money, and explain (orally and in writing) the calculation strategy	
		Estimate sums, differences, products, and quotients of decimals in the context of money, and explain (orally) the estimation strategy.	
	Adding and Subtracting Decimals		

	Lesson 2 (Optional)	Compare and contrast (orally and in writing) vertical calculations and base-ten diagrams that represent adding and subtracting decimals.	
		Explain (in words and through other representations) that adding and subtracting decimals requires combining digits that represent like baseten units.	

		Interpret and create diagrams that represent 10 like base-ten units being composed into 1 unit of higher place value, e.g., 10 tenths as 1 one, and comprehend the word “bundle” to refer to this concept.	
	Lesson 3	Add or subtract decimals, and explain the reasoning (using words and other representations).	
		Comprehend the term “unbundle” means to decompose a larger baseten unit into 10 units of lower place value (e.g., 1 tenth as 10 hundredths).	
		Recognize and explain (orally) that writing additional zeros or removing zeros after the last non-zero digit in a decimal does not change its value.	
	Lesson 4	Add or subtract decimals with multiple non-zero digits, and explain (orally) the solution method.	

		Interpret a description (in written language) of a real-world situation involving decimals, and write an addition or subtraction problem to represent it.	
		Recognize and explain (orally) that vertical calculation is an efficient strategy for adding and subtracting decimals, especially decimals with multiple non-zero digits.	
	Multiplying Decimals		
	Lesson 5	Generalize (orally and in writing) that the number of decimal places in a product is related to the number of decimal places in the factors.	
		Justify (orally) the product of two decimals, which each have only one non-zero digit, by multiplying equivalent fractions that have a power of ten in the denominator.	
	Lesson 6	Interpret different methods for computing the product of decimals, and evaluate (orally) their usefulness.	
		Justify (orally, in writing, and through other representations) where to place the decimal point in the product of two decimals with multiple non-zero digits.	

	Lesson 7	Comprehend how the phrase “partial products” (in spoken and written language) refers to decomposing a multiplication problem.	

		Coordinate area diagrams and vertical calculations that represent the same decimal multiplication problem.	
		Use an area diagram to represent and justify (orally and in writing) how to find the product of two decimals.	
	Lesson 8	Draw and label a diagram to check the answer to a decimal multiplication problem.	
		Interpret a description (in written language) of a real-world situation involving multiplication of decimals, and write a multiplication problem to represent it.	
		Use an algorithm to calculate the product of two decimals, and explain (orally) the solution method.	
	Dividing Decimals		
	Lesson 9	Comprehend that the phrase “partial quotients” (in spoken and written language) refers to decomposing a division problem.	

		Divide whole numbers that result in a whole-number quotient, and explain the reasoning (using words and other representations).	

		Interpret different methods for computing the quotient of whole numbers, i.e., base-ten diagrams and partial quotients, and evaluate (orally) their usefulness.	
	Lesson 10	Interpret the long division method, and compare and contrast it (orally) with other methods for computing the quotient of whole numbers.	
		Recognize and explain (orally) that long division is an efficient strategy for dividing numbers, especially with multi-digit dividends.	
		Use long division to divide whole numbers that result in a wholenumber quotient, and multiply the quotient by the divisor to check the answer.	
	Lesson 11	Interpret different methods for computing a quotient that is not a whole number, and express it (orally and in writing) in terms of “unbundling.”	

		Use long division to divide whole numbers that result in a quotient with a decimal, and explain (orally) the solution method.	
	Lesson 12	Compare and contrast (orally and using other representations) division problems with whole-number and decimal dividends	
		Divide decimals by whole numbers, and explain the reasoning (orally and using other representations).	

		Generalize (orally and in writing) that multiplying both the dividend and the divisor by the same factor does not change the quotient.	
	Lesson 13	Compare and contrast (orally and using other representations) division problems with whole-number and decimal divisors.	
		Divide whole numbers or decimals by decimals, and explain the reasoning (orally and using other representations), including choosing to divide a different expression that gets the same quotient.	
		Generate another division expression that has the same value as a given expression, and justify (orally) that they are equal.	

	Let's Put it to Work		
	Lesson 14	Apply operations with decimals to solve problems about the dimensions of a sports field or court, and explain (orally, in writing, and using other representations) the solution method.	
		Choose whether an exact answer or an estimate is appropriate for a given problem.	
		Interpret a verbal description or diagram that represents the dimensions of a sports field or court.	

	Lesson 15 (Optional)	Apply operations with decimals to calculate the surface area of paper boxes.	
		Describe (orally) sources of measurement error, and justify an appropriate level of precision for reporting the answer.	
		Measure and compare (orally and in writing) the dimensions of paper boxes.	

Unit 6: Expressions and Equations			20 - 22 Days of Instruction -- 4 Weeks
	Equations in One Variable		
	Lesson 1	Draw tape diagrams to represent equations of the forms x+p=q and px=q.	
		Interpret (orally and in writing) tape diagrams that represent equations of the form p+x=q or px=q.	
		Use tape diagrams to find unknown values in equations of the forms x+p=q and px=q and explain (orally) the solution method.	
	Lesson 2	Comprehend the word “variable” to refer to a letter standing in for a number and recognize that a coefficient next to a variable indicates multiplication (in spoken and written language).	

		Generate values that make an equation true or false and justify (orally and in writing) whether they are “solutions” to the equation.	

		Use substitution to determine whether a given number makes an equation true.	
	Lesson 3	Interpret hanger diagrams (orally and in writing) and write equations that represent relationships between the weights on a balanced hanger diagram.	
		Use balanced hangers to explain (orally and in writing) how to find solutions to equations of the form x+p=q or px=q.	
	Lesson 4	Interpret and coordinate sentences, equations, and diagrams that represent the same addition or multiplication situation.	
		Solve equations of the form x+p=q or px=q and explain (in writing) the solution method.	
	Lesson 5	Comprehend that the notation ab can be used to represent division generally, and the numerator and denominator can include fractions, decimals, or variables.	
		Describe (orally) a situation that could be represented by a given equation of the form x+p=q or px=q.	

		Express division as a fraction (in writing) when solving equations of the form px=q.	
	Equal and Equivalent		
	Lesson 6	Explain (orally) how to create and solve an equation that represents a situation with an unknown amount.	
		Write an expression with a variable to generalize the relationship between quantities in a situation.	
	Lesson 7	State explicitly what the chosen variable represents when creating an equation.	
		Use equations to solve problems involving percentages and explain (orally) the solution method.	
		Write equations of the form px=q or equivalent to represent situations where the amount that corresponds to 100% is unknown.	
	Lesson 8	Draw a diagram to represent the value of an expression for a given value of its variable.	

		Explain (in writing) that some pairs of expressions are equal for one value of their variable but not for other values.	

		Justify (orally, in writing, and through other representations) whether two expressions are “equivalent”, i.e., equal to each other for every value of their variable.	
	Lesson 9	Generate equivalent numerical expressions that are related by the distributive property, and explain (orally or using other representations) the reasoning.	
		Use an area diagram to make sense of equivalent numerical expressions that are related by the distributive property.	
	Lesson 10	Generate algebraic expressions that represent the area of a rectangle with an unknown length.	
		Justify (orally and using other representations) that algebraic expressions that are related by the distributive property are equivalent.	

	Lesson 11 (Optional)	Draw a diagram to justify that two expressions that are related by the distributive property are equivalent.	
		Explain (orally) how to use the distributive property to identify or generate equivalent algebraic expressions.	
		Use the distributive property to write equivalent algebraic expressions, including where the common factor is a variable.	

	Expressions with Exponents		
	Lesson 12	Describe (orally and in writing) a pattern that could be expressed using repeated multiplication.	
		Generate and evaluate numerical expressions involving whole-number exponents.	
		Interpret expressions with exponents larger than 3, and comprehend the phrase “to the power” or “to the” (in spoken language).	

	Lesson 13	Critique (orally and in writing) arguments that claim two different numerical expressions are equal.	
		Justify (orally and in writing) whether numerical expressions involving whole-number exponents are equal.	
	Lesson 14	Evaluate numerical expressions that have an exponent and one other operation, and justify (orally) the process.	
		Explain (orally and in writing) that the convention is to evaluate the exponent before the other operations in an expression with no grouping symbols.	
		Interpret expressions with exponents that represent the surface area or volume of a cube.	

	Lesson 15	Describe (orally) the values that result from evaluating expressions in which a fraction is raised to a power.	

	Determine whether a given value is a solution to an equation that includes an exponent.	
	Evaluate expressions that have a variable, an exponent, and one other operation for a given value of the variable, carrying out the operations in the conventional order.	
Relationships Between Quantities		
Lesson 16	Compare and contrast (orally) graphs and equations that represent a relationship between the same quantities but have the independent and dependent variables switched.	
	Comprehend the terms “independent variable” and “dependent variable” (in spoken and written language).	
	Create a table, graph, and equation to represent the relationship between quantities in a set of equivalent ratios.	

	Lesson 17	Create a table, graph, and equation to represent the relationship between distance and time for an object moving at a constant speed.	

		Identify (in writing) the independent and dependent variable in an equation.	
		Interpret (orally and in writing) an equation that represents the relationship between distance and time for an object moving at a constant speed.	
	Lesson 18 (Optional)	Coordinate (orally and in writing) graphs, tables, and equations that represent the same relationship.	
		Create an equation and a graph to represent the relationship between two variables that are inversely proportional.	
		Describe and interpret (orally and in writing) a graph that represents a nonlinear relationship between independent and dependent variables.	
	Let's Put it to Work		

	Lesson 19	Create a verbal description and a graph to represent the relationship shown in an equation and table.	
		Identify tables and equations that represent the same relationship and justify (orally) the match.	
		Interpret and critique (orally) different representations of the same relationship, i.e. table, equation, graph, and verbal description	

Unit 7: Rational Numbers			**21 Days of Instruction -- 4 Weeks**
	Negative Numbers and Absolute Value		
	Lesson 1	Comprehend the words “positive” and “negative” (in spoken and written language) and the symbol “-” (in written language). Say “negative” when reading numbers written with the “-” symbol.	
		Interpret positive and negative numbers that represent temperature or elevation, and understand the convention of what “below zero” typically means in each of these contexts.	

		Recognize that the number line can be extended to represent negative numbers.	
	Lesson 2	Comprehend that two numbers are called “opposites” when they are the same distance from zero, but on different sides of the number line.	
		Interpret a point on the number line that represents a positive or negative rational number.	
		Plot a point on a number line to represent a positive or negative rational number.	
	Lesson 3	Compare rational numbers in the context of temperature or elevation, and express the comparisons (in writing) using the symbols > and <.	

		Comprehend the word “sign” (in spoken language) to refer to whether a number is positive or negative.	

		Critique (orally and in writing) statements comparing rational numbers, including claims about relative position and claims about distance from zero.	
	Lesson 4	Compare rational numbers without a context and express the comparisons using the terms “greater than,” “less than,” and “opposite” (orally and in writing).	
		Comprehend that all negative numbers are less than all positive numbers.	
		Order rational numbers from least to greatest, and explain (orally and through other representations) the reasoning.	
	Lesson 5	Interpret a table of signed numbers that represent how a quantity changed.	
		Recognize that signed numbers can be useful to represent changes in a quantity in opposite directions, e.g., money received and money paid, inventory bought and inventory sold, etc.	
	Lesson 6	Compare rational numbers and their absolute values, and explain (orally and in writing) the reasoning.	

		Comprehend the phrase “absolute value” and the symbol || to refer to a number’s distance from zero on the number line.	
		Interpret rational numbers and their absolute values in the context of elevation or temperature.	
	Lesson 7	Critique comparisons (expressed using words or symbols) of rational numbers and their absolute values.	
		Generate values that meet given conditions for their relative position and absolute value, and justify the comparisons (using words and symbols).	
		Recognize that the value of -a	
		-a can be positive or negative, depending on the value of a	

	Inequalities		
	Lesson 8	Coordinate verbal, algebraic, and number line representations of inequalities.	
		Critique (orally and in writing) possible values given for a situation with a constraint, including determining whether the boundary value is included and making sense of situations with discrete quantities.	

		Interpret phrases that describe a quantity constrained by a maximum or minimum acceptable value, e.g. “at least,” “at most,” “up to,” “more than,” “less than”, etc., and write an inequality statement to represent the constraint.	
	Lesson 9	Draw and label a number line diagram to represent the solutions to an inequality.	
		Recognize and explain (orally and in writing) that an inequality may have infinitely many solutions.	
		Use substitution to justify (orally) whether a given value is a “solution” to a given inequality.	

	Lesson 10	Critique (orally and in writing) possible values given for a situation with more than one constraint, including whether fractional or negative values are reasonable.	
		Interpret unbalanced hanger diagrams (orally and in writing) and write inequality statements to represent relationships between the weights on an unbalanced hanger diagram.	
		Write and interpret inequality statements that include more than one variable.	
	The Coordinate Plane		

	Lesson 11	Generalize about the signs of coordinates that represent locations in each “quadrant” of the coordinate plane.	
		Plot a point given its coordinates or identify the coordinates of a given point on the coordinate plane.	

		Recognize that the axes of the coordinate plane can be extended to represent negative numbers.	
	Lesson 12	Choose and label appropriate scales for the axes of the coordinate plane, based on the coordinates to be plotted, and explain (orally and in writing) the choice.	
		Compare and contrast different scales for the axes of the coordinate plane.	
	Lesson 13	Compare points on a graph, including statements about relative position and the vertical distance between points.	
		Describe (using words and inequality symbols) and interpret the range of coordinates on a graph, including the meaning of y-values that are negative.	

		Identify and interpret points on a graph to answer questions about situations involving temperature or money.	

	Lesson 14	Compare and contrast (orally and in writing) the coordinates for points in different locations on the coordinate plane.	
		Determine the vertical or horizontal distance between two points on the coordinate plane that share the same x- or y-coordinate.	
		Generalize (orally) about the coordinates of points that are reflected across the x- or y-axis.	
	Lesson 15	Determine the total length of multiple horizontal and vertical segments in the coordinate plane that are connected end-to-end.	
		Draw a polygon in the coordinate plane given the coordinates for its vertices.	

		Explain (orally) that coordinates can be a useful way of describing geometric figures or modeling real-world locations.	
	Common Factors and Common Multiples		
	Lesson 16	Comprehend (orally and in writing) the terms “factor,” “common factor,” and “greatest common factor.”	
		Explain (orally and in writing) how to determine the greatest common factor of two whole numbers less than 100.	

		List the factors of a number and identify common factors for two numbers in a real-world situation.	
	Lesson 17	Comprehend (orally and in writing) the terms “multiple,” “common multiple,” and “least common multiple.”	
		Explain (orally and in writing) how to calculate the least common multiple of 2 whole numbers.	

		List the multiples of a number and identify common multiples for two numbers in a real-world situation.	
	Lesson 18	Choose to calculate the greatest common factor or least common multiple to solve a problem about a real-world situation, and justify (orally) the choice.	
		Present (orally, in writing, and using other representations) the solution method for a problem involving greatest common factor or least common multiple.	
	Lesson 19	Generate a list of ordered pairs to create an image in the coordinate plane, and explain (orally) the reasoning.	
Unit 8: Data Sets and Distributions			**21 Days of Instruction -- 4 Weeks**
	Data, Variability, and Statistical Questions		
	Lesson 1	Ask survey questions (orally) and record responses (in writing). Include units of measurement when reporting numerical data (orally and in writing).	
		Comprehend and use the terms “numerical” and “categorical” to describe data sets (orally and in writing).	

		Interpret various representations of data sets and determine whether it is reasonable that a verbal description represents a given numerical data set.	
	Lesson 2	Justify (orally) whether a question is “statistical” based on whether variability is expected in the data that could be collected.	
		Match survey questions to data sets representing possible responses and justify (in writing) why they match.	
	Dot Plots and Histograms		
	Lesson 3	Comprehend the word “frequency” to refer to the number of times a particular value occurs in a data set.	
		Create and interpret a dot plot to answer statistical questions about a numerical data set.	

		Justify (in writing) whether a dot plot is an appropriate way to display a given data set, paying attention to whether the data set is numerical or categorical.	

Lesson 4	Describe (orally and in writing) a distribution represented by a dot plot, including informal observations about its center and spread.
	Interpret a dot plot to answer (in writing) statistical questions about a data set and to identify (orally) what values are “typical” for the distribution.
Lesson 5	Compare and contrast (orally and in writing) dot plots that represent two different data sets measuring the same quantity, paying attention to the “center” and “spread” of each distribution.
	Critique or justify (orally and in writing) claims about the center of a distribution represented on a dot plot.
Lesson 6	Compare and contrast (orally) dot plots and histograms in terms of how useful they are for answering different statistical questions.
	Create a histogram to represent a data set.

		Interpret a histogram to answer (in writing) statistical questions about a data set.	

	Lesson 7	Compare and contrast (in writing) histograms that represent two different data sets measuring the same quantity.	
		Critique (orally) a description of a distribution, recognizing that there are multiple valid ways to describe its center and spread.	
		Describe (orally and in writing) the distribution shown on a histogram, including making claims about the center and spread.	
	Lesson 8	Compare and contrast (orally) bar graphs and histograms, recognizing that descriptions of shape, center, and spread don't pertain to bar graphs.	
		Describe (orally and in writing) the overall shape and features of a distribution represented on a histogram, including peaks, clusters, gaps, and symmetry.	
		Identify histograms that display distributions with specific features.	

	Mean and MAD		
	Lesson 9	Comprehend the words “mean” and “average” as a measure of center that summarizes the data using a single number.	
		Explain (using words and other representations) how to calculate the mean for a numerical data set.	

		Interpret diagrams that represent finding the mean as a process of leveling out the data to find a “fair share.”	
	Lesson 10	Calculate and interpret (orally and in writing) distances between data points and the mean of the data set.	
		Interpret diagrams that represent the mean as a “balance point” for both symmetrical and non-symmetrical distributions.	

		Represent the mean of a data set on a dot plot and interpret it in the context of the situation.	
	Lesson 11	Calculate the mean absolute deviation (MAD) for a data set and interpret what it tells us about the situation.	
		Compare and contrast (in writing) distributions that have the same mean, but different amounts of variability.	
		Comprehend that “mean absolute deviation (MAD)” is a measure of variability, i.e., a single number summarizing how spread out the data set is.	
	Lesson 12	Compare (orally and in writing) the means and mean absolute deviations of different distributions, specifically those with the same MAD but different means.	

		Interpret the mean and mean absolute deviation (MAD) in the context of the data.	
	Median and IQR		
	Lesson 13	Comprehend that the “median” is another measure of center, which uses the middle of all the values in an ordered list to summarize the data.	
		Identify and interpret the median of a data set given in a table or on a dot plot.	
		Informally estimate the center of a data set and then compare (orally and in writing) the mean and median with this estimate.	
	Lesson 14	Choose which measure of center to use to describe a given data set and justify (orally and in writing) the choice.	

		Explain (orally) that the median is a better estimate of a typical value than the mean for distributions that are not symmetric or contain values far from the center.	
		Generalize how the shape of the distribution affects the mean and median of a data set.	

	Lesson 15	Calculate the range and interquartile range (IQR) of a data set and interpret (orally and in writing) what they tell us about the situation.	
		Comprehend that “interquartile range (IQR)” is another measure of variability that describes the span of the middle half of the data.	
		Identify and interpret (in writing) the numbers in the five-number summary for a data set, i.e., the minimum, first quartile (Q1), median (Q2), third quartile (Q3), and maximum.	
	Lesson 16	Compare and contrast (orally) a dot plot and a box plot that represent the same data set.	
		Create a box plot to represent a data set.	

		Describe (orally) the parts of a box plot that correspond with each number in the five-number summary, the range, and the IQR of a data set.	
	Lesson 17	Compare and contrast (orally and in writing) box plots that represent different data sets, including ones with the same median but very different IQRs and vice versa.	
		Determine what information is needed to solve problems about comparing box plots. Ask questions to elicit that information.	

		Interpret a box plot to answer (orally) statistical questions about a data set.	
	Let's Put it to Work		
	Lesson 18	Recognize that different graphical displays offer different insights into a distribution. Choose an appropriate graphical display to represent a data set, and justify the choice (orally and in writing).	

		Recognize that different measures of center and variability offer different insights into a data set. Choose an appropriate measure of center and variability to describe a data set, and justify the choice (orally and in writing).	
Unit 9: Putting It All Together (Optional Unit)			**0 - 18 Days of Instruction -- 4 Weeks**
	Making Connections		
	Lesson 1 - Optional	Estimate quantities in a real-world situation and explain (orally and in writing) the estimation strategy.	
		Justify (orally) why it is unreasonable to have an exact answer for a situation that involves estimation, and critique (orally) different estimates.	

		Make simplifying assumptions and determine what information is needed to solve a Fermi problem about distance, volume, or surface area.	

	Lesson 2 - Optional	Apply reasoning about percentages and equivalent ratios to analyze and approximate characteristics of the world's population.	
		Generate (orally and in writing) mathematical questions about the world's population, e.g., "How many people . . . ?"	
		Present (using words and other representations) a comparison that uses the number of students in the class to represent the proportion of the world's population with a particular characteristic.	
	Lesson 3 - Optional	Coordinate diagrams and expressions involving equivalent fractions.	
		Interpret and create diagrams involving a rectangle decomposed into squares.	
		Recognize that decomposing rectangles into squares is a geometric way to determine the greatest common factor of two numbers.	
	Voting		

	Lesson 4 - Optional	Apply reasoning about ratios and percentages to analyze (orally and in writing) voting situations involving two choices.	

		Comprehend the terms “majority” and “supermajority” (in spoken and written language).	
		Critique (using words and other representations) a statement reporting the results of a vote.	
	Lesson 5 - Optional	Apply reasoning about ratios and percentages to analyze (orally and in writing) voting situations involving more than two choices.	
		Choose and justify (orally) which voting system seems the fairest for dealing with more than two choices.	
		Compare and contrast (orally and in writing) different voting systems for dealing with more than two choices, i.e., plurality, runoff, and instant runoff.	
	Lesson 6 - Optional	Critique (orally and in writing) whether a method for distributing representatives is fair.	
		Compare and contrast different ways to distribute representatives, and recognize that changing the way the votes are grouped can affect the outcome.	
		Suggest a method for distributing representatives and justify (orally) why is it fair.	

Supports of Diversity, Equity and Inclusion

Please provide any information relative to supporting culturally responsive instruction, multi-language learners, and students with disabilities

Review Site Information:

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Culturally Responsive Instruction:

Illustrative Mathematics includes culturally relevant materials and culturally responsive teaching and instructional practices. Materials are inclusive of a variety of cultures and ethnicities and are free from bias in the portrayal of ethnic groups, gender, age, class, cultures, religions, and people with disabilities.

We address racial, cultural, and religious bias in the following ways:

- The materials contain racial/ethnic balance in the main characters and illustrations.
- Minorities are represented as central figures in text and illustrations.
- Minority figures reflect qualities such as leadership, intelligence, imagination, and courage.
- The materials provide an opportunity for a variety of racial, ethnic, and cultural perspectives.
- The vocabulary or depiction of racism is avoided (i.e., insulting overtones).
- Race/culture stereotyping language is avoided.
- Biographical or historical content includes minority figures and their discoveries and contributions to society.

Multi-Language Learners:

In a problem-based mathematics classroom, sense-making and language are interwoven. Mathematics classrooms are language-rich, and therefore language demanding learning environments for every student. The linguistic demands of doing mathematics include

reading, writing, speaking, listening, conversing, and representing (Aguirre & Bunch, 2012). Students are expected to say or write mathematical explanations, state assumptions, make conjectures, construct mathematical arguments, and listen to and respond to the ideas of others. In an effort to advance the mathematics and language learning of all students, the materials purposefully engage students in sense-making and using language to negotiate meaning with their peers. To support students who are learning English in their development of language, this curriculum includes instruction devoted to fostering language development alongside mathematics learning, fostering language-rich environments where there is space for all students to participate.
This interwoven approach is grounded in four design principles that promote mathematical language use and development:

Principle 1. Support sense-making: Scaffold tasks and amplify language so students can make their own meaning. Students need multiple opportunities to talk about their mathematical thinking, negotiate meaning with others, and collaboratively solve problems with targeted guidance from the teacher. Teachers can make language more accessible by amplifying rather than simplifying speech or text. Simplifying includes avoiding the use of challenging words or phrases. Amplifying means anticipating where students might need support in understanding concepts or mathematical terms and providing multiple ways to access them.

Principle 2. Optimize output: Strengthen opportunities for students to describe their mathematical thinking to others, orally, visually, and in writing. All students benefit from repeated, strategically optimized, and supported opportunities to articulate mathematical ideas into linguistic expression, to communicate their ideas to others. Opportunities for students to produce output should be strategically optimized for both (a) important concepts of the unit or course, and (b) important disciplinary language functions (for example, explaining reasoning, critiquing the reasoning of others, making generalizations, and comparing approaches and representations).

Principle 3. Cultivate conversation: Strengthen opportunities for constructive mathematical conversations. Conversations are backand-forth interactions with multiple turns that build up ideas about math. Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning, communicate that meaning, and refine the way content understandings are communicated. During effective discussions, students pose and answer questions, clarify what is being asked and what is happening in a problem, build common understandings, and share experiences relevant to the topic. Meaningful conversations depend on the teacher using activities and routines as opportunities to build a classroom culture that motivates and values efforts to communicate.

Principle 4. Maximize meta-awareness: Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language. Meta-awareness, consciously thinking about one's own thought processes or language use, develops when students

consider how to improve their communication and reasoning about mathematical concepts. When students are using language in ways that are purposeful and meaningful for themselves, in their efforts to understand—and be understood by—each other, they are motivated to attend to ways in which language can be both clarified and clarifying. Students learning English benefit from being aware of how language choices are related to the purpose of the task and the intended audience, especially if oral or written work is required. Both metacognitive and metalinguistic awareness are powerful tools to help students self-regulate their academic learning and language acquisition.

These design principles and related mathematical language routines, described below, ensure language development is an integral part of planning and delivering instruction. Moreover, they work together to guide teachers to amplify the most important language that students are expected to know and use in each unit.

Mathematical Language Routines

Mathematical Language Routines (MLRs) are instructional routines that provide structured but adaptable formats for amplifying, assessing, and developing students' language. The MLRs included in this curriculum were selected because they simultaneously support students' learning of mathematical practices, content, and language. They are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English. These routines are flexible and can be adapted to support students at all stages of language development in using and improving their English and disciplinary language use.

These routines are included in the Curriculum Guide and noted below:

- MLR 1: Stronger and Clearer Each Time
- MLR 2: Collect and Display
- MLR 3: Clarify, Critique, Correct
- MLR 4: Information Gap
- MLR 5: Co-Craft Questions
- MLR 6: Three Reads
- MLR 7: Compare and Connect

- MLR 8: Discussion Supports

MLRs are included in select activities in each unit to provide all students with explicit opportunities to develop mathematical and academic language proficiency. These "embedded" MLRs are described in the teacher notes for the lessons in which they appear.

Each lesson also includes optional, suggested MLRs that can be used to support access and language development for English learners, based on the language demands students will encounter. They are described in the activity narrative, under the heading "Access for English Learners." Teachers can use the suggested MLRs and language strategies as appropriate to provide students with access to an activity without reducing the mathematical demand of the task. When using these supports, teachers should take into account the language demands of the specific activity and the language needed to engage the content more broadly, in relation to their students' current ways of using language to communicate ideas as well as their students' English language proficiency. Using these supports can help maintain student engagement in mathematical discourse and ensure that struggle remains productive. All of the supports are designed to be used as needed, and use should fade out as students develop understanding and fluency with the English language.

In addition to the comprehensive pedagogical design of the program, Spanish translations are available for the educator components, including teacher slides, and the student components, including the student workbook (print version).

Materials are also available in Spanish as follows:

What's in Spanish for IM?

K-5	6-8	AGA
• Printed: Student Workbooks • eBook/PDF: Student, Teacher, Teacher Resource Pack • Spanish Lesson Cards **Other Materials** *(no solutions translated)* • Task Statements (PDF) • Cool-Down (PDF) • Practice Problems (PDF) • Unit Assessments (PDF) • Section Checkpoint Quizzes (PDF) • Family Supports (PDF) • Center Materials (PDF) • Glossary entries	**6-8 Courses Only (Not Acc.)** • Printed: Student Workbooks • eBook/PDF: Student **Other Materials** *(no solutions translated)* • Task Statements (PDF) • Cool-Down (PDF) • Practice Problems (PDF) • Unit Assessments Option B, (PDF) • Glossary entries	**Algebra 1 Only** eBook/PDF: Student Workbook *Print coming for BTS 2023 **Other Materials** *(no solutions translated)* • Task Statements (PDF) • Cool-Down (PDF) • Practice Problems (PDF) • Unit Assessments (PDF) • Modeling prompts • Glossary entries

Exceptional Learners:

Imagine Learning Illustrative Mathematics materials empower all students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students, positioning all learners as competent, valued contributors. When planning to support access, teachers should consider the strengths and needs of their particular students.

Each lesson is carefully designed to maximize engagement and accessibility for all students. Purposeful design elements that support access for all learners, but that are especially helpful for students with disabilities, include:

Lesson Structures are Consistent

The structure of every lesson is the same: warm-up, activities, synthesis, cool-down. By keeping the components of each lesson similar from day to day, the flow of work in class becomes predictable for students. This reduces cognitive demand and enables students to focus on the mathematics at hand rather than the mechanics of the lesson.

Concepts Develop from Concrete to Abstract
Mathematical concepts are introduced simply, concretely, and repeatedly, with complexity and abstraction developing over time. Students begin with concrete examples, and transition to diagrams and tables before relying exclusively on symbols to represent the mathematics they encounter.

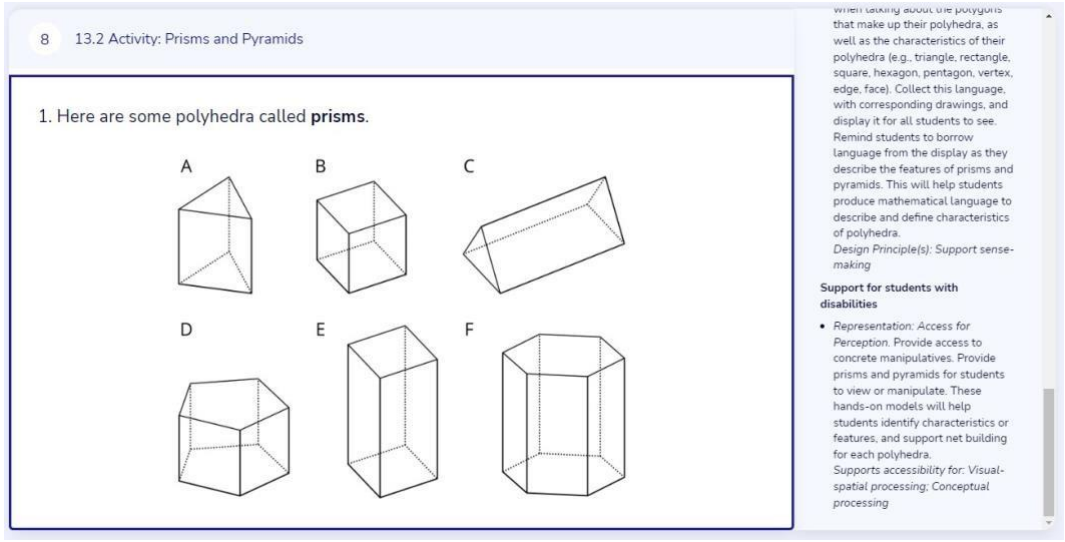
Individual to Pair, or Small Group to Whole Class Progression
Providing students with time to think through a situation or question independently before engaging with others allows students to carry the weight of learning, with support arriving just in time from the community of learners. This progression allows students to first activate what they already know, and continue to build from this base with others.

Opportunities to Apply Mathematics to Real-World Contexts
Giving students opportunities to apply the mathematics they learn clarifies and deepens their understanding of core math concepts and skills and provides motivation and support. Mathematical modeling is a powerful activity for all students, but especially students with disabilities. Each unit has a culminating activity designed to explore, integrate, and apply all the big ideas of the unit. Centering instruction on these contextual situations can provide students with disabilities an anchor on which to base their mathematical understandings.

Supplemental instructional strategies that can be used to increase access, reduce barriers and maximize learning are included in each lesson, listed in the activity narratives under "*Access for Students with Disabilities.*" Each support is aligned to the Universal Design for Learning Guidelines and based on one of the three principles of UDL, to provide alternative means of *engagement, representation,* or *action and expression*. These supports provide teachers with additional ways to adjust the learning environment so that students can access activities, engage in content, and communicate their understanding. Supports are tagged with the areas of cognitive functioning they are designed to address to help teachers identify and select appropriate supports for their students. Designed to facilitate access to Tier 1 instruction by capitalizing on student strengths to address challenges related to cognitive functions or disabilities, these strategies and supports are appropriate for any students who need additional support to access rigorous, grade-level content.

Teachers are encouraged to use what they know about their students' IEPs, strengths and challenges, and a UDL approach to ensure access.

There are embedded supports for exceptional students in most lessons. Teachers will find these in the **Teaching Notes** section. As of June 2020, Illustrative Mathematics 6-8 student facing materials meet Section 508 compliance standards, meaning that students can use assistive technology to navigate the site. Illustrative Mathematics K-5 digital materials were added during the 21-22 School Year and are 508 compliant as well. Outlined in the Curriculum Guide, there are features, supports, and strategies available.



The curriculum authors drew heavily on the UDL framework in the design of these materials. A number one design principle of the curriculum is "Access for all." This foundational principle draws from the UDL framework and shapes the instructional goals, recommended practices, lesson plans, and assessments to support a flexible approach to instruction, ensuring all students have an equitable opportunity to learn.

Imagine Learning software is browser-based so it will work with any browser-based text-to-speech tools. Fonts can be adjusted in type and size. Non-text navigation elements can be adjusted in size. Math equation editing is available on assessment items and practice problems.

Imagine Learning can provide a NIMAS-compatible version of Illustrative Mathematics content. These files may be used for the production of alternate formats as permitted under the law for students with disabilities.