## Eureka Math ${ }^{2}$ Level 7 Correlation to Connecticut Model Curriculum

## Level 7: Ratios and Proportionality

| Model Unit Name | Model Unit Standards | Lessons | Pacing <br> Lessons that address concepts in more than one unit are only counted once |
| :---: | :---: | :---: | :---: |
| Operating with Rational Numbers (Addition \& Subtraction) | 7.NS.A. 1 | Module 2: Operations with Rational Numbers <br> Topic A: Adding Rational Numbers <br> Lesson 1: Combining Opposites <br> Lesson 2: Adding Integers <br> Lesson 3: Adding Integers Efficiently <br> Lesson 4: KAKOOMA® <br> Lesson 5: Decomposing Rational Numbers to Make Addition More Efficient <br> Lesson 6: Adding Rational Numbers <br> Module 2: Operations with Rational Numbers <br> Topic B: Subtracting Rational Numbers <br> Lesson 7: What Subtraction Means <br> Lesson 8: Subtracting Integers, Part 1 <br> Lesson 9: Subtracting Integers, Part 2 <br> Lesson 10: Subtracting Rational Numbers, Part 1 <br> Lesson 11: Subtracting Rational Numbers, Part 2 <br> Lesson 12: The Integer Game <br> Module 2: Operations with Rational Numbers <br> Topic E: Numerical Expressions with Rational Numbers <br> Lesson 25: Writing and Evaluating Expressions with Rational Numbers, Part 1 <br> Lesson 26: Writing and Evaluating Expressions with Rational Numbers, Part 2 | 14 days |



| Operating with Rational Numbers (Multiplication \& Division) (cont.) | 7.EE.B. 3 | Lesson 5: Factoring Expressions <br> Lesson 6: Comparing Expressions <br> Module 3: Expressions, Equations, and Inequalities <br> Topic B: Unknown Angle Measurements Lesson 9: Solving Equations to Determine Unknown Angle Measures <br> Module 3: Expressions, Equations, and Inequalities <br> Topic B: Unknown Angle Measurements Lesson 9: Solving Equations to Determine Unknown Angle Measures Lesson 10: Solving with Unknown Angle Measures <br> Module 3: Expressions, Equations, and Inequalities <br> Topic C: Solving Equations <br> Lesson 11: Dominoes and Dominoes <br> Lesson 16: Using Equations to Solve Rate Problems <br> Lesson 17: Using Equations to Solve Problems |  |
| :---: | :---: | :---: | :---: |
| Two and Three Dimensional Geometry | 7.G.A. 2 | Module 4: Geometry <br> Topic A: Constructing Geometric Figures <br> Lesson 1: Sketching, Drawing, and Constructing Geometric Figures <br> Lesson 2: Constructing Parallelograms and Other Quadrilaterals <br> Lesson 3: Side Lengths of a Triangle <br> Lesson 4: Angles of a Triangle <br> Lesson 5: Constructing Quadrilaterals and Triangles <br> Module 4: Geometry <br> Topic B: Constructing Triangles <br> Lesson 6: Unique Triangles <br> Lesson 7: Two Angles and One Side <br> Lesson 8: Two Sides and One Angle <br> Module 4: Geometry <br> Topic C: Circumference and Area of Circles <br> Lesson 9: Constructing a Circle | 27 days |


| Two and Three Dimensional Geometry (cont.) | 7.G.A. 3 | Module 4: Geometry <br> Topic E: Cross Sections and Volume <br> Lesson 22: Understanding Planes and Cross Sections <br> Lesson 23: Cross Section Scavenger Hunt <br> Module 4: Geometry <br> Topic C: Circumference and Area of Circles <br> Lesson 10: The Outside of a Circle <br> Lesson 11: The Inside of a Circle <br> Lesson 12: Exploring the Area and Circumference of a Circle <br> Lesson 13: Finding Areas of Circular Regions <br> Lesson 14: Composite Figures with Circular Regions <br> Lesson 15: Watering a Lawn <br> Module 3: Expressions, Equations, and Inequalities <br> Topic B: Unknown Angle Measurements <br> Lesson 7: Angle Relationships and Unknown Angle Measures <br> Lesson 8: Strategies to Determine Unknown Angle Measures <br> Lesson 10: Problem Solving with Unknown Angle Measures <br> Module 4: Geometry <br> Topic D: Area and Surface Area <br> Lesson 16: Solving Area Problems by Composition and Decomposition <br> Lesson 17: Surface Area of Right Rectangular and Right Triangular Prisms <br> Lesson 18: Surface Area of Right Prisms <br> Lesson 20: Surface Areas of Right Pyramids <br> Lesson 21: Surface Area of Other Solids <br> Module 4: Geometry <br> Topic E: Cross Sections and Volume <br> Lesson 24: Volume of Prisms <br> Lesson 25: Volume of Composite Solids <br> Lesson 26: Designing a Fish Tank |  |  |
| :---: | :---: | :---: | :---: | :---: |
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| Proportional Reasoning (cont.) |  | Lesson 11: Constant Rates <br> Lesson 12: Multi-Step Ratio Problems, Part 1 <br> Lesson 13: Multi-Step Ratio Problems, Part 2 <br> Module 5: Percent and Applications of Percent <br> Topic A: Proportions and Percent <br> Lesson 2: Racing for Percents <br> Lesson 3: Percent as a Rate per 100 <br> Lesson 4: Proportion and Percent <br> Lesson 5: Common Denominators or Common Numerators <br> Topic B: Common Denominators or Common Numerators <br> Lesson 6: Finding Commission <br> Lesson 7: Finding Discounts <br> Lesson 8: Determining Fees <br> Lesson 9: Tax as a Fee <br> Topic C: More or Less Than 100\% <br> Lesson 10: Percent Increase <br> Lesson 11: Percent Decrease <br> Lesson 12: More Discounts <br> Lesson 13: What Is the Best Deal? <br> Lesson 14: Scale Factor—Percent Increase and Decrease <br> Topic D: Applications of Percent <br> Lesson 15: Tips and Taxes <br> Lesson 16: Markups and Discounts <br> Lesson 17: Simple Interest and Proportionality <br> Lesson 18: Simple Interest—Solving for Unknown Values <br> Lesson 19: Applying Percent Error <br> Topic E: Problems Involving Percent <br> Lesson 20: Making Money, Day 1 <br> Lesson 21: Making Money, Day 2 |
| :---: | :---: | :---: |




| Algebraic Reasoning II (cont.) |  | Lesson 22: Solving Problems Involving Inequalities Lesson 23: Inequalities vs. Equations |  |
| :---: | :---: | :---: | :---: |
| Probability | 7.SP.C. 5 | Module 6: Probability and Populations Topic A: Calculating and Interpreting Probabilities Lesson 1: What is Probability? | 10 days |
|  | 7.SP.C. 6 | Module 6: Probability and Populations <br> Topic A: Calculating and Interpreting Probabilities <br> Lesson 2: Empirical Probability <br> Lesson 3: Outcomes of Chance Experiments <br> Lesson 6: Outcomes That Are Not Equally Likely <br> Module 6: Probability and Populations <br> Topic B: Estimating Probabilities <br> Lesson 7: The Law of Large Numbers <br> Lesson 8: Picking Blue |  |
|  | 7.SP.C. 7 | Module 6: Probability and Populations <br> Topic A: Calculating and Interpreting Probabilities <br> Lesson 4: Theoretical Probability <br> Lesson 5: Multistage Experiments <br> Module 6: Probability and Populations <br> Topic B: Estimating Probabilities <br> Lesson 7: The Law of Large Numbers <br> Lesson 8: Picking Blue |  |
|  | 7.SP.C. 8 | Module 6: Probability and Populations <br> Topic A: Calculating and Interpreting Probabilities Lesson 5: Topic A Lesson 5: Multistage Experiments <br> Module 6: Probability and Populations <br> Topic B: Estimating Probabilities <br> Lesson 7: The Law of Large Numbers |  |


| Probability (cont.) |  | Lesson 8: Picking Blue <br> Lesson 9: Probability Simulations <br> Lesson 10: Simulations with Random Number Tables |  |
| :---: | :---: | :---: | :---: |
| Inferences and Populations | 7.SP.A. 1 | Module 6: Probability and Populations <br> Topic C: Random Sampling <br> Lesson 11: Populations and Samples <br> Lesson 12: Selecting a Sample <br> Lesson 13: Variability Between Samples <br> Lesson 14: Sampling Variability When Estimating a Population Mean | 9 days |
|  | 7.SP.A. 2 | Module 6: Probability and Populations <br> Topic C: Random Sampling <br> Lesson 13: Variability Between Samples <br> Lesson 14: Sampling Variability When Estimating a Population Mean <br> Lesson 15: Sampling Variability and the Effect of Sample Size <br> Lesson 16: Sampling Variability When Estimating a Population Proportion |  |
|  | 7.SP.B. 3 | Module 6: Probability and Populations <br> Topic D: Comparing Populations <br> Lesson 17: Comparing Sample Means <br> Lesson 18: Comparing Population Means <br> Lesson 19: Memory Games |  |
|  | 7.SP.B. 4 | Module 6: Probability and Populations <br> Topic D: Comparing Populations <br> Lesson 17: Comparing Sample Means <br> Lesson 18: Comparing Population Means <br> Lesson 19: Memory Games |  |

Level 7: Ratios and Proportionality

## If a district uses this resource to implement the state model curriculum for grade 7 , the following scope and sequence should be followed to ensure alignment and attention to the progressions of mathematics.

| Module 1 <br> Ratios and Proportional Relationships | Module 2 <br> Operations with Rational Numbers | Module 3 <br> Expressions, Equations, and Inequalities | Module 4 <br> Geometry | Module 5 <br> Percent and Applications of Percent | Module 6 <br> Probability and Populations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Topic A: Understanding Proportional Relationships <br> Lesson 1: An Experiment with Ratios and Rates <br> - Compare different relationships in situations by using ratio and rate reasoning. <br> 7.RP.A.1, 7.RP.A.2.a, MP8, <br> 7.Mod1.AD1, 7.Mod1.AD2 <br> Lesson 2: Exploring Tables of Proportional Relationships <br> - Identify proportional relationships represented in tables by calculating constant unit rates. <br> 7.RP.A.1, 7.RP.A.2.a, 7.RP.A.2.c, MP2, 7.Mod1.AD1, 7.Mod1.AD2, 7.Mod1.AD4 <br> Lesson 3: Identifying Proportional Relationships in Tables <br> - Analyze tables to identify proportional relationships. <br> - Determine the unit rate associated with a ratio of fractions by evaluating a complex fraction. | Topic A: Adding Rational Numbers <br> Lesson 1: Combining Opposites <br> - Represent positive and negative numbers on a number line. <br> - Recognize that opposite integers sum to zero. <br> 7.NS.A.1.a, 7.NS.A.1.b, MP8, 7.Mod2.AD2, 7.Mod2.AD4 <br> Lesson 2: Adding Integers <br> - Write addition expressions involving integers. <br> - Add integers by using a model. <br> 7.NS.A.1.b, MP8, 7.Mod2.AD3 <br> Lesson 3: Adding Integers <br> Efficiently <br> - Describe a number and its opposite as additive inverses because they sum to zero. <br> - Evaluate addition expressions with two or more addends. <br> 7.NS.A.1.b, MP8, 7.Mod2.AD3, <br> 7.Mod2.AD4, 7.Mod2.AD5 | Topic A: Equivalent Expressions <br> Lesson 1: Equivalent Expressions <br> - Generate equivalent expressions by using properties of operations. <br> 7.EE.A.1, MP3, 7.Mod3.AD1 <br> Lesson 2: The Distributive Property and the Tabular Model <br> - Generate equivalent expressions containing rational numbers by using the tabular model to represent the distributive property. <br> 7.EE.A.1, 7.EE.A.2, MP3, <br> 7.Mod3.AD1, 7.Mod3.AD2 <br> Lesson 3: The Distributive Property and Combining Like Terms <br> - Generate equivalent expressions by applying the distributive property to combine like terms. <br> 7.EE.A.1, MP6, 7.Mod3.AD1 | Topic A: Constructing Geometric Figures <br> Lesson 1: Sketching, Drawing, and Constructing Geometric Figures <br> - Construct geometric figures with given conditions. <br> - Construct geometric figures by using technology. <br> 7.G.A.2, MP5, 7.Mod4.AD1 <br> Lesson 2: Constructing <br> Parallelograms and Other <br> Quadrilaterals <br> - Construct parallelograms and other quadrilaterals, given conditions. <br> 7.G.A.2, MP6, 7.Mod4.AD1 <br> Lesson 3: Side Lengths of a <br> Triangle <br> - Determine whether a triangle with three given side lengths exists. <br> - Determine the relationship between the sum of two side lengths of a triangle and its third side length. <br> 7.G.A.2, MP2, 7.Mod4.AD1, <br> 7.Mod4.AD2 | Topic A: Proportion and Percent <br> Lesson 1: Proportionality and Scale Factor <br> - Identify the scale factor of cross sections. <br> 7.G.A.1, 7.RP.A.2.c, MP8, 7.Mod5.AD2, 7.Mod5.AD7 <br> Lesson 2: Racing of Percents <br> - Identify proportional relationships and write the constant of proportionality as a percent. <br> - Identify percent as a rate per . <br> 7.RP, 7.RP.A.3, MP7, 7.Mod5.AD1, 7.Mod5.AD3 <br> Lesson 3: Percent as a Rate per <br> - Interpret percent as a rate per when solving percent problems. <br> 7.RP.A.3, MP5, 7.Mod5.AD3 | Topic A: Calculating and Interpreting Probabilities <br> Lesson 1: What Is Probability? <br> - Find a number between and that represents the likelihood that an event will occur. <br> 7.SP.C.5, MP2, 7.Mod6.AD5 <br> Lesson 2: Empirical Probability <br> - Calculate empirical probabilities by collecting data from a chance experiment. <br> 7.SP.C.6, MP6, 7.Mod6.AD6 <br> Lesson 3: Outcomes of Chance <br> Experiments <br> - Determine the sample space for chance experiments. <br> - Given a description of a chance experiment and an event, determine for which outcomes in the sample space the event will occur. <br> 7.SP.C.6, MP2, 7.Mod6.AD6 <br> Lesson 4: Theoretical Probability <br> - Calculate theoretical probabilities of events for chance experiments that have equally likely outcomes. <br> 7.SP.C.7.a, MP6, 7.Mod6.AD8 |

7.RP.A.1, 7.RP.A.2.a, 7.RP.A.2.c, MP8, 7.Mod1.AD1, 7.Mod1.AD2, 7.Mod1.AD4

Lesson 4: Exploring Graphs of
Proportional Relationships

- Identify proportional relationships represented as graphs.
- Interpret and makes sense of the - interpret and mat.
7.RP.A.2.a, 7.RP.A.2.b,
7.RP.A.2.d, MP8, 7.Mod1.AD2, 7.Mod1.AD3, 7.Mod1.AD5

Lesson 5: Analyzing Graphs of Proportional Relationships

- Analyze graphs or sets of ratios to determine whether they represent proportional relationships.
- Identify the point on a graph that best shows the constant of proportionality and explain the meaning of the point in context.
7.RP.A.2.a, 7.RP.A.2.b,
7.RP.A.2.d, MP2, 7.Mod1.AD2,
7.Mod1.AD3, 7.Mod1.AD5

Lesson 6: Identifying Proportional Relationships in Written Descriptions

- Determine whether a written description represents a proportional relationship.
7.RP.A.2.a, 7.RP.A.2.b, MP2,
7.Mod1.AD2, 7.Mod1.AD3

Topic B: Working with Proportional Relationships

## Lesson 7: Handstand Sprint

- Model a situation by using a
proportional relationship to solve a problem.
7.RP.A.3, MP4, MP5, 7.Mod1.AD6

Lesson 4: KAKOOMA ${ }^{\oplus}$

- Add integers to solve and create - Add inte
7.NS.A.1.d, MP1, 7.Mod2.AD8

Lesson 5: Decomposing Rational Numbers to Make Addition More Efficient

- Add rational numbers by decomposing them.
7.NS.A.1.b, 7.NS.A.1.d, MP3 7.Mod2.AD3, 7.Mod2.AD8


## Lesson 6: Adding Rational

Numbers

- Fluently add rational numbers. 7.NS.A.1.b, 7.NS.A.1.d, MP5 7.Mod2.AD3, 7.Mod2.AD8

Topic B: Subtracting Rational Numbers

Lesson 7: What Subtraction Means

- Show that the distance between two integers on the number line is the - Evaluate integer subtraction expressions by finding the unknown expression
addends.
7.NS.A.1.c, MP7, 7.Mod2.AD7

Lesson 8: Subtracting Integers, Part 1

- Use expressions, number lines, and patterns to model contextual problems involving subtraction
- Write subtraction expressions as equivalent addition expressions. 7.NS.A.1.b, 7.NS.A.1.c, MP2, 7.Mod2.AD5, 7.Mod2.AD6

Lesson 4: Adding and Subtracting Expressions

- Generate equivalent expressions by using properties of operations to add nd subtract expressions.
7.EE.A.1, 7.EE.A.2, MP7,
7.Mod3.AD1, 7.Mod3.AD2

Lesson 5: Factoring Expressions

- Generate equivalent expressions by using the distributive property to factor.
7.EE.A.1, 7.EE.A.2, MP2,
7.Mod3.AD1, 7.Mod3.AD2

Lesson 6: Comparing Expressions - Use properties of operations to determine whether expressions are equivalent.
7.EE.A.1, 7.EE.A.2, MP7,
7.Mod3.AD1, 7.Mod3.AD2

Topic B: Unknown Angle Measurements

Lesson 7: Angle Relationships and Unknown Angle Measures

- Identify and describe angle
relationships given in diagrams.
- Write and solve equations that use angle relationships to find unknown angle measures
7.G.B.5, 7.EE.B.4.a, MP5,
7.Mod3.AD8, 7.Mod3.AD12

Lesson 8: Strategies to Determine Unknown Angle Measures

- Identify and describe angle relationships given in diagrams. - Write and solve two-step equation - Write and solve two-step equation unknown angle measures. 7.G.B.5, 7.EE.B.4.a, MP6 7.Mod3.AD8, 7.Mod3.AD12

Lesson 4: Angles of a Triangle - Determine whether a triangle can be formed with two given angle

## measures

## 7.G.A.2, MP3, 7.Mod4.AD

## 7.Mod4.AD2

## Lesson 5: Constructing

Quadrilaterals and Triangles

- Construct quadrilaterals given four side lengths and determine whether a side lengths and determine whe
unique quadrilateral is formed.
- Construct triangles given three side

Construct triangles given three side unique triangle is formed.
7.G.A.2, MP8, 7.Mod4.AD1,

## 7.Mod4.AD2

Topic B: Constructing Triangles

Lesson 6: Unique Triangles - Determine that at least three conditions are needed to guarantee a unique triangle.

- Determine that three angle measures alone do not guarantee a unique triangle.
7.G.A.2 ,MP3, 7.Mod4.AD1
7.Mod4.AD2

Lesson 7: Two Angles and One Side

- Determine whether two angle measures and an included side length - guarantee a unique triangle.

Determine whether two angle measures and a non-included side lengh guarantee a unique trin .G.A 2, MP3, 7.Mod4.AD1 7.Mod4.AD2

Lesson 4: Proportion and Percent - Solve percent problems by using equations in the forms $y=k x$ and $\stackrel{a}{a}=\underline{c}$.
7.RP.A.2.c, 7.RP.A.3, MP3 7.Mod5.AD2, 7.Mod5.AD3

Lesson 5: Common Denominators or Common Numerators

- Solve percent problems by using strategies that involve finding common denominators or common
numerators to solve proportions.
7.RP.A.2.c, 7.RP.A.3, MP5,
7.Mod5.AD2, 7.Mod5.AD3


## Topic B: Part of

Lesson 6: Finding Commission - Apply percents in the real-world context of commission. 7.RP.A.3, MP1, 7.Mod5.AD3,

## 7.Mod5.AD4

Lesson 7: Finding Discounts

- Apply percents in the real-world context of discounts
7.RP, 7.RP.A.3, MP1, 7.Mod5.AD1, 7.Mod5.AD3, 7.Mod5.AD4

Lesson 8: Determining Fees

- Apply percents in the real-world
- Apply percents
7.RP.A.3, MP3, 7.Mod5.AD3, 7.Mod5.AD4

Lesson 9: Tax as a Fee

- Apply percents in the real-world context of taxes.
7.RP.A.3, MP1, 7.Mod5.AD3, 7.Mod5.AD4

Lesson 5: Multistage Experiments - Use tree diagrams to organize and represent the outcomes in the sample space of a multistage experiment
7.SP.C.8.a, 7.SP.C.8.b, MP7
7.Mod6.AD10

Lesson 6: Outcomes That Are Not Equally Likely

- Calculate probabilities of events for chance experiments that do not have equally likely outcomes. 7.SP.C.6, MP7, 7.Mod6.AD6

Topic B: Estimating
Probabilities
Lesson 7: The Law of Large

## Numbers

- Use empirical probability to estimate theoretical probability
- Compare probabilities from a theoretica model to observed relative frequencies 7.SP.C.7, 7.SP.C.7.a, 7.SP.C.7.b, MP8, 7.Mod6.AD7, 7.Mod6.AD8, 7.Mod6.AD9

Lesson 8: Picking Blue

- Use empirical probabilities to create a probability model.
7.SP.C.6, 7.SP.C.7.b, MP2
7.Mod6.AD6, 7.Mod6.AD9

Lesson 9: Probability Simulations - Use a simulation to generate empirical probabilities for events. 7.SP.C.8.c, MP1, 7.Mod6.AD11

Lesson 10: Simulations with Random Number Tables - Conduct simulations with a random number table.
7.SP.C.8.c, MP5, 7.Mod6.AD11

## Lesson 8: Relating Representation

 of Proportional Relationships - Relate information among tables, graphs, equations, and situations to Id play a proportonal dontio proportionality in different representations of a proportional relationship
## 7.RP.A.2.b, 7.RP.A.2.c, MP7

## 7.Mod1.AD3, 7.Mod1.AD4

Lesson 9: Comparing Proportiona Relationships

- Explain how to use the point $(1, r)$ to find the unit rate of a proportional relationship.
- Relate the unit rate to the steepness of the line representing the proportional relationship by using the unit rate triangle with vertices $(0,0)$ $(1,0)$, and ( $1, r$ )
7.RP.A.2.b, 7.RP.A.2.d, MP7
7.Mod1.AD3, 7.Mod1.AD5


## Lesson 10: Applying Proportional

 Reasoning- Represent proportional relationships as equations
- Solve problems by applying
proportional reasoning
7.RP.A.2.c, 7.RP.A.3, MP2,
7.Mod1.AD4, 7.Mod1.AD6

Lesson 11: Constant Rates

- Represent rate problems as proportional relationships with equations.
- Solve rate problems.
7.RP.A.2.b, 7.RP.A.2.c, 7.RP.A.3 MP1, 7.Mod1.AD3, 7.Mod1.AD4, 7.Mod1.AD6

Lesson 9: Subtracting Integers, Part 2

- Express subtraction of a number as addition of its
- Subtract integers by using equivalent addition expressions.
7.NS.A.1.c, 7.NS.A.1.d, MP8


## 7.Mod2.AD6, 7.Mod2.AD8

## Lesson 10: Subtracting Rationa

 Numbers, Part- Evaluate expressions involving subtraction of rational numbers.
- Use properties of operations to make a simpler expression.
7.NS.A.1.c, 7.NS.A.1.d, MP7, 7.Mod2.AD6, 7.Mod2.AD8

Lesson 11: Subtracting Rational Numbers, Part 2

- Subtract rational numbers by writing equivalent addition expressions and evaluating them.
- Use properties of operations to make a simpler expression.
7.NS.A.1.c, 7.NS.A.1.d, MP1, 7.Mod2.AD6, 7.Mod2.AD8

Lesson 12: The Integer Gam - Apply strategies of integer addition and subtraction.

- Recognize when opposites combine to make zero.
7.NS.A.1.a, 7.NS.A.1.d, MP6 7.Mod2.AD2. 7.Mod2.AD8

Topic C: Multiplying Rational Numbers

Lesson 13: Understanding Multiples of Negative Numbers

- Interpret multiplication as repeated addition by using the distributive property.

Lesson 9: Solving Equations to Determine Unknown Angle

## Measures

- Identify and describe angle
relationships given in diagrams.
- Write and solve two-step equations that use angle relationships to find unknown angle measures.
7.Mod3.AD2, 7.Mod3.AD3

Lesson 10: Problem Solving with Unknown Angle Measures

- Solve multi-step problems to determine unknown angle measures by using all known angle relationships.
7.EE.B.3, 7.G.B.5, MP1,
7.Mod3.AD3, 7.Mod3.AD12


## Topic C: Solving Equations

## Lesson 11: Dominoes and

Dominoes

- Compare different ways of solving a problem.
- Use equations as mathematical models to estimate the number of models to estimate


## 7.EE.B.3, 7.EE.B.4, MP1, MP4

7.Mod3.AD3, 7.Mod3.AD4, 7.Mod3.AD5

Lesson 12: Solving Problem Algebraically and Arithmetically - Use if-then moves to solve word problems leading to equations of the forms $p x+q=r$ and $p(x+q)=r$, where $p, q$, and $r$ are specific rational numbers.
7.EE.B.4, 7.EE.B.4.a, MP2, 7.Mod3.AD5, 7.Mod3.AD7, 7.Mod3.AD8

Lesson 8: Two Sides and One Angle

- Determine whether two side lengths and an included angle measure guarantee a unique triangle.
Determine whether two side lengths and a non-included angle measure guarantee a unique triangle.
7.G.A.2, MP8, 7.Mod4.AD1
7.Mod4.AD2

Topic C: Circumference and Area of Circles

Lesson 9: Constructing a Circle

- Define and construct circles given radius or a diameter


## 7.G.A.2, MP6, 7.Mod4.AD

Lesson 10: The Outside of a Circle - Describe the relationship between the circumference and diameter of any circle as a proportional relationship. - Find the approximate circumference of a circle by using the value as the constant of proportionality.
7.G.B.4, MP8, 7.Mod4.AD4

Lesson 11: The Inside of a Circle - Estimate the area of a circle. 7.G.B.4, MP7, 7.Mod4.AD4

Lesson 12: Exploring the Area and Circumference of a Circle

- Model and describe the relationship between the circumference and the area of a circle.
7.G.B.4, MP7, 7.Mod4.AD4,
7.Mod4.AD5

Topic C: More or Less Than

Lesson 10: Percent Increase

- Solve percent problems in a real-
world context that involves percent increase.
7.RP.A.3, 7.EE.A.2, MP2
7.Mod5.AD4, 7.Mod5.AD5 7.Mod5.AD6


## Lesson 11: Percent Decrease

- Solve percent problems in a real world context that involves percent decrease.
7.Mod5.AD4, 7.Mod5.AD5 7.Mod5.AD6

Lesson 12: More Discounts

- Use equations to solve percent
problems that involve the real-world
context of discounts.
7.RP.A.3, 7.EE.A.2, MP6,
7.Mod5.AD4, 7.Mod5.AD5
7.Mod5.AD6

Lesson 13: What Is the Best Deal?

- Use equations to calculate multiple discounts and discounted prices. 7.RP.A.3, MP1, MP2, 7.Mod5.AD4

Lesson 14: Scale Factor-Percent Increase and Decrease

- Apply scale factor expressed as a
percent, a percent decrease, or a percent increase.
- Construct a scale drawing by using a scale factor given as a percent, a percent decrease, or a percent increase.
7.RP, 7.EE.A.2, 7.G.A.1, MP1,
7.Mod5.AD1, 7.Mod5.AD6
7.Mod5.AD7

Topic C: Random Sampling
Lesson 11: Populations and Samples

- Distinguish populations and their characteristics from samples and their statistics.
7.SP.A.1, MP6, 7.Mod6.AD1

Lesson 12: Selecting a Sample

- Take a random sample from a population.
- Describe the importance of a random sample in drawing conclusions about a population.
7.SP.A.1, MP2, 7.Mod6.AD1

Lesson 13: Variability Between Samples

- Observe the variability between different random samples taken from the same population.
7.SP.A.1,7.SP.A.2, MP6,
7.Mod6.AD1, 7.Mod6.AD2

Lesson 14: Sampling Variability When Estimating a Population Mean

- Describe sampling variability in the context of estimating a population mean.
- Use data from a random sample to estimate a population mean.
7.SP.A.1, 7.SP.A.2, MP2,
7.Mod6.AD1, 7.Mod6.AD2

Lesson 15: Sampling Variability and the Effect of Sample Size

- Observe that increasing the sample size decreases the sampling variability of the sample mean.
7.SP.A.2, MP1, 7.Mod6.AD2


## Lesson 12: Multi-Step Ratio

Problems, Part 1

- Solve multi-step ratio problems by using proportional reasoning
7.RP.A.2.b, 7.RP.A.2.c, 7.RP.A.3

MP7, 7.Mod1.AD3, 7.Mod1.AD4,
7.Mod1.AD6

Lesson 13: Multi-Step Ratio
Problems, Part 2

- Solve multi-step ratio problems by using proportional reasoning.
7.RP.A.2.b, 7.RP.A.2.c, 7.RP.A.3, MP5, 7.Mod1.AD3, 7.Mod1.AD4, 7.Mod1.AD6

Topic C: Scale Drawings and Proportional Relationships

Lesson 14: Extreme Bicycles

- Compare objects of different sizes by using proportional reasoning.


## 7.RP.A.2.a, MP1, MP5,

## 7.Mod1.AD2

## Lesson 15: Scale Drawing

- Determine one-to-one correspondence of points in related figures
- Recognize that corresponding lengths in scale drawings are in proportional relationship with a constant of proportionality called a scale factor.


## 7.GA.1, MP7, 7.Mod1.AD7

- Informally verify that multiplying two numbers with opposite signs results in a negative product.
7.NS.A.2.a, 7.NS.A.2.c, MP2 7.Mod2.AD9, 7.Mod2.AD12

Lesson 14: Understanding the Product of Two Negative Numbers - Informally verify that multiplying two numbers with the same sign results in a positive product.

- Predict the sign of a product with multiple factors
7.NS.A.2.a, 7.NS.A.2.c, MP3, 7.Mod2.AD9, 7.Mod2.AD11, 7.Mod2.AD12


## Lesson 15: Multiplying Rationa

 Numbers- Extend knowledge of multiplying integers to multiply rational numbers. 7.NS.A.2.a, 7.NS.A.2.c, MP7 7.Mod2.AD9, 7.Mod2.AD12

Lesson 16: Exponential Expressions with Rational Numbers

- Extend knowledge of multiplying integers to multiply rational numbers in all forms.
- Evaluate exponential expressions containing rational bases.
7.NS.A.2.a, 7.NS.A.2.c, MP6, 7.Mod2.AD9, 7.Mod2.AD12

Topic D: Dividing Rationa Numbers

Lesson 17: Understanding Negative Dividends

- Model division and recognize limitations of the models whe limitations of models when 7.NS.A.2.c, MP7,

Lesson 13: Solving EquationsPuzzles
Use if-then moves to solve equations of the forms $p x+q=r$ and $p(x+$ $q)=r$, where $p, q$, and $r$ are specific rational numbers.

## 7.EEB4, 7.EEB.4a, MP7

7.Mod3.AD5, 7.Mod3.AD7

Lesson 14: Solving Equations-
Scavenger Hunt

- Solve equations of the forms $p x+q=$ $r$ and $p(x+q)=r$, where $p, q$ and $r$ are specific rational numbers.
7.EE.B.4.a, MP7, 7.Mod3.AD7

Lesson 15: Solving Equations Fluently

- Fluently solve equations of the forms $p x+q=r$ and $p(x+q)=r$, where $p, q$, and $r$ are specific rational numbers


## 7.EE.B.4.a, MP1, 7.Mod3.AD7

Lesson 16: Using Equations to Solve Rate Problems

- Create and solve word problems containing rates by using equations of the forms $p x+q=r$ and $p(x+q)=$ $r$, where $p, q$, and $r$ are specific rational numbers.
7.EE.B.3, 7.EE.B.4, 7.EE.B.4.a, MP2, 7.Mod3.AD3, 7.Mod3.AD5, 7.Mod3.AD8

Lesson 17: Using Equations to Solve Problems

- Write and solve equations in the form $\underline{a}=\frac{c}{d}$, where either $a, b, c$, or $d$ is unknown and the other three are specific rational numbers.


## 7.EE.B.3, 7.EE.B.4, MP7

7.Mod3.AD3, 7.Mod3.AD4, 7.Mod3.AD5

Lesson 13: Finding Areas o Circular Regions

- Solve problems by using the formula for the area of a circle.
- Model and describe the relationship between the areas of circles and the areas of semicircular and quartercircular regions.


## 7.G.B.4, MP7, 7.Mod4.AD4

7.Mod4.AD5

Lesson 14: Composite Figures with

## Circular Regions

- Solve problems involving area and
perimeter of composite figures
G.B.4 7 G B.6. MP7
7.Mod4.AD4, 7.Mod4.AD6

Lesson 15: Watering a Law - Model a situation by using
rectangular, circular, semicircular,
and quarter-circular regions and
calculate area to solve problems.
7.G.B.4, MP1, MP4, 7.Mod4.AD4

Topic D: Area and Surface Area

Lesson 16: Solving Area Problems by Composition and
Decomposition

- Calculate the area of composite
figures in real-world and
mathematical problems by using mathematical problems by usin 7.G.B.6, MP1, 7.Mod4.AD6

Lesson 17: Surface Area of Right Rectangular and Right Triangula Prisms

- Calculate the surface area of right rectangular and right triangular prisms. 7.G.B.6, MP6, 7.Mod4.AD7

Topic D: Applications of Percent

Lesson 15: Tips and Taxes

- Calculate percent increases such as tax and tip.
- Calculate the total from the subtotal, tax, and tip.
7.RP.A.3, 7.EE.A.2, MP7,
7.Mod5.AD4, 7.Mod5.AD5
7.Mod5.AD6

Lesson 16: Markups and Discount

- Determine retail prices by using
markups.
- Determine discounted prices by using discounts.
7.RP.A.3, 7.EE.A.2, MP7,
7.Mod5.AD4, 7.Mod5.AD5
7.Mod5.AD6

Lesson 17: Simple Interest and
Proportionality

- Calculate simple interest given principal, time, and interest rate. 7.RP.A.3, MP7, 7.Mod5.AD4

Lesson 18: Simple Interest-
Solving for Unknown Values

- Calculate simple interest, principal, time, and interest rate. 7.RP.A.3, MP8, 7.Mod5.AD4

Lesson 19: Applying Percent Error

- Use absolute error to define percent error.
- Apply percent error to real-world contexts.
7.RP.A.3, MP2, 7.Mod5.AD4

Lesson 16: Sampling Variability When Estimating a Population Proportion

- Observe that increasing the sample size decreases the sampling variability of the sample proportion
7.SP.A.2, MP6, 7.Mod6.AD2

Topic D: Comparing Populations

Lesson 17: Comparing Sample Means

- Determine whether there is convincing evidence to conclude that two population means differ based on sample estimates.
7.SP.B.3, 7.SP.B.4, MP3,
7.Mod6.AD3, 7.Mod6.AD4

Lesson 18: Comparing Population Means

- Express the difference in sample means as a multiple of a measure of variability.
7 SP B 3, 7 SP B.4, MP7
7.Mod6.AD3, 7.Mod6.AD4


## Lesson 19: Memory Game

- Make conclusions about a difference in population means by using sample means and mean absolute deviations 7.SP.B.3, 7.SP.B.4, MP4, 7.Mod6.AD3, 7.Mod6.AD4


## Lesson 16: Using a Scale Facto

- Determine whether a scale factor produces an enlargement or a reduction. - Create a scale drawing by using the proportional relationship that exists between corresponding distances.


## 7GA1 7RPA 2a, 7 RP A 2

 MP3, 7 Mod1AD3 7.Mod1.AD8Lesson 17: Finding Actual Distances from a Scale Drawing

- Find measurements of a figure when given a scale factor and either the scale drawing or the original figure 7.G.A.1, MP6, 7.Mod1.AD8

Lesson 18: Relating Areas of Scale Drawings

- Describe the area of a scale drawing with scale factor $r$ as $r^{2}$ times the area of the original figure.


## 7.G.A.1, 7.RP A.2.b, MP8

## 7.Mod1.AD3, 7.Mod1.AD8

Lesson 19: Scale and Scale Factors

- Describe the difference between a
scale and a scale factor.
- Find unknown measurements in scale drawings through the appropriate use of scales and scale factor


## 7.G.A.1, MP4, 7.Mod1.AD7,

## 7.Mod1.AD8

Lesson 20: Creating Multiple Scale Drawings

- Draw a scale drawing of another scale drawing by using a new scale factor. -Write an equation for the proportional relationship relating scale drawings that have different scale factors and use the equation to find unknown distances.
7.G.A.1, MP3, 7.Mod1.AD7,
7.Mod1.AD8


## Lesson

## Divisors

Write division expressions as unknown factor equations to determine the value of the quotient.
al numbers as quotients of integers.
7.NS.A.2.b, 7.NS.A.2.c, MP7

## 7.Mod2.AD10, 7.Mod2.AD12

## Lesson 19: Rational Numbers as

 Decimals, Part 1- Calculate quotients of integers where the divisor is a product of 's and/or's and express them as terminating decimals.
7.NS.A.2.d, MP8, 7.Mod2.AD13

Lesson 20: Rational Numbers as Decimals, Part 2

- Calculate quotients where the diviso contains factors other than and and express those quotients as repeating decimals.
Write rational numbers as either terminating decimals or repeating decimals.
7.NS.A.2.d, MP8, 7.Mod2.AD13,


## 7.Mod2.AD14

Lesson 21: Comparing and
Ordering Rational Numbers

- Compare and order rational numbers, including those written as repeating decimals.
7.NS.A.2.b, 7.NS.A.2.d, MP5, 7.Mod2.AD11, 7.Mod2.AD13 7.Mod2.AD14

Topic D: Inequalities
Lesson 18: Understanding Inequalities and Their Solutions

- Find solutions to inequalities by testing numbers and graphing them on a number line.
7.EE.B.4, 7.EE.B.4.b, MP6, 7.Mod3.AD6, 7.Mod3.AD10 7.Mod3.AD11


## Lesson 19: Using Equations to

 Solve Inequalities- Solve inequalities and graph their
solution sets on number lines.
- Describe similarities and differences between inequalities and equations. 7.EE.B.4, 7.EE.B.4.b, MP7, 7.Mod3.AD9, 7.Mod3.AD10,


## 7.Mod3.AD11

Lesson 20: Preserving and Reversing
Solve one-step inequalities and graph
their solution sets on number lines.

- Identify when to reverse the
inequality symbol in an inequality to produce an equivalent inequality 7.EE.B.4.b, MP8, 7.Mod3.AD9 7.Mod3.AD10,


## Lesson 21: Solving Two-Step

 Inequalities- Write and solve inequalities to represent context problems and identify restrictions to their solutio sets
7.EE.B.4, 7.EE.B.4.b, MP2, 7.Mod3.AD6, 7.Mod3.AD9 7.Mod3.AD11

Lesson 18: Surface Area of Right Prisms

- Calculate the surface area of right prisms by determining an efficient strategy for finding the sum of the ral faces and base


## 7.G.B.6, MP7, 7.Mod4.AD7

Lesson 19: Surface Area of
Cylinders (Optional)

- Calculate the surface area of right circular cylinders.


## MP8

Lesson 20: Surface Area of Righ Pyramids
Calculate the surface area of right pyramids.
7.G.B.6, MP6, 7.Mod4.AD7

Lesson 21: Surface Area of Othe Solids

- Calculate the surface area of solids composed of right prisms and righ pyramids.
7.G.B.6, MP6, 7.Mod4.AD7

Topic E: Cross Sections and Volume

Lesson 22: Understanding Planes and Cross Sections

- Sketch cross sections of right prisms and right pyramids cut by a plane parallel or perpendicular to the base. 7.G.A.3, MP7, 7.Mod4.AD3


## Lesson 23: Cross Section

Scavenger Hunt

- Explore cross sections formed when a right prism or a right pyramid is cut by a plane at an angle other than $90^{\circ}$ to the base.
7.G.A. 3 MP7, 7.Mod4.AD3

Topic E: Problems Involving Percent

Lesson 20: Making Money, Day 1

- Model and solve a real-world problem
involving percent.
7.RP.A.3, MP4, 7.Mod5.AD4

Lesson 21: Making Money, Day 2

- Model and solve a real-world problem involving percent.


## 7.RP.A.3, MP1, 7.Mod5.AD4

Lesson 22: Making Mixtures

- Develop and compare mixtures made from percents of two or more liquids 7.RP.A.3, MP7, 7.Mod5.AD4

Lesson 23: Percents of Percents

- Solve context problems involving percents related to a percent of the whole or unknown.
7.RP.A.3, 7.EE.A.2, MP2,
7.Mod5.AD4, 7.Mod5.AD6

Lesson 24: Counting Problems

- Solve counting problems related to computing percent. 7.RP, MP6, 7.Mod5.AD1

Lesson 22: Multiplication and Division Expressions

- Calculate quotients of rational numbers, including non-integer rational numbers. Write expressions with division as equivalent expressions with multiplication by using multiplicative inverses.


## NSA. 2c, MP7, 7 Mod2 AD12

## Topic E: Numerical

Expressions with Rational
Numbers

## Lesson 23: Properties of

Operations with Rational Numbers

- Evaluate expressions involving rational numbers by applying properties of operations.
7.NS.A, MP7, 7.Mod2.AD1


## Lesson 24: Order of Operations

## with Rational Numbers

- Evaluate expressions containing exponents.
Use the order of operations to evaluate numerical expressions
containing rational numbers.
7.NS.A, 7.NS.A.2.c, MP6
7.Mod2.AD1, 7.Mod2.AD12

Lesson 25: Writing and Evaluating Expressions with Rational
Numbers, Part
Write numerical expressions given mathematical and real-world contexts.
Evaluate expressions and interpret
their value in context.
7.NS.A.3, 7.EE.B.3, MP2,
7.Mod2.AD15

Lesson 22: Solving Problems Involving Inequalities
Write and solve inequalities comparing $p x+q$ and $r$, where $p, q$, and $r$ are specific rational nu wite and solve ins.
-reart contert po pentify restrictio to in r solution
7.EE.B.4, 7.EE.B.4.b, MP6, 7.Mod3.AD6, 7.Mod3.AD9 7.Mod3.AD11

## Lesson 23: Inequalities vs

 EquationsDetermine whether a situation should be modeled with an equation or with an inequality.

- Write a context that can be modeled by a given inequality.
EEB.4, 7EE B.4.b, MP2
7.Mod3.AD5, 7.Mod3.AD6,
7.Mod3.AD11

Lesson 24: Volume of Prisms - Determine a formula for finding the volume of any right prism.

## the volume of a right prism

 7.G.B.6, MP7, 7.Mod4.AD7
## Lesson 25: Volume of Composit

 Solids- Find the volume of composite solids. 7.G.B.6, MP7, 7.Mod4.AD7

Lesson 26: Designing a Fish Tank - Model real-world problems involving
surface area and volume.

|  | Lesson 26: Writing and Evaluating <br> Expressions with Rational <br> Numbers, Part 2 <br> - Write and evatuate numerical <br> expressions and interpret their value <br> in ontext. <br> 7.NS.A.3, 7.EE.B.3, MP4, <br> 7.Mod2.AD15 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

## Year-Long Curriculum Overview: Levels 6-8

Trimester and quarter indicators are provided as a guide for pacing. A few optional lessons in each grade level are included in total number of lessons. About thirty additional days are allotted at each level for assessment and responsive teaching.


## Supports of Diversity, Equity and Inclusion

## Providing Culturally Responsive Instruction

Eureka Math ${ }^{2}$ values the funds of knowledge that students bring into the classroom and acknowledges that deep learning happens when all students are able to leverage their diverse life experiences while learning mathematics.

One of the ways Eureka Math ${ }^{2}$ invites students into mathematics and celebrates the diversity present in every classroom is by highlighting for teachers those specific lesson moments that can be tailored to bring students' experiences from their home and communities into the classroom. For example, a strategically placed Universal Design for Learning (UDL) margin note in grade 7 module 5 lesson 15 highlights that providing students with a restaurant menu allows them to choose the meal they would like to order and promotes relevance because students can draw on their own experiences to understand the problem.

In grade 6 module 6 lesson 6, students work in pairs to choose a statistical question and decide how to display their data set. They create a poster and participate in a gallery walk to provide feedback to their peers. This UDL margin note suggests that teachers promote relevance by leveraging students' life experiences and allowing them to choose the context for the statistical question.

In grade 6 module 1 lesson 4, students complete a digital

## UDL: Engagement

Including a variety of data set contexts provides an opportunity for student choice. Allowing students to select a statistical Allowing students to select a statist challenging puts them in charge of their learning and promotes relevance. lesson in which they use tape diagrams to understand how to make specific color batches of paint. This content provides the teacher with the opportunity to connect to students' home lives and learn more about the experiences they've had with painting.

Students are diverse, and any one classroom can have students from either an individualist frame of reference or a collectivist frame of reference. The teacher-writers of Eureka Math ${ }^{2}$ considered both frames of reference in intentionally balancing activities that build off individualism as well as collectivism.


Example of placed UDL margin note

In her book Culturally Responsive Teaching and the Brain, Zaretta Hammond references collectivism as emphasizing relationships, interdependence within a community, and cooperative learning (page 25). In Eureka Math ${ }^{2}$, a collectivist approach to learning mathematics is present in the embedded cooperative learning structures in open-middle and open-ended tasks. Specifically, the instructional routines Numbered Heads and Co-construction are rooted in students working cooperatively in groups to deepen their mathematical conceptual understanding. See grade 6 module 6 lesson 16 for an example of how students use the Co-construction routine to write statistical questions.

Beyond the instructional routines, Eureka Math ${ }^{2}$ leverages the power of student relationships and interdependence through frequent partner and group work. For any partner or group work referenced in the instructional materials, teachers may make use of strategic, flexible groupings that build off students' strengths, including home language. A Language Support margin note in the first lesson of every module serves to remind teachers to leverage students' cultural perspectives when strategically placing students in partners.

Hammond references individualism as emphasizing individual achievement and independence (page 25). In Eureka Math ${ }^{2}$, an individualist approach to learning mathematics may be seen in the embedded systems for independent practice in every lesson, such as Exit Tickets and Practice Sets. Additionally, the instructional routines Critique a Flawed Response and Take a Stand both start with students working on a math problem individually before engaging in student discourse. See grade 7 module 1 lesson 4 for an example of students engaging in the Take a Stand routine to discuss whether the statement "Graphed lines represent proportional relationships" is always, sometimes, or never true.

Beyond balancing individualism and collectivism, Eureka Math ${ }^{2}$ activities and problems provide students with mirrors in which to see their own cultural perspectives reflected, as well as windows through which to view others' cultural perspectives.

Eureka Math ${ }^{2}$ is an inclusive mathematics curriculum that represents diverse doers of math. The curriculum's images, fine art, and pictures of people represent diversity through problems and exercises related to real-life experiences, perspectives, and contributions of people from various cultures, ethnicities, and gender identities. These representations affirm student identities while rejecting the stereotypes and biases that have excluded many students from mathematical learning in favor of a more robust and inclusive perspective. Representing a diverse array of doers of mathematics in the curriculum inspires all students to think of themselves as mathematicians.

For example, Eureka Math ${ }^{2}$ includes various mathematical activities that involve counting on hands or simulating a number line with one's fingers. In images throughout the curriculum, care was taken to include a variety of body types and skin tones.

The names used in word problems and for sample students in the lesson vignettes are intentionally diverse to represent the wide variety of students who use the curriculum. The names in student-facing word problems are also designed for readability to ensure that they are not a barrier to accessing the math.

Story of Ratios ${ }^{\text {® }}$ and Story of Functions ${ }^{\text {® }}$

Logan, Ava, Noor, Nora, So-chee, Tiah, Zara, Fin, Huan, Kadir, Lucas, Riku, Bahar, Maya, Ali, Haru, Pia, Yooni, Amir, Ji-ho, Kota, Marco, Preet, Theo

According to CAST, "individuals are engaged by information and activities that are relevant and valuable to their interests and goals." (UDL Guidelines, Engagement, Checkpoint 7.2) Eureka Math ${ }^{2}$ also leverages students' experiences, goals, and interests through Math Pasts (described below), art connections, and wordless context videos.

To honor the diverse contributions to the development of the field of mathematics, to build knowledge about our shared math history, and to empower every child to see themselves as able to do mathematics, nearly every module in Eureka Math ${ }^{2}$ includes a feature called Math Past. Each Math Past tells the history of some big ideas in the module, recounting the story of the mathematics through artifacts, discoveries, and other contributions from cultures around the world. Math Past also provides ideas about how to engage students in the history of mathematics. Math Past counters the traditional Eurocentric perspective and celebrates the many contributions of Black, Indigenous, and People of Color communities to the history of mathematics.
 students are highly engaged in studyin area by examining a sketch from the
Codex Vergara, a document written around 1540 CE to show the landholdings of families in Aztec villages. Students decode the symbols to determine the side lengths, perimeters, and areas
of the fields. Students then work in pairs to draw polygons composed of rectangles in the coordinate plane. Partners find more than one way to determine the area of various polygons shown in the grid. When students determine the area of a polygon by decomposing it into rectangles and apply their knowledge of the area of a rectangle, they look for and make use of structure, addressing a mathematical habit of mind. The Math Past Teacher Resource includes information about how the Aztecs recorded the perimeters and areas of fields. It also includes other ancient area problems for students to engage in and solve.

Later, in grade 8 module 5 , students learn to write and solve systems of equations for problems that use ancient Chinese numbers. The problems in the lessons were translated from one of the most influential mathematical texts of all time: Jiuzhang Suanshu, translated as Nine Chapters on the Mathematical Art. The chapter titled "Fangcheng," or "Rectangular Arrays," contains 18 problems that address issues of trade, crop yield, number of animals, and other situations that can be solved with systems of linear equations. When students use systems of equations to represent real-world contexts involving comparison of weights and the trading of livestock, they reason abstractly and quantitatively (another mathematical habit of mind). The Math Past Teacher Resource guides teachers to prompt students to explore answers to questions such as the following: Are the techniques used today for solving systems of linear equations like those used in the past? What are some alternative ways to represent a system of linear equations? Are there alternative ways to solve systems of linear equations?

In a similar vein, Eureka Math ${ }^{2}$ connects works of fine art to the standards of each grade level. Each Teach book opens with a stunning work of fine art that has a connection to the math learned in the grade. There is also a wide variety of additional pieces of art embedded in each grade's lessons. For example, in grade 8 module 3 lesson 13 (pages 248-270), students examine Composition 8, a piece by Wassily Kandinsky. Students hypothesize whether the figures in the painting are similar figures, and then use what they know about rigid motions and dilations to justify their thinking.

Wordless context-building videos highlight how we use math to solve problems in our everyday lives and make sense of the world around us. Eureka Math ${ }^{2}$ lessons include more than 190 videos. The curriculum offers three types of highly engaging, wordless math context videos: character animation, collage animation, and live action.

Students can identify with the diverse set of actors and characters in the videos, which helps them visualize how math is part of everyday life. Through these videos, students will more readily realize that math surrounds them and that they, too, can engage in mathematical pursuits. The videos allow students to see themselves in the math problems they encounter, which lowers the barrier to engagement and makes the math classroom a more welcoming place.

Wordless videos in lessons serve many other purposes as well, such as the following:

- They make the context for a given problem come alive, putting all students on the same footing by giving them the requisite background knowledge.
- They remove any language and reading barriers to the written word problem.
- They raise the accessibility of mathematics through accurate and inclusive representation.
- They show the many ways in which we interact with math in the world around us and how these interactions spark curiosity and joy.
- They help students see the delight and wonder associated with being a mathematician.
- They create excitement and buzz in the classroom about the content of the new word problem.
- They invite students to tell the story of the math problem, to notice, to wonder, and to drive the discussion.
- Examples include:
- Grade 6 module 1 lesson 3: Batches of Paint Part 1
- Grade 7 module 1 lesson 4: Bulk Almonds Part 1
- Grade 8 module 3 lesson 14: Mirror Height

Specific instructional prompts, engaging word problems, accessible and engaging tasks, art connections, Math Past connections, and context videos throughout Eureka Math ${ }^{2}$ work together to create a powerful curriculum that welcomes all students and invites them to become doers of mathematics.

## Addressing Learner Variance

To ensure success of all learners, every Eureka Math ${ }^{2}$ lesson includes Universal Design for Learning (UDL) strategies and scaffolds that address learner variance. These suggestions promote flexibility with engagement, representation, and action and expression, the three UDL principles described by CAST. These strategies and scaffolds are complements to the curriculum's overall alignment with the UDL Guidelines and were designed to support educators in effectively teaching students who experience difficulty in mathematics. The strategies are based on research specific to mathematics instruction.

According to Teaching Mathematics Meaningfully: Solutions for Reaching Struggling Learners, Second Edition, (page 71) "Students who have learning difficulties that affect their ability to do well in mathematics come from a variety of backgrounds and experiences. Although each of these students is individual and unique, students often demonstrate one or more of the nine learning characteristics..." The nine learning characteristics described include: learned helplessness, passive learning, knowledge and skills gaps, math anxiety, memory disabilities, attention disabilities, metacognitive thinking disabilities, processing disabilities, and reading disabilities. Some of these characteristics can affect all students who may be struggling in math regardless of whether they have learning-related disabilities (learned helplessness, passive learning, knowledge and skills gaps, math anxiety). Other characteristics result from learning-related disabilities (memory disabilities, attention disabilities, metacognitive thinking disabilities, processing disabilities, and reading disabilities). These learning characteristics as well as curriculum factors can result in common mathematics performance traits of students who struggle in mathematics.

According to Allsopp et. al (2018), "Mathematics visuals appear to be most effective when used in conjunction with other effective instructional practices. An example of this is the use of explicit instruction techniques in conjunction with visuals. (page 192)." "Explicit cueing techniques can be utilized with visuals in ways that help students attend to the visual's most important features and its representation of the mathematical idea. Simple techniques, such as color-coding, using

## UDL: Action \& Expression

Before beginning the Scavenger Hunt, prompt students to engage in strategic planning by asking partners to recall the types of equations they have been solving and the strategies they have used.

- How can we solve equations that contain both fractions and decimals?

What steps can we take to make the equation simpler before we use if-then moves?

## UDL: Representation

To help students identify different ways they To help students identify different ways they can group the treats, consider providing them nd candies. Making the activity a concrete ererience for students promotes conceptual understanding about the factors of each new ndersion xpression and what the factors represent in the situation.
directional arrows, and highlighting, can help students focus on what is most relevant." An example of this is found in grade 7 module 5 lesson 7 .

A variety of other strategies suggested in the literature are the foundation of all UDL margin notes found in Eureka Math ${ }^{2}$. Each margin note is aligned to a strategy found to minimize the impacts of one of the nine learning characteristics listed above. Strategies include, but are not limited to:

- Break down tasks into manageable chunks.
- Demonstrate the belief that students can be successful.
- Visually organize to cue student to important aspects of concept.
- Teach students to change their frame of thinking.
- Embed math in relevant contexts.
- Help students make connections to prior knowledge
- Engage students by addressing interests.
- Celebrate progress and success.
- Cultivate a growth mindset.
- Relate math to students' lives.
- Use concrete materials
- Associate content with meaningful context.
- Use a variety (visual, auditory, tactile or kinesthetic) of strategies.
- Provide visual organizers.
- Provide think alouds.

UDL: Representation
To activate students' prior knowledge about ratios, rates, and percents, consider using the following prompts:

How do you know whether a relationship is a ratio relationship?
What is the value of the ratio and how do you find it?

What is speed and how do you determine it?
What strategies can you use to determine the percent of a number?

In problem 3, consider drawing attention to the fact that the coefficient of $p$ in the expression 3.5 p is the value of the ratio.

- Use novel learning contexts.
- Help students focus on what is important rather than on things that are irrelevant.

Eureka Math ${ }^{2}$ embeds differentiation through the simple-to-complex sequencing of lesson and Practice problems. This logical sequence gradually reduces scaffolds and builds in complexity, allowing teachers to differentiate assignments for either individual or small-group work. For all students, including those working above grade level, the gradual reduction of support and increase in complexity builds independent thinking and encourages productive struggle. Problems toward the end of the Problem Set (a lesson's daily independent practice) are often open-ended, at Depth of Knowledge (DOK) levels 2 and 3, and integrate two or more standards and/or Standards for Mathematical Practice. Teachers can assign problems of different complexities to students according to their needs or allow students to select problems in the 10-minute (approximate) timeframe. Lessons provide differentiation suggestions at the point of instruction to support a wide variety of learners. Differentiation margin notes found in the Teach book offer guidance for adapting instruction so that all students can successfully access grade-level content. There are two types of Differentiation margin notes: Support and Challenge. Challenge boxes suggest ways to keep students working at a more advanced level engaged by providing opportunities for extension.

In this example from grade 8 module 6 lesson 2 the Differentiation margin note offers a suggestion for students to interact with the purpose of the Learn segment of determining whether tables represent functions at a deeper level of complexity by having students create their own tables and trade with a partner.

## Supporting Multilanguage Learners

Eureka Math ${ }^{2}$ writers relied on language development research to outline and build in the language support needed for multilanguage learners to engage with the language-rich lessons. With the goal of supporting the clear, concise, and precise use of reading, writing, speaking, and listening in English, Eureka Math ${ }^{2}$ supports multilanguage learners through each lesson's instructional design. It does this by including instructional best practices, support for mathematical discourse, and support for the different tiers of terminology. Additionally, Language Support margin notes provide just-intime, targeted instructional recommendations to support multilanguage learners.

## Instructional Best Practices

The following table outlines the instructional best practices included in Eureka Math².

| Practice | Eureka Math ${ }^{2}$ |
| :--- | :--- |
| Activate prior knowledge <br> (mathematics content, terminology, contexts) | The daily Fluency and Launch lesson components activate prior <br> knowledge to prepare students for new learning. Context videos <br> demonstrate math concepts in a concrete or real-world context. |
| Provide multiple entry points to the mathematics | Recurring Notice and Wonder routines and frequent open-middle and <br> open-ended tasks provide multiple points of entry for students to <br> participate. The inclusion of fine art and Math Past history components <br> engages students with math in the real world. |
| Use clear, concise student-facing language | Readability guidelines ensure that words are never an obstacle to math <br> learning. |
| Provide strategic active processing time | Frequent mathematical discourse, core instructional routines, and the <br> 10/2 principle expand topportunities for students to synthesize and <br> process new information. |
| Illustrate multiple modes and formats | Varied physical and visual models, such as digital interactives, context <br> videos, and graphic organizers, help students make connections and <br> deepen understanding. |
| Provide opportunities for strategic review | Daily fluency activities, distributed practice Remember problems, Exit <br> Tickets, and comprehensive assessments provide frequent <br> opportunities for strategic review. |

## Mathematical Discourse

To support all learners, lessons provide ample authentic and engaging opportunities for students to read, write, speak, and listen. Eureka Math ${ }^{2}$ supports teachers in creating language-rich classrooms by modeling teacher-student discourse and by providing suggestions for supported student-tostudent discourse. Because curricula in general have an abundance of receptive language experiences (reading and listening), Eureka Math ${ }^{2}$ focuses specific supports on language production (speaking and writing) in mathematics.

The instructional routines that promote discourse are aligned with Stanford's Language Design Principles of supporting sense-making, optimizing output, cultivating conversation, and maximizing linguistic and cognitive meta-awareness.

Eureka Math ${ }^{2}$ periodically includes Language Support notes that suggest specific sentence frames and sentence starters to support multilanguage learners in student-tostudent discussions, such as those used in instructional routines. General sentence frames and sentence starters are provided in the Talking Tool which is referenced often during times of student-to-student discourse.


## Terminology

Eureka Math ${ }^{2}$ lessons give students experience with a new mathematical concept before naming it with a precise mathematical term. Students may see a mathematical concept come to life in a digital interactive, manipulate counters in groups, or use an instructional routine to engage in mathematical discourse before the teacher gives that concept a name. In addition, teachers are provided with educative guidance, either in the body of the lesson or in a Language Support margin note, to support students in pairing the written term with a visual representation. Eureka Math ${ }^{2}$ highlights domain-specific terms from previous lessons in the current lesson, along with instructional recommendations for supporting those terms. These instructional recommendations focus on previewing the meaning of the terms before students are expected to interact with them in the mathematics of the lesson. Additionally, domain-specific terms from previous lessons are also supported by pairing the written term with a visual representation. For each grade, the
academic verbs needed to engage with the mathematics were considered. Each grade in Eureka Math ${ }^{2}$ offers a carefully curated list of targeted academic verbs that appear in the lessons for students to preview before they are expected to understand and use the language. For example, before students are asked to verify in grade 8 , lessons preview the meaning of the academic verb, supporting the meaning of the term in a class discussion emphasizing the use of synonyms of that verb.

Multiple-meaning terms encompass homophones like very and vary, and homographs, like scale and scale, (see image from grade 7 module 1 lesson 19) and other pronunciation-based challenges, like the difference between approximate (as an adjective, as in, "What is the approximate value?") and approximate (as a verb, as in, "Approximate the sum."). Lessons call out multiplemeaning terms that could affect multilanguage learners' understanding of the mathematics. Lessons also include Language Support notes to preview the meaning of the term in the lesson. These previews include pairing the term with a visual, with real items, or with a video to highlight the different meanings of the term and emphasize the specific meaning used in the lesson.

## Language Support Boxes

2. Consider the following figures, which all depict the werd scal. Which one best
relateses to the work ef the module? Whry What doess the figure how?
C.

A Language Support margin note appears in the first lesson of every module to prompt teachers to consider using strategic, flexible grouping in each activity of the module to support multilanguage learners. These grouping suggestions invite teachers to use students' knowledge and home language by pairing students in different ways. Each of these different ways of pairing students has specific benefits for multilanguage learners. The Language Support margin notes highlight either discourse, language or terminology supports.

To learn more, please visit the Great Minds MLL blog: https://gm.greatminds.org/how-to-support-multilingual-learners-in-engaging-in-math-conversations-in-the-classroom

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